A Study on Single Server Fuzzy Queuing Model using DSW Algorithm

S. Shanmugasundaram
Assistant Professor
Department of Mathematics
Government Arts College, Salem-636007,Tamil Nadu, India

S.Thamotharan
Assistant Professor
Department of Mathematics
AVS Engineering College, Salem-636003,Tamil Nadu, India

M.Ragapriya
Assistant Professor
Department of Mathematics
Annapoorana Engineering College, Salem-636308,Tamil Nadu, India

Abstract - In this paper,we study DSW algorithm for triangular and trapezoidal fuzzy numbers. The performance measure of this queuing model is analyzed and also we study a fuzzy nature of a single server queue. The numerical example is also given to test the feasibility of this model.

Keywords - Membership function, triangular and trapezoidal fuzzy number, α cuts, Standard interval analysis, DSW algorithm.

I. INTRODUCTION

The queuing theory or waiting line theory is development by A.K.Erlang in 1903 based on congestion of telephone traffic. He directed his first efforts at finding the delay for one operator and later on the results were extended to find the delay for many operators. Moline and Thornton D- Fry who was further developed this telephone traffic in 1927 and 1928 respectively. The inter arrival time and service times are required to follow particular distribution in traditional queuing theory.

But in fuzzy queuing theory best described arrival rate and service rates through linguistic terms very high, high. Low and very low or moderate.

Here fuzzy set can be splits in to leaven distinct points through alpha cut method also DSW algorithm is used to define a membership function of the performance measure in queuing models.
Fuzzy queues are developed by many authors such as J.J.Buckely (1990), S.P.Chen (2005), R.J.Li and E.S.Lee (1989), D.S.Negi and E.S.Lee (1992).

II. FUZZY SET THEORY

Definition: 1

Let X be a classical set of objects, called the universe, whose generic elements are denoted by x. Membership in a classical subject A of X is often denoted as characteristic function μ_A from X to $\{0,1\}$ such that

$$\mu_A(x) = \begin{cases} 1 \text{ iff } x \in A, \\ 0 \text{ iff } x \notin A \end{cases}$$

$\{0,1\}$is called a valuation set. If the valuation set is allowed to be the real interval $[0,1]$. $A$is called a fuzzy set.

Definition: 2

For any set X a membership function on X is any function from X to the real unit interval $[0,1]$

$$i.e., \mu_{\tilde{A}} : X \rightarrow [0,1]$$

Membership function on X represent fuzzy subsets of X. The membership function which represents a fuzzy set $\tilde{A}$ is usually denoted by μ_A. For an element $x$of X. The value $\mu_{\tilde{A}}(x)$ is called the membership degree of x in the fuzzy set $\tilde{A}$.

Definition: 3

If a fuzzy set A is defined on X for any $\alpha \in [0,1]$ the αcuts α_A is represented by the following crisp set.

Strong α - cuts

$$\alpha^+{}_A = \{x \in X \,/\, \mu_A(x) > \alpha \,;\, \alpha \in [0,1]\}$$

Weak α - cuts

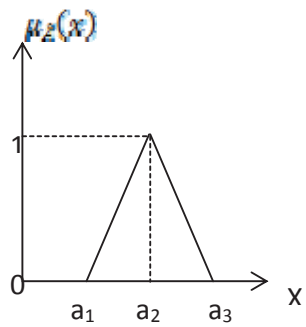
$$\alpha_A = \{x \in X \,/\, \mu_A(x) \geq \alpha \,;\, \alpha \in [0,1]\}$$

Therefore, it is inferred that fuzzy set A can be treated as crisp set α_A in which all the members have their membership values greater than or at least equal to α.

Definition: 4

A triangular fuzzy number A can be defined as a triplet $[a_1, a_2, a_3]$. Its membership function is defined as

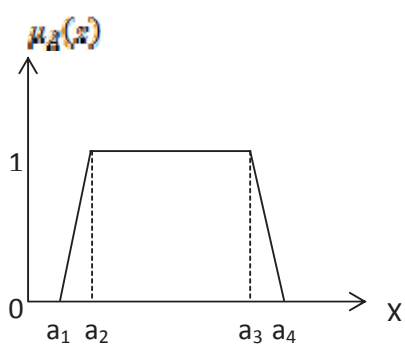
$$\mu_A(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\[2ex] \dfrac{a_3 - x}{a_3 - a_2}, & a_2 \leq x \leq a_3 \end{cases}$$



Definition: 5

A trapezoidal fuzzy number A can be defined as $[a_1, a_2, a_3, a_4]$. Its membership function is defined as

$$\mu_A(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\[2ex] 1, & a_2 \leq x \leq a_3 \\[2ex] \dfrac{a_4 - x}{a_4 - a_3}, & a_3 \leq x \leq a_4 \end{cases}$$



III. FORMULATION OF THE PROBLEM

We consider a single server queuing system with infinite source population. The inter arrival time αA and service time S are represented by the following fuzzy sets

$$A = \{(a, \mu_A(a)) \ / \ a \in X\}$$

$$S = \{(s, \mu_S(s)) \ / \ s \in Y\}$$

Where X & Y are the crisp universal sets of the inter arrival time and service time. $\mu(a)$and$\mu(s)$ are the corresponding membership functions.

The membership functions of A and S are defined by

$$A(\alpha) = \{a \in X \ / \ \mu_A(a) \geq \alpha\}$$

$$S(\alpha) = \{s \in Y \ / \ \mu_S(s) \geq \alpha\}$$

Where $A(\alpha)$ and $S(\alpha)$ are crisp sets using α cuts, the inter arrival time and service time can be represented by different levels of confidence intervals.
The triangular membership function $F(A, S)$ is

$$\mu_{F(A,S)} = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 \leq x \leq a_3 \end{cases}$$

Where $a_1 \leq a_2 \leq a_3$. An approximate method of extension is propagating fuzziness for continuous valued mapping determined the membership functions for the output variables.

The trapezoidal membership function $F(A, S)$ is

$$\mu_{F(A,S)} = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ 1, & a_2 \leq x \leq a_3 \\ \dfrac{a_4 - x}{a_4 - a_3}, & a_3 \leq x \leq a_4 \end{cases}$$

Where $a_1 < a_2 < a_3 < a_4$. An approximate method of extension is propagating fuzziness for continuous valued mapping determined the membership functions for the output variables.

IV. THE (FM/FM/1):(∞/FCFS) QUEUE

In this model we consider an infinite source population with first come first served discipline where both the inter arrival time λ and the service time μfollow an exponential distributions.

The expected number of customer in the system

$$L_s = \frac{\lambda}{\mu - \lambda}$$

The expected number of customers in the queue

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

The average waiting time in the system

$$W_s = \frac{1}{\mu - \lambda}$$

The average waiting time of a customer in the queue

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)}$$

V. STANDARD INTERVAL ANALYSIS ARITHMETIC

Let I_1 and I_2 be two interval numbers defined by ordered pairs of real numbers with lower and upper bounds.

$$I_1 = [a,b]\ ,\ a \leq b \qquad I_2 = [c,d]\ ,\ c \leq d$$

Define a general arithmetic property with the symbol *, where * = [+, -, ×, ÷] symbolically the operation.

$$I_1 * I_2 = [a,b] * [c,d]$$

represents another interval. The interval calculation depends on the magnitudes and signs of the element a, b, c, d.

$$[a, b] + [c, d] = [a + c, b + d]$$

$$[a, b] - [c, d] = [a - d, b - c]$$

$$[a, b] .[c, d] = [\min (ac, ad, bc, bd), \max (ac, ad, bc, bd)]$$

$$[a, b] \div [c, d] = [a,b].\left[\frac{1}{d}, \frac{1}{c}\right] \text{ provided that } 0 \notin [c, d]$$

$$\alpha[a, b] = \begin{cases} [\alpha a, \alpha b] \text{ for } \alpha \geq 0 \\ [\alpha b, \alpha a] \text{ for } \alpha < 0 \end{cases}$$

VI. DSW ALGORITHM

DSW (Dong, Shah, Wong) is one of the approximate methods make use of intervals at various α cut levels in defining membership functions. It was the full α cut intervals in a standard interval analysis.

The DSW algorithm greatly simplifies manipulation of the extension principle for continuous valued fuzzy variables, such as fuzzy numbers defined on the real line. It prevent abnormality in the output membership function due to application of the discrimination reaching on the fuzzy variable domain, it can prevent the widening of the resulting functional expression by conventional interval analysis methods.

Any continuous membership function can be represented by a continuous sweep of α cut in term from $\alpha = 0$ to $\alpha = 1$ suppose we have single input mapping given by $y = f(x)$ that is to be extended for fuzzy sets $\tilde{B} = f(\tilde{A})$ and we want to decompose $\tilde{A}$ into the series of α cut intervals say I_α. It uses the full α cut intervals in a standard interval analysis.

The DSW algorithm consists of the following steps:

1. Select a α cut value where $0 \leq \alpha \leq 1$.
2. Find the intervals in the input membership functions that correspond to this α.
3. Using standard binary interval operations compute the interval for the output membership function for the selected α cut level.
4. Repeat steps 1 – 3 for different values of α to complete α cut representation of the solution.

VII. NUMERICAL EXAMPLE

(i) Triangular fuzzy number

Consider a FM/FM/1 queue, where the both the arrival rate and service rate are triangular fuzzy numbers represented by $\tilde{\lambda}$ = [1,2,3] and $\tilde{\mu}$ = [11,12,13]. The interval of confidence at possibility level α as

$$[1+\alpha\ ,\ 3-\alpha] \text{ and } [11+\alpha\ ,\ 13-\alpha]$$

The expected number of customer in the system

$$L_s = \frac{x}{y-x}$$

The expected number of customers in the queue

$$L_q = \frac{x^2}{y(y-x)}$$

The average waiting time in the system

$$W_s = \frac{1}{y-x}$$

The average waiting time of a customer in the queue

$$W_q = \frac{x}{y(y-x)}$$

Where $x = [1+\alpha\ ,\ 3-\alpha]$ and $y = [11+\alpha\ ,\ 13-\alpha]$

TABLE: 1 The α cuts of L_s, L_q, W_s, W_q at α values

Alpha	L_s	L_q	W_s	W_q
0	[0.0833, 0.375]	[0.0064, 0.1022]	[0.0833, 0.125]	[0.0064, 0.0340]
0.1	[0.0932, 0.3536]	[0.0079, 0.0923]	[0.0847, 0.1219]	[0.0072, 0.0318]
0.2	[0.1034, 0.3333]	[0.0096, 0.0833]	[0.0862, 0.1190]	[0.0080, 0.0297]
0.3	[0.1140, 0.3139]	[0.0116, 0.0750]	[0.0877, 0.1162]	[0.0089, 0.0277]
0.4	[0.125, 0.2954]	[0.0138, 0.0673]	[0.0892, 0.1136]	[0.0099, 0.0259]
0.5	[0.1363, 0.2777]	[0.0163, 0.0603]	[0.0909, 0.1111]	[0.0109, 0.0241]
0.6	[0.1481, 0.2608]	[0.0191, 0.0539]	[0.0925, 0.1086]	[0.0119, 0.0224]
0.7	[0.1603, 0.2446]	[0.0221, 0.0480]	[0.0943, 0.1063]	[0.0130, 0.0209]
0.8	[0.1730, 0.2291]	[0.0255, 0.0427]	[0.0961, 0.1041]	[0.0141, 0.0194]
0.9	[0.1862, 0.2142]	[0.0292, 0.0378]	[0.0980, 0.1020]	[0.0153, 0.0180]
1	[0.2, 0.2]	[0.0333, 0.0333]	[0.1, 0.1]	[0.0166, 0.0166]

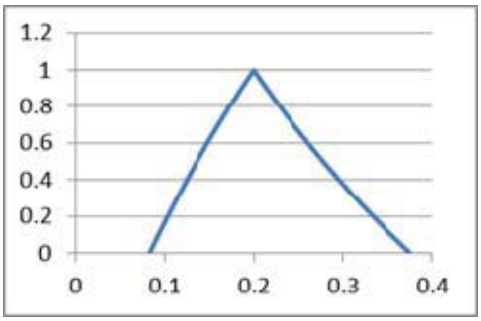


Fig 1: Expected number of customers in the system (Ls)

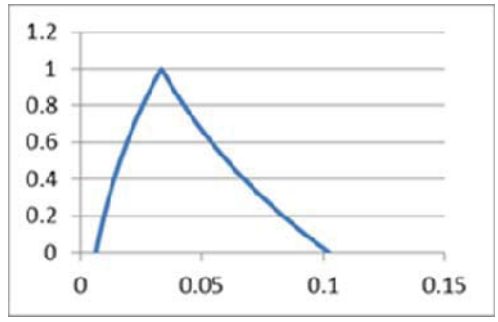


Fig 2: Expected number of customers in the queue (Lq)

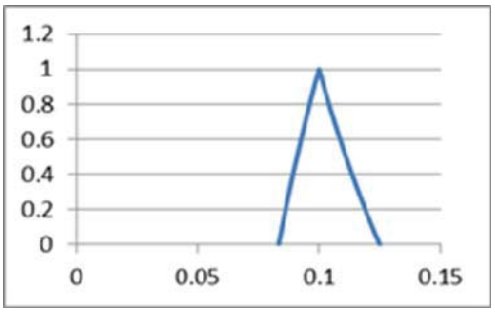


Fig 3 : Average waiting time of a customer in the system (Ws)

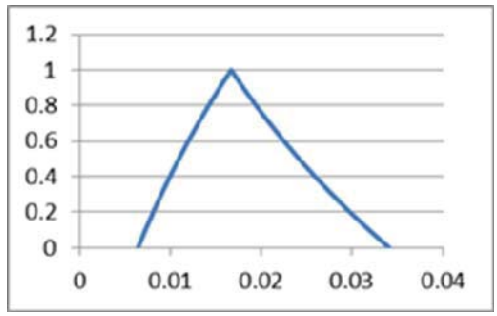


Fig 4: Average waiting time of a customer in the queue (Wq)

(ii) Trapezoidal fuzzy number

Take the both the arrival rate and service rate are trapezoidal fuzzy number represented by $\tilde{\lambda}$ = [5, 6, 7, 8] and $\tilde{\mu}$ = [16, 17, 18, 19]. The interval of confidence at possibility level α as

$$[5+\alpha\ ,\ 8-\alpha] \text{ and } [16+\alpha\ ,\ 19-\alpha]$$

The expected number of customer in the system

$$L_s = \frac{x}{y-x}$$

The expected number of customers in the queue

$$L_q = \frac{x^2}{y(y-x)}$$

The average waiting time in the system

$$W_s = \frac{1}{y-x}$$

The average waiting time of a customer in the queue

$$W_q = \frac{x}{y(y-x)}$$

Where $x = [5+\alpha\ ,\ 8-\alpha]$ and $y = [16+\alpha\ ,\ 19-\alpha]$

TABLE:2The α cuts of L_s, L_q, W_s, W_q at α values

ALPHA	L_s	L_q	W_s	W_q
0	[0.3571, 1]	[0.0939, 0.5]	[0.0714, 0.125]	[0.0187, 0.0625]
0.1	[0.3695, 0.9634]	[0.0997, 0.4727]	[0.0724, 0.1219]	[0.0195, 0.0598]
0.2	[0.3823, 0.9285]	[0.1057, 0.4470]	[0.0735, 0.1190]	[0.0203, 0.0573]
0.3	[0.3955, 0.8953]	[0.1120, 0.4229]	[0.0746, 0.1162]	[0.0211, 0.0549]
0.4	[0.4090, 0.8636]	[0.1187, 0.4002]	[0.0757, 0.1136]	[0.0219, 0.0526]
0.5	[0.4230, 0.8333]	[0.1257, 0.3787]	[0.0769, 0.1111]	[0.0228, 0.0505]
0.6	[0.4375, 0.8043]	[0.1331, 0.3585]	[0.0781, 0.1086]	[0.0237, 0.0484]
0.7	[0.4523, 0.7765]	[0.1409, 0.3394]	[0.0793, 0.1063]	[0.0247, 0.0465]
0.8	[0.4677, 0.75]	[0.1490, 0.3214]	[0.0806, 0.1041]	[0.0257, 0.0446]
0.9	[0.4836, 0.7244]	[0.1576, 0.3043]	[0.0819, 0.1020]	[0.0267, 0.0428]
1	[0.5, 0.7]	[0.1666, 0.2882]	[0.0833, 0.1]	[0.0277, 0.0411]

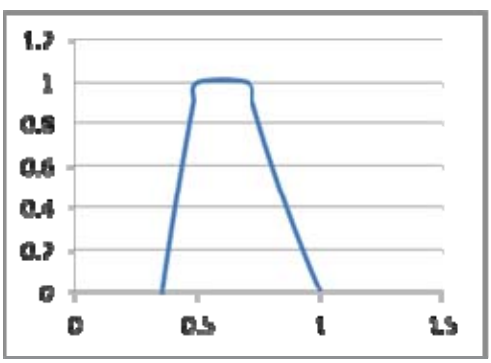


Fig 1: Expected number of customers in the system (Ls)

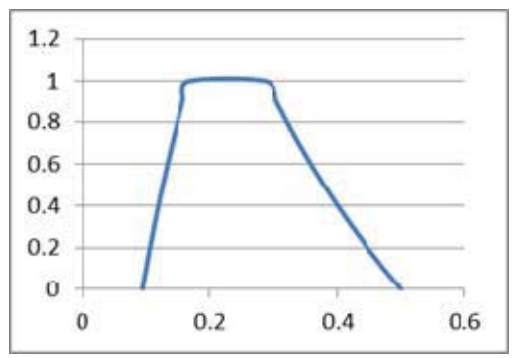


Fig 2: Expected number of customers in the queue (Lq)

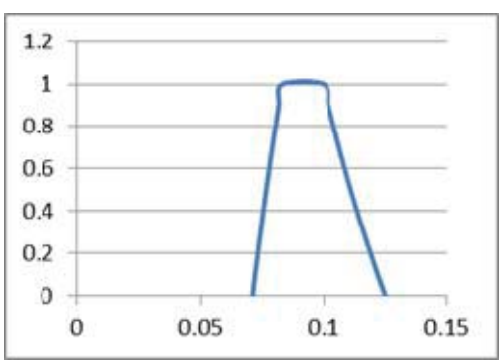


Fig 3 : Average waiting time of a customer in the system (Ws)

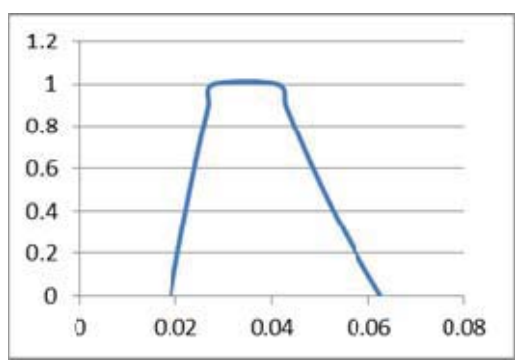


Fig 4: Average waiting time of a customer in the queue (Wq)

Using Microsoft Excel we perform α cuts of arrival rate and service rate and fuzzy expected number of jobs in queue at eleven distinctα levels: 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1. Crisp intervals for fuzzy expected number of jobs in queue at different possibility α levels are presented in table1and 2. The performance measures such as expected number of customers in the system (L_s), expected length of queue (L_q), the average waiting time in the system (W_s) and the average waiting time of a customer in the queue (W_q) also derived in table 1and 2.

The α cut represent, while these four performance measures are fuzzy. Using triangular fuzzy number the most likely value of the expected queue length (L_q) is 0.0333 and its value impossible to fall outside the range of 0.0064 and 0.1022; it is definitely possible that the average waiting time in the system (L_s) is 0.2 and its value will never fall below 0.0833 or exceed 0.375. In the same way using trapezoidal fuzzy number the expected queue length (L_q) falls between 0.1666 and 0.2882 and its value impossible to fall outside the range of 0.0939 and 0.5; it is definitely possible that the average waiting time in the system (L_s) falls between 0.5 and 0.7 and it value will never fall below 0.3571 or exceed 1.The above information will be very useful for defining a queuing system.

VIII. CONCLUSION

In this paper we infer that fuzzy set theory has been applied to queuing theory also. The inter arrival time and service time are fuzzy nature. The performance measure such as system length, queue length, system time, queue time etc. are also in fuzzy nature. Numerical example shows that the efficiency of DSW algorithm.

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