Intuitionistic Fuzzy sg-open and sg-closed Mappings

Jyoti Pandey Bajpai
Department of Applied Mathematics
Jabalpur Engineering College
Jabalpur (M.P.) 482011

S.S. Thakur
Department of Applied Mathematics
Jabalpur Engineering College
Jabalpur (M.P.) 482011

Abstract - The aim of this paper is to extend the concepts of sg-open and sg-closed mappings in intuitionistic fuzzy topological spaces and obtain some of its basic properties.

Keywords – Intuitionistic fuzzy sg-closed sets and Intuitionistic fuzzy sg-open sets, Intuitionistic fuzzy sg- continuous mappings , Intuitionistic fuzzy sg-open mapping and intuitionistic fuzzy sg-closed mapping, 2000, Mathematics Subject Classification: 54A99, 03E99.

I. INTRODUCTION

Mapping plays an important role in study of modern mathematics, especially in Topology and Functional Analysis. Closed and open mapping are one such mapping which are studied for different type of closed sets by various mathematicians for the past many years. . The concept of intuitionistic fuzzy sets was introduced by Atanassov [1] as a generalization of fuzzy sets. In 1997 Coker [5] introduced the concept of intuitionistic fuzzy topological spaces. In 2008, Thakur and Chaturvedi [12] introduced the notion of intuitionistic fuzzy generalized closed set in intuitionistic fuzzy topological space. Since then different mathematicians worked and studied in different forms of intuitionistic fuzzy g-closed set and related topological properties. The authors of the paper introduced the concepts of intuitionistic fuzzy sg-closed sets [7] , intuitionistic fuzzy sg-continuous mappings[15], intuitionistic fuzzy sg-irresolute mappings[16], intuitionistic fuzzy rw-closed sets[18], intuitionistic fuzzy w-closed sets[17], intuitionistic fuzzy rgα-closed sets[19], intuitionistic fuzzy rgα continuity [8] , intuitionistic fuzzy gpr-closed sets[20], intuitionistic fuzzy gpr open and gpr-closed mappings [9] and intuitionistic fuzzy g-open and g-closed mappings [14] in intuitionistic fuzzy topology. In the present paper we introduce weak form of open and closed mapping namely intuitionistic fuzzy sg-open mappings and intuitionistic fuzzy sg-closed mappings using intuitionistic fuzzy sg-closed set and obtain some of their characterization and properties.

II. PRELIMINARIES

Definition 2.1: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space $(X,\Im)$ is called:

(a) Intuitionistic fuzzy g-closed if cl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy open.[12]
(b) Intuitionistic fuzzy g-open if its complement A^c is intuitionistic fuzzy g-closed.[12]
(c) Intuitionistic fuzzy sg-closed if cl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy regular open.[7]
(d) Intuitionistic fuzzy sg-open if its complement A^c is intuitionistic fuzzy rg-closed.[7]
(e) Intuitionistic fuzzy w-closed if cl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy semi open.[17]
(f) Intuitionistic fuzzy w -open if its complement A^c is intuitionistic fuzzy w-closed.[17]
(g) Intuitionistic fuzzy rw-closed if cl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy regular semi open.[18]

(h) Intuitionistic fuzzy rw -open if its complement A^c is intuitionistic fuzzy rw-closed.[18]

(i) Intuitionistic fuzzy gpr-closed if pcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy regular open.[20]

(j) Intuitionistic fuzzy gpr -open if its complement A^c is intuitionistic fuzzy gpr-closed.[20]

(k) Intuitionistic fuzzy rgα -closed if αcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy regular alpha open.[19]

(l) Intuitionistic fuzzy rgα-open if its complement A^c is intuitionistic fuzzy rgα-closed.[19]

Definition 2.2 [7] The sg- interior and sg- closure of an intuitionistic fuzzy set A of a intuitionistic fuzzy topological space (X,$\Im$) respectively denoted by sgint(A) and sgcl(A) are defined as follows:

$$\text{sgint}(A) = \cup \{ V : V \subseteq A , V \text{ is intuitionistic fuzzy sg- open}\}$$

$$\text{sgcl}(A) = \cap \{ F: A \subseteq F, F \text{ is intuitionistic fuzzy sg- closed} \}$$

Definition 2.3 [6]: Let (X, $\Im$) and (Y, σ) be two intuitionistic fuzzy topological spaces and let f : X→Y be a function. Then f is said to be

(a) Intuitionistic fuzzy continuous if the pre image of each intuitionistic fuzzy open set of Y is an intuitionistic fuzzy open set in X.

(b) Intuitionistic fuzzy closed if the image of each intuitionisic fuzzy closed set in X is an intuitionistic fuzzy closed set in Y.

(c) Intuitionistic fuzzy open if the image of each intuitionisic fuzzy open set in X is an intuitionistic fuzzy open set in Y.

Definition 2.4: Let (X,$\Im$) and (Y, σ) be two intuitionistic fuzzy topological spaces and let f: X→Y be a function. Then *f* is said to be

(a) Intutionistic fuzzy g-continuous if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy g –closed in X.[13]

(b) Intuitionistic fuzzy g-open if image of every open set of X is intuitionistic fuzzy g-open in Y.[14]

(c) Intuitionistic fuzzy g-closed if image of every closed set of X is intuitionistic fuzzy g-closed in Y.[14]

(d) Intutionistic fuzzy sg-continuous if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy sg –closed in X.[15]

(e) Intutionistic fuzzy sg- irresolute if the pre image of every intuitionistic fuzzy sg- closed set in Y is intuitionistic fuzzy sg –closed in X.[16]

(f) Intutionistic fuzzy w-continuous if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy w –closed in X.[17]

(g) Intuitionistic fuzzy w-open if image of every open set of X is intuitionistic fuzzy w-open in Y.[17]

(h) Intuitionistic fuzzy w-closed if image of every closed set of X is intuitionistic fuzzy w-closed in Y.[17]

(i) Intutionistic fuzzy gpr-continuous if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy gpr –closed in X.[20]

(j) Intuitionistic fuzzy gpr-open if image of every open set of X is intuitionistic fuzzy gpr-open in Y.[9]

(k) Intuitionistic fuzzy gpr-closed if image of every closed set of X is intuitionistic fuzzy gpr-closed in Y.[9]

III. INTUITIONISTIC FUZZY SG- OPEN MAPPING

Definition 3.1: A mapping f : (X, $\Im$) →(Y, σ) is intuitionistic fuzzy sg-open if image of every intuitionistic fuzzy open set of X is intuitionistic fuzzy sg-open set in Y.

Remark 3.1 **:** Every intuitionistic fuzzy open mapping is intuitionistic sg-open but converse may not be true. For,

*Example 3.1***:** Let X = {a ,b} , Y = {x , y} and the intuitionistic fuzzy set U and V are defined as follows :

U = {< a, 0.3, 0/6> <b , 0.4, 0.6> }
V = { < x, 0.5, 0.4 > , (y, 0.6, 0.3) >}

Then $\Im = \{ \tilde{0}, U, \tilde{1} \}$ and $\sigma = \{ \tilde{0}, V, \tilde{1} \}$ be intuitionistic fuzzy topologies on X and Y respectively . Then the mapping $f : (X,\Im). \rightarrow (Y, \sigma)$ defined by $f(a) = x$ and $f(b) = y$ is intuitionistic fuzzy sg-open but it is not intuitionistic fuzzy open.

Theorem 3.1: A mapping $f : (X, \Im) \rightarrow (Y, \sigma)$ is intuitionistic fuzzy sg-open if and only if for every intuitionisic fuzzy set U of X $f(\mathrm{Int}(U)) \subseteq \mathrm{sgint}(f(U))$.

Proof: Necessity Let f be an intuitionistic fuzzy sg-open mapping and U is an intuitionistic fuzzy open set in X. Now $\mathrm{Int}(U) \subseteq U$ which implies that $f(\mathrm{Int}(U) \subseteq f(U)$. Since f is an intuitionistic fuzzy sg-open mapping, $f(\mathrm{Int}(U))$ is intuitionistic fuzzy sg-open set in Y such that $f(\mathrm{Int}(U) \subseteq f(U)$ therefore $f(\mathrm{Int}(U) \subseteq \mathrm{sgint}\, f(U)$.

Sufficiency: For the converse suppose that U is an intuitionistic fuzzy open set of X. Then $f(U) = f(\mathrm{Int}(U)) \subseteq \mathrm{sgint}\, f(U)$. But $\mathrm{sgint}\,(f(U)) \subseteq f(U)$. Consequently $f(U) = \mathrm{sgint}(f(U))$ which implies that $f(U)$ is an intuitionistic fuzzy sg-open set of Y and hence f is an intuitionistic fuzzy sg-open mapping.

*Theorem 3.2***:** If $f : (X, \Im) \rightarrow (Y, \sigma)$ is an intuitionistic fuzzy sg-open mapping then $\mathrm{int}\,(f^{-1}(G)) \subseteq f^{-1}(\mathrm{sgint}(G))$ for every intuitionistic fuzzy set G of Y.

Proof: Let G is an intuitionistic fuzzy set of Y . Then $\mathrm{Int}\, f^{-1}(G)$ is an intuitionistic fuzzy open set in X. Since f is intuitionistic fuzzy sg-open $f(\mathrm{Int}\, f^{-1}(G))$ is intuitionistic fuzzy sg-open in Y and hence $f(\mathrm{Int}\, f^{-1}(G)) \subseteq \mathrm{sgint}(f(f^{-1}(G)) \subseteq \mathrm{sgint}(G)$. Thus $\mathrm{int}(f^{-1}(G)) \subseteq f^{-1}(\mathrm{sgint}(G))$.

Theorem 3.3: A mapping $f : (X, \Im) \rightarrow (Y, \sigma)$ is intuitionistic fuzzy sg-open if and only if for each intuitionistic fuzzy set S of Y and for each intuitionistic fuzzy closed set U of X containing $f^{-1}(S)$ there is an intuitionistic fuzzy sg-closed V of Y such that $S \subseteq V$ and $f^{-1}(V) \subseteq U$.

Proof: Necessity: Suppose that f is an intuitionistic fuzzy sg- open mapping. Let S be the intuitionistic fuzzy closed set of Y and U is an intuitionistic fuzzy closed set of X such that $f^{-1}(S) \subseteq U$. Then $V = (f^{-1}(U^c))^c$ is intuitionistic fuzzy sg- closed set of Y such that $f^{-1}(V) \subseteq U$.

Sufficiency: For the converse suppose that F is an intuitionistic fuzzy open set of X. Then $f^{-1}((f(F))^c \subseteq F^c$ and F^c is intuitionistic fuzzy closed set in X. By hypothesis there is an intuitionistic fuzzy sg-closed set V of Y such that $(f(F))^c \subseteq V$ and $f^{-1}(V) \subseteq F^c$ .Therefore $F \subseteq (f^{-1}(V))^c$. Hence $V^c \subseteq f(F) \subseteq f((f^{-1}(V))^c) \subseteq V^c$ which implies $f(F) = V^c$. Since V^c is intuitionistic fuzzy sg-open set of Y. Hence $f(F)$ is intuitionistic fuzzy sg-open in Y and thus f is intuitionistic fuzzy sg-open mapping.

*Theorem 3.4***:** A mapping $f : (X, \Im) \rightarrow (Y, \sigma)$ is intuitionistic fuzzy g-open if and only $f^{-1}(\mathrm{sgcl}(B) \subseteq \mathrm{cl}\, f^{-1}(B)$ for every intuitionistic fuzzy set B of Y.

Proof: Necessity: Suppose that f is an intuitionistic fuzzy sg- open mapping. For any intuitionistic fuzzy set B of Y $f^{-1}(B) \subseteq \mathrm{cl}(f^{-1}(B))$ Therefore by theorem 3.3 there exists an intuitionistic fuzzy sg-closed set F in Y such that $B \subseteq F$ and $f^{-1}(F) \subseteq \mathrm{cl}(f^{-1}(B))$. Therefore we obtain that $f^{-1}(\mathrm{sgcl}(B)) \subseteq f^{-1}(F) \subseteq \mathrm{cl}\, f^{-1}((B))$.

Sufficiency: For the converse suppose that B is an intuitionistic fuzzy set of Y. and F is an intuitionistic fuzzy closed set of X containing f^{-1}(B). Put W= cl(B) ,then we have B $\subseteq$ W and W is sg-closed and $f^{-1}(W) \subseteq cl(f^{-1}(B)) \subseteq F$. Then by theorem 3.3 f is intuitionistic fuzzy sg-open .

Theorem 3.5: If f: (X, $\Im$) $\to$(Y, σ) and g :(Y, σ) $\to$(Z, μ) be two intuitionistic fuzzy mappings and gof : (X,$\Im$) $\to$(Z, μ) is intuitionistic fuzzy sg-open**.** If g :(Y, σ) $\to$(Z, μ) is intuitionistic fuzzy sg-irresolute then f: (X,$\Im$) $\to$(Y, σ) is intuitionistic fuzzy sg-open mapping.

Proof: Let H be an intuitionistic fuzzy open set of intuitionistic fuzzy topological space(X,$\Im$). Then (go f) (H) is intuitionistic fuzzy sg-open set of Z because gof is intuitionistic fuzzy sg- open mapping. Now since g :(Y, σ) $\to$(Z, μ) is intuitionistic fuzzy sg-irresolute and (go f) (H) is intuitionistic fuzzy sg-open set of Z therefore g^{-1} (gof(H)) = f(H) is intuitionistic fuzzy sg-open set in intuitionistic fuzzy topological space Y. Hence f is intuitionistic fuzzy sg-open mapping.
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IV. INTUITIONISTIC FUZZY SG-CLOSED MAPPING

*Definition 4.1:*A mapping f : (X, $\Im$) $\to$(Y, σ) is intuitionistic fuzzy sg-closed if image of every intuitionistic fuzzy closed set of X is intuitionistic fuzzy sg-closed set in Y .

Remark 3.1 Every intuitionistic fuzzy closed mapping is intuitionistic sg-closed but converse may not be true. For,

Example 4.1: Let X = {a ,b} , Y = {x , y} and the intuitionistic fuzzy set U and V are defined as follows :

U = {< a, 0.3, 0/6> <b , 0.4, 0.6> }

V = { < x, 0.5, 0.4 > , (y, 0.6, 0.4) >}

Then $\Im$ = { $\tilde{\mathbf{0}}$,**U, 1**$\square$ } and σ = { $\tilde{\mathbf{0}}$ **,V, 1**$\square$ } be intuitionistic fuzzy topologies on X and Y respectively . Then the mapping f : (X,$\Im$) $\to$(Y, σ) defined by f(a) = x and f(b) =y is intuitionistic fuzzy sg- closed but it is not intuitionistic fuzzy closed.

Theorem 4.1: A mapping f : (X, $\Im$) $\to$(Y, σ) is intuitionistic fuzzy sg-closed if and only if for each intuitionistic fuzzy set S of Y and for each intuitionistic fuzzy open set U of X containing f^{-1}(S) there is an intuitionistic fuzzy sg-open set V of Y such that S $\subseteq$ V and $f^{-1}(V) \subseteq U$.

Proof: Necessity: Suppose that f is an intuitionistic fuzzy sg- closed mapping. Let S be the intuitionistic fuzzy closed set of Y and U is an intuitionistic fuzzy open set of X such that $f^{-1}(S) \subseteq U$. Then V =Y **-** $f^{-1}(U^c)$ is intuitionistic fuzzy sg- open set of Y such that $f^{-1}(V) \subseteq U$.

Sufficiency: For the converse suppose that F is an intuitionistic fuzzy closed set of X. Then $(f(F))^c$ is an intuitionistic fuzzy set of Y and F^c is intuitionistic fuzzy open set in X such that

$f^{-1}((f(F))^c) \subseteq F^c$. By hypothesis there is an intuitionistic fuzzy sg-open set V of Y such that $(f(F))^c \subseteq V$ and $f^{-1}(V) \subseteq F^c$ .Therefore $F \subseteq (f^{-1}(V))^c$. Hence $V^c \subseteq f(F) \subseteq f((f^{-1}(V))^c) \subseteq V^c$ which implies $f(F) = V^c$. Since V^c is intuitionistic fuzzy g-closed set of Y. Hence f (F) is intuitionistic fuzzy sg-closed in Y and thus f is intuitionistic fuzzy closed mapping.

Theorem 4.2: If f: (X, $\Im$) $\to$(Y, σ) is intuitionistic fuzzy continuous and intuitionistic sg-closed mapping and A is an intuitionistic fuzzy sg-closed set of X ,then f (A) intuitionistic fuzzy sg-closed.

Proof : Let $f(A) \subseteq O$ where O is an intuitionistic fuzzy open set of Y. Since f is intuitionistic fuzzy continuous therefore $f^{-1}(O)$ is an intuitionistic fuzzy open set of X such that $A \subseteq f^{-1}(O)$. Since A is intuitionistic fuzzy sg-closed of X which implies that $cl(A) \subseteq (f^{-1}(O))$ and hence $f(cl(A) \subseteq O$ which implies that $cl(f(cl(A)) \subseteq O$ therefore $cl(f((A)) \subseteq O$ whenever $f(A) \subseteq O$ where O is an intuitionistic fuzzy open set of Y. Hence $f(A)$ is an intuitionistic fuzzy sg-closed set of Y.

Corollary 4.1 : If $f: (X, \Im) \to (Y, \sigma)$ is intuitionistic fuzzy sg-continuous and intuitionistic fuzzy closed map and A is an intuitionistic fuzzy sg-closed set of X ,then $f(A)$ intuitionistic fuzzy sg-closed.

Theorem 4.3: If $f: (X, \Im) \to (Y, \sigma)$ is intuitionistic fuzzy closed mapping and $g: (Y, \sigma) \to (Z, \mu)$ is intuitionistic fuzzy g-closed mapping. Then $gof: (X,\Im) \to (Z,\mu)$ is intuitionistic fuzzy g-closed mapping**.**

Proof: Let H be an intuitionistic fuzzy closed set of intuitionistic fuzzy topological space$(X,\Im)$. Then $f(H)$ is intuitionistic fuzzy closed set of (Y, σ) because f is intuitionistic fuzzy closed mapping. Now $(gof)(H) = g(f(H))$ is intuitionistic fuzzy sg-closed set in intuitionistic fuzzy topological space Z because g is intuitionistic fuzzy sg-closed mapping. Thus $gof: (X,\Im) \to (Z,\mu)$ is intuitionistic fuzzy g-closed mapping.

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