A Study on Quadratic Assignment Problem in Wireless Sensor Networks

Vinoba.V
Department of Mathematics, K.N. Government Arts college, Tamil Nadu, India

Indhumathi.A
Department of Mathematics, R.M.K College of Engineering and Technology, Research Scholar, Bharathidasan University, Tamil Nadu, India

Abstract - This paper focuses on the occurrence of Quadratic Assignment problem and their applications in Wireless Sensor Networks. The QAP is one of the classical combinatorial optimization problems and is known for its diverse applications. This paper aims at describing the state of the art on quadratic assignment problems(QAPs) in wireless sensor networks. In recent years, wireless communication has experienced exponential growth caused by the need for connectivity. Wireless sensor networking is a broad research area, and many researchers have done research in the area of power efficiency to extend network lifetime. A node can easily transmit data to a distance node, if it has sufficient battery power. For maximizing the lifetime of network, the data should be forwarded such that energy consumption is balanced among the nodes in proportion to their energy reserved, instead of routing to minimize consumed power. And this can be achieved using QAP.

Keywords: Quadratic Assignment problem, NP-complete, formulation, Lower Bounds, Exact Algorithm, Heuristics and Wireless Sensor Networks.

I. INTRODUCTION

The Quadratic Assignment Problem (QAP) is one of the most interesting and most challenging combinatorial optimization problems in existence. Koopmans and Beckman first introduced the Quadratic Assignment problem QAP in 1957 as a mathematical model for the assignment of n "invisible economic activities" (i.e. plants) to n locations. The general Quadratic Assignment problem QAP is known to be NP-Complete. In its basic interpretation, the problem seeks to assign n facilities to n locations with the cost being proportional to the flow between the facilities multiplied with their distances. The objective is to allocate each facility to a location such that the total cost is minimized. Many scientists including mathematicians, computer scientists, operations research analysts, and economists have used the QAP to model a variety of optimization problems. Sahni and Gonzalez (1976) showed that the QAP is *NP*-hard and that there is no ε-approximation polynomial algorithm for the QAP unless $P = NP$. Other optimization combinatorial problems such as the traveling salesman problem, maximal clique, isomorphism and graph partitioning can be formulated as a QAP.

II. MATHEMATICAL FORMULATION OF QAP

Optimization deals with problems of maximizing or minimizing functions with several variables, usually subject to equality and/or inequality constraints. An optimization algorithm is a procedure which is executed iteratively till the optimum or satisfactory solutions are found.

An optimization problem can be represented in the following way .Given a function f: A $\rightarrow R$ from some set A to the real numbers, an element x_0 in A such that $f(x_0) \leq f(x)$ for all x in A ("minimization") or such that $f(x_0) \geq f(x)$ for all x in A ("maximization").A is some subset of the Euclidean space R^n, often specified by a set of constraints, equalities or inequalities that the members of A have to satisfy. A feasible solution that minimizes (or maximizes) the objective function is called an optimal solution.

Consider the set N = {1,2,3,……n} and three nxn matrices F = (f_{ij}), D = (d_{ij}) and C =(c_{ij}). The Quadratic Assignment problem with coefficient matrices F,D, and C shortly denoted by QAP can be stated as follows :

$$\underset{x \in X}{Min} \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} f_{ik} d_{jl} x_{ij} x_{kl} + \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} x_{ij}$$

$$such\ that\ \sum_{i=1}^{n} x_{ij} = 1,\ i \in N, \sum_{j=1}^{n} x_{ij} = 1,\ j \in N,\ x_{ij} \in \{0,1\}, i, j \in N$$

f_{ik} denotes the amount of flow between facilities i and k, d_{jl} denotes the distance between locations j and l and c_{ij} denotes the cost of locating facility i at location j.

$$x_{ij} = \begin{cases} 1, & \text{if facility } i \text{ is assigned to location } j \\ 0, & \text{otherwise} \end{cases}$$

III. EXACT ALGORITHMS

An exact algorithm for a combinatorial optimization problem provides the global optimal solution to the problem. There are three main methods used to find the global optimal solution for a given QAP: dynamic programming, cutting plane techniques, and branch and bound procedures. Research has shown that the latter is the most successful for solving instances of the QAP.

Branch and bound algorithms have been applied successfully to many hard combinatorial optimization problems, and they appear to be the most efficient exact algorithms for solving the QAP. The basic ingredients of branch and bound algorithms are bounding, branching, and the selection rule. Although many bounding techniques have been developed for the QAP the most efficient branch and bound algorithms for this problem employ the Gilmore-Lawler bound (GLB). The reason is that other bounds which outperform GLB in terms of bound quality are simply expensive in terms of computation time.

Branch and bound algorithms receive their name from an intuitive description of how they are executed. First, a heuristic procedure is used to generate a suboptimal, but suitable, initial feasible solution. This initial solution is used as an upper bound. Next, the problem is separated into a finite number of sub problems, with a lower bound being established for each. A so-called *search tree* is formed by the repetition of the decomposition/lower bounding process being applied to each sub problem. Branch and bound techniques have evolved greatly over the past 40 years, starting with Gilmore who in 1962 solved a QAP of size n = 8, to the solution of the nug30, a QAP of size *n* = 30 in 2000 by Anstreicher, *et al.*

IV. HEURISTIC ALGORITHMS

Heuristic algorithms do not give any guarantee of optimality for the best solution obtained. As a matter of fact, it is usual to find approximate algorithms treated as heuristic algorithms in the Combinatorial Optimization literature, as in Osman and Laporte (1996). There are five basic categories of heuristics for the QAP:

1. Construction Methods.
2. Limited Enumeration Methods.
3. Improvement Methods.
4. Simulated Annealing Methods.
5. Genetic Algorithms.
6. Greedy Randomized Adaptive Search Procedures
7. Tabu Search

Construction Methods

Construction methods create suboptimal permutations by starting with a partial permutation which is initially empty. The permutation is expanded by repetitive assignments based on set selection criterion until the permutation is complete. One of the oldest heuristics in use is a construction method algorithm. The CRAFT (Computerized

Relative Allocation of Facilities Technique), used for the layout of facilities was first introduced by Armour and Bura in 1963.

Limited Enumeration Methods

Enumeration methods can guarantee that the obtained solution is optimum only if they can go to the end of the enumerative process. However, it is possible that a good solution, or even an optimal solution, is found by the beginning of the process. It can be observed that the best the information used to guide the enumeration, the bigger the chances to find prematurely good quality solutions.

Improvement methods

These methods belong to the larger class of local search algorithms. A local search procedure starts with an initial feasible solution and iteratively tries to improve the current solution. This is done by substituting the latter with a (better) feasible solution from its neighborhood. This iterative step is repeated until no further improvement can be found. Improvement methods are local search algorithm which allow only improvements of the current solution in each iteration. Improvement methods correspond to local search algorithms. An improvement method begins with a feasible solution and tries to improve it, searching for other solutions in its neighborhood. The process is repeated until no improvement can be found.

Simulated Annealing Methods

This group of heuristics, which is also used for overcoming local optima, receives its name from the physical process which it imitates. This process, called *annealing* moves high energy particles to lower energy states with the lowering of the temperature, thus cooling a material to a steady state. Initially, in the initial state of the heuristic, the algorithm is lenient and capable of moving to a *worse* solution. However, with each iteration the algorithm becomes stricter requiring a better solution at each step.

Genetic Algorithms

Genetic algorithms also receive their names from an intuitive explanation of the manner in which they behave. This explanation is based on Darwin's Theory of Natural Selection. Genetic algorithms store a set of solutions and then work to replace these solutions with better ones based on some fitness criterion, usually the objective function value. Genetic algorithms are parallel and are helpful when applied in such an environment.

Greedy Randomized Adaptive Search Procedure

The greedy randomized adaptive search procedure (GRASP) was introduced by Feo and Resende and has been applied successfully to different hard combinatorial optimization problems and among them to the QAP and the BiQAP. GRASP is a relatively new heuristic used to solve combinatorial optimization problems. At each iteration, a solution is computed. The final solution is taken as the one which is best after all GRASP iterations are performed. The GRASP was first applied to the QAP in 1994 by Li,Pardalos, and Resende. They applied the GRASP to 88 instances of the QAP, finding the best known solution in almost every case, and improved solutions for a few instances. GRASP is a combination of greedy elements with random search elements in a two phase heuristic.

Tabu search

Tabu search is a local search algorithm that was introduced by Glover to find good quality solutions for integer programming problems. Its main feature is an updated list of the best solutions that were found in the search process. Each solution receives a priority value or an aspiration criterion. Their basic ingredients are: a *tabu list*, used to keep the history of the search process evolution; a mechanism that allows the acceptance or rejection of a new allocation in the neighborhood, based on the tabu list information and on their priorities; and a mechanism that allows the alternation between neighborhood diversification and intensification strategies.

V. APPLICATIONS OF QAP

This model finds applications in parallel and distributed computing, combinatorial data analysis, facility location, computer manufacturing, scheduling, building layout design and process communications. Other optimization combinatorial problems such as travelling salesman problem, maximal clique isomorphism and graph partitioning can be formulated as a QAP. Additional applications of QAP include (i) the allocation of plants to candidate locations; (ii) layout of plants; (iii) backboard wiring problem; (iv) design of control panels and typewriter keyboards; (v) balancing.

VI. WIRELESS SENSOR NETWORKS (WSN)

NETWORK

In Telecommunications, a network refers to a connection of devices such as telephones, computer switches and printers. A node is where a number of connections meet at a common point within the network. Nodes in general are complex structures and in order to ensure the smooth flow of information through the node, the equipment at the node must operate to well defined rules or what is referred to as a protocol.

Some of the networks in common use are :

i) Local Area Network (LAN) : connects devices that are close geographically.eg. in the same building.
ii) Wide Area Network (WAN) : connects devices that are well separated geographically.
eg. long distances telephone lines or radio may have to be used for the connections.
iii) Metropolitan Area Network (MAN) : Designed for use in a town or city.
iv) Campus Area Network (CAN) : Designed for operations in a campus such as a military campus or the campus of an educational establishment.
v) Home Area Network (HAN) : Designed to connect together devices in a person's home.eg.computers and printers.

Wireless Sensor Networks (WSN)

A Wireless Sensor Network (WSN) is a network of thousands of small low-cost sensor nodes whose communications with a central station are conveyed by means of wireless signals. The sensor network consists of the sensor fields, sensor nodes, sink and Task manager.

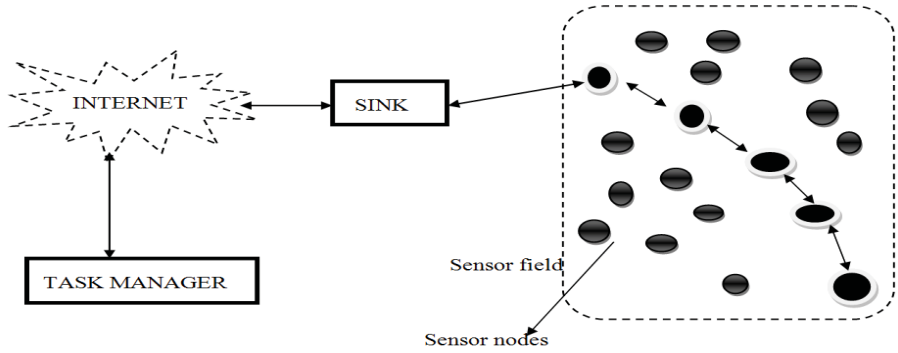

Fig-1 Components of a wireless sensor network

i) Sensor field : It is the region where the sensor nodes are scattered.
ii) Sensor Nodes: They collect data and route data back to the sink and the end users. Data are routed back to the end user by a mulithop infrastructureless architecture through the sink. Therefore they are the heart of the network.
iii) Sink : It is a sensor node with the specific task of receiving, processing and storing data from the other sensor nodes. The sink may communicate with the task manager node via Internet or satellite.
iv) Task Manager : It is also known as base station which extract information from the network and disseminates control information back into the network. The base station is either a laptop or a workstation.

Sensor nodes in WSN mainly use a broadcast communication paradigm where the sensor signals are used in further analysis of the sensed environment. WSN is preferred as the sensor system architecture with regard to its inherent redundancy but is susceptible to disadvantages caused by limited operation life-time.

The first WSN was designed and used in 70s, in military held during the Vietnam war. WSN consist of nodes, from few to several one, which work together to capture data from an environment region and send this data to a base station. These sensor nodes use to track and monitor heat, temperature, vibratory movement, etc. They are small with limited computing resources and base on arouting algorithm, they can transmit data to the user.

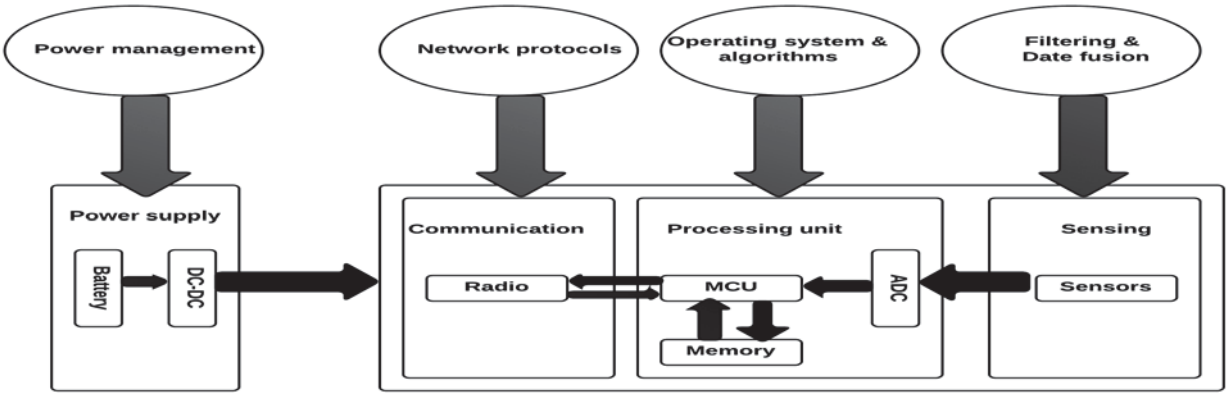

fig-2 Wsn sensor node system architecture

Models of Wireless sensor networks

Sensor nodes used in wsn can be fixed or mobile. So, accordingly to this wsn's can be classified into two types:

a) Static Wireless Sensor Networks b) Mobile Wireless sensor networks

a) Static wireless sensor networks

Static wireless sensor network, have all nodes fixed at one place. i.e there is no motion among the nodes placed in the sensor networks. This type of network model is reliable, easy to implement. To communicate between two nodes is simple as all the nodes are static.

b) Mobile wireless sensor networks

In mobile wireless sensor networks nodes are mobile .i.e. nodes can move from place to place. Due to which communication between two nodes can be very complicated. Routes selected for communication also have to change with respect to movement of nodes. Node which has to transfer the data called source node and node to which the data has to be sent is called sink node. But MWSNs are more advantageous over static WSNs .

VII. MOTIVATIONS

Wireless Sensor Networks (WSN s) are large-scale, dynamic and limited in Power. These WSN s can be used for various applications such as military, environmental, health, home and other commercial applications. With the high degree of deployment flexibility, applications of WSN are vast and can be broadly classified into the monitoring and tracking categories.

Monitoring applications include environmental monitoring such as forest fire detection, bio complexity mapping of the environment, flood detection, precision agriculture, health monitoring contains rele-monitoring of human physiological data, monitoring doctors and patients conditions and drug administration in hospitals.

Tracking applications include objects, animals, humans, vehicles and military enemy tracking. For example, in a battle field, a commander can be aware of the status of Friendly troops or the availability of equipment by sing the sensor networks. Another application is forest fire early detection system. Smoke or temperature sensors can be Deployed into a fire-susceptible forest area to detect a forest fire on its early stage. Lastly the technology of WSN also can be used in health applications. The physiological data are collected by wireless sensors are stored for a long period and used for medical exploration. Hence in the future this wide range of applications will make sensor networks an integral part of our lives.

We categorize the applications into military; environmental, health and home application.

A. *Military Applications:* Wireless sensor networks can be an integral part of military command, control, communication, computing, intelligence, surveillance and targeting (C4ISRT) systems. The rapid deployment, fault tolerance and self-organization characteristics of sensor networks make them a very promising sensing technique for military (C4ISRT). Since sensor networks are based on dense deployment of disposable and low cost sensor nodes, destruction of some nodes by hostile actions does not affect military applications as much as the destruction of traditional sensor, which makes sensor networks concept a better approach for battlefield.

B. *Environmental Applications:* Some environmental applications of sensor network include tracking the movement of birds, small animals and insects; monitoring environmental conditions that affect crops and livestock, irrigation, macro instruments for large scale earth monitoring and planetary exploration; chemical/biological detection; precision agriculture; biological, Earth and environmental monitoring in marine, soil and atmospheric contexts; forest fire detection and meteorological and geophysical research; flood detection; bio complexity mapping of the environment and pollution study.

C. *Health Application:* Some of the applications are providing interfaces for the disabled; integrated patient monitoring; diagnostics; drug administration in hospital; monitoring the movements and internal process of insects or others mall animals; telemonitoring of human physiological data; and tracking and monitoring doctors and patients inside a hospital.

D. *Home Applications:* Home automation; as technology advances, smart sensor nodes and actuators can be buried appliances, such as vacuum cleaners, micro wave ovens, refrigerators and VCRs. These sensor nodes inside the domestic devices can interact with each other and with external network via the internet or satellite. They allow end users to manage home devices locally and remotely more easily.

VIII.CONCLUSION

The Quadratic Assignment Problem (QAP) is a discrete optimization problem which can be found in economics, operations research, and engineering. Since its first formation the QAP has been drawing researchers attention all over the world, not only because of its practical and theoritical importance but also mainly because of its complexity. The main objective of this paper is to minimize the energy consumption of the sensor nodes during transmission and reception by using optimal path algorithm which is possible by means of Quadratic Assignment problem. The interested readers can go through these references and see how QAP is applied in WSN. Further advances and development in these areas will certainly lead to the solution of even more difficult instances in the near future.

REFERENCES

[1] Akkaya, K.; Younis, M. A survey on routing protocols for wireless sensor networks [J]. Ad Hoc Networks, 2005, 3(3):325-349.

[2] Akyildiz.I, W. Su, Sankarasubramaniam Y., and. Cayirci E, (August 2002), A survey on sensor networks," IEEE Communications Magazine, Volume: 40 Issue: 8, pp.102-114.

[3] Krishnamachari B and Orid F, (2003), Analysis of Energy efficient Fair Routing in Wireless Sensor Network Through Non Linear optimization in the proceedings of workshop on wireless Ad hoc sensor and Wearable Networks in IEEE Vehicular technology Conference, Florida.PP 2844-2848.

[4] J.H. Chang, L. Tassiulas, Maximum lifetime routing in wireless sensor networks, Journal on IEEE/ACM Transaction. and Networking, Vol.12, No.4, pp.609–619, 2004.

[5] J.H. Chang, L. Tassiulas, Energy conserving routing in wireless ad-hoc networks, in: INFOCOM, 2000, pp. 22–31.

[6] S. Coleri, P. Varaiya, Fault tolerant and energy efficient routing for sensor networks, in: GlobeCom, 2004.

[7] O. Yilmaz, K. Erciyes, Distributed weighted node shortest path routing for wireless sensor networks, in: Recent Trends in Wireless and Mobile Networks, Communications in Computer and Information Science, pp. 304–314, 2010.

[8] K.B. Lakshmanan, K. Thulasiraman, M.A. Comeau, An efficient distributed protocol for finding shortest paths in networks with negative weights, IEEE Trans. Softw. Eng. 15 (1989) 639–644.
[9] A. Boukerche, R. Werner Nelem Pazzi, R. Borges Araujo, Fault-tolerant wireless sensor network routing protocols for the supervision of context-aware physical environments, J. Parallel Distrib. Comput. 66 (2006) 586–599.
[10] A. Boukerche, Algorithms and Protocols for Wireless Sensor Networks, Wiley-IEEE Press, 2008.
[11] D. Ganesan, R. Govindan, S. Shenker, D. Estrin, Highly-resilient, energy-efficient multipath routing in wireless sensor networks, in: Proceedings of the 2nd ACM International Symposium on Mobile ad Hoc Networking & Computing, MobiHoc'01, 2001, pp. 251–254.
[12] M. S. Bazaraa and H. D. Sherali. On the Use of Exact and Heuristic Cutting Plane Methods for the Quadratic Assignment Problem. The Journal of the Operational Research Society, Volume 33, Issue 11, Novmber 1982.
[13] R.E. Burkard, Quadratic assignment problems, European Journal of Operational Research 15 (3) (1984) 283–289.
[14] E. Cela, The Quadratic Assignment Problem: Theory and Algorithms, Kluwer Academic Publishers, Dordrecht, The Netherlands, 1998.
[15] P.C.Gilmore. Optimal and Suboptimal Algorithms for the Quadratic Assignment Problem. SIAM Journal on Applied Mathematics, Volume 10, 1962.
[16] P.M. Pardalos, F. Rendl, and H. Wolkowicz. The Quadratic Assignment Problem: A Survey and Recent Developments, in Quadratic Assignment and Related Problems, DIMACS Series in Discrete Mathematics and Theoretical Computer Science, Volume 16, 1994.