Design of an IIR by Impulse Invariance and Bilinear Transformation

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Abstract- A fundamental aspect of signal processing is filtering. Filtering involves the manipulation of the spectrum of a signal by passing or blocking certain portions of the spectrum, depending on the frequency of those portions. Discrete time IIR filter design can be a very complex procedure in discrete time domain. Therefore, transformations have been developed to use well known design methods in continuous time domain. The filter specifications are made in discrete time domain, shifted into continuous time domain where the filter is designed in the old fashioned way. That continuous time filter is then transformed back into discrete time domain.The result is a discrete time filter which matches the given specifications in a certain way. In this paper impulse invariant design is compared with bilinear transformation.

Keywords – IIR filter, impulse invariance, Bilinear Transformation, filter design

I. INTRODUCTION

IIR filter design primarily concentrates on the magnitude response of the filter and regards the phase response as secondary. The most common design method for digital IIR filters is based on designing an analogue IIR filter and then converting it to an equivalent digital filter. A digital filter with infinite impulse response (IIR), can be designed by first transforming it into a prototype analog filter and then design this analog filter using a standard procedure. Once the analog filter is properly designed, it is then mapped back to the discrete-time domain to obtain a digital filter that meets the specifications. The commonly used analog filters are

1.Butterworth filters – no ripples at all,
2. Chebychev filters - ripples in the passband OR in the stopband, and
3. Elliptical filters - ripples in BOTH the pass and stop bands.

The design of these filters are well documented in the literature. A disadvantage of IIR filters is that they usually have nonlinear phase. Some minor signal distortion is a result.

There are two main techniques used to design IIR filters:

1. The Impulse Invariant method, and
2. The Bilinear transformation method.

Both methods will lead to a discrete time IIR filter which matches the constraints for different applications accurately. We will show advantages and disadvantages of impulse invariant design and Bilinear Transformation.

II. METHODS

A. *Specifications*

The design process of a filter consists of three steps

1) Specificationof the filter
2)Approximationof the specifications by a discrete time system
3)Realizationof the designed filter

This article will focus on the approximation process of given specifications especially on low-pass filters. All of the commonly used filter types are well described in continuous time frequency domain. Therefore it is easier to transform the specifications from discrete to continuous time domain and design the filter as a continuous system.

The specification for passband can be
$\delta_1 = 0.01$ which is the tolerance range in the passband
$\Omega_P = 2\pi(2000)$ which is the cut off frequency of the passband .

The specification for passband can be
$\delta_2 = 0.001$ which is the tolerance range in the passband
$\Omega_S = 2\pi(3000)$ which is the cut off frequency of the passband .

The δ can understood as upper and lower limit of gain in passband and stopband. It means that

$(1 - \delta_{1}) \leq |H(e^{jw})| \leq (1+ \delta_{1})$ (1)

With

$$\Omega = wT \quad (2)$$

Where w is the dicrete frequency , Ω is the continous frequency and T is the sampling Time.
$W_P = 2\pi(2000) . 10^{-4} = 0.4 \pi$ radians
For Stop Band
$|H(e^{jw})| \leq \delta_2 \quad w_S \leq |w| \leq w_P$ (3)
$W_S = 2\pi(3000).10^{-4} = 0.6 \pi$ radians

With this method it is easy to transform discrete time specifications to continous time domain. For all calculation a bandlimited input signal is assumed and therefore aliasing is negligible[4].

B. *Impulse invariant Technique*

In the impulse invariant method, the impulse response of the digital filter, $h[n]$, is made (approximately) equal to the impulse response of an analog filter, $h_C(t)$, evaluated at $t = nT_d$, where T_d is an (abitrary) sampling period. Specifically

$$h(n) = T_d h_C(nT_d)$$

This method is based on the principle that the behavior of the discrete time system to the discrete time impulse function should be equal to the response of the continous time system to the continous time impulse functionwhen sampled with a given T_d.1 gives an example of periodic sampling. The sampling interval Td has no relevance but for better understanding, we take it in our derivations. The relationship between continuous time frequency response and discrete time frequency response is

$$H(e^{j\omega}) = \sum_{k=-\infty}^{\infty} H_c\left(j\frac{\omega}{T_d} + j\frac{2\pi}{T_d}k \right) \quad (4)$$

and for band limited signals the relationship is shorter.

$$H(e^{j\omega}) = H_c\left(j\frac{\omega}{T_d} \right), \qquad |\omega| \leq \pi \quad (5)$$

The discrete and continous frequency response are linearly related by $w = \Omega T_d$ for $|w| \leq \pi$. If the input signal is not ideally bandlimited, aliasing occur if the transit frequency in continous time domain is reasonably low, aliasing can be neglected. If we assume that all poles of $H_c(s)$ are single order. We can write the transfer function as

$$H_c(s) = \sum_{k=1}^{N} \frac{A_k}{s - s_k}, \quad (6)$$

For $t \geq 0$ the impulse response is equal to

$$h_c(t) = \sum_{k=1}^{N} A_k e^{s_k t}, \quad (7)$$

Now the continous time impulse response is sampled at equal distances T_d the discrete Impulse response is

$$h[n] = T_d h_c(nT_d) = \sum_{k=1}^{N} T_d A_k (e^{s_k T_d}) u[n], \quad (8)$$

With this impulse response the discrete transfer function can be calculated to

$$H(z) = \sum_{k=1}^{N} \frac{T_d A_k}{1 - e^{s_k T_d} z^{-1}}, \quad (9)$$

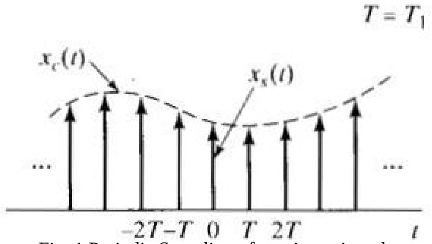

Fig. 1 Periodic Sampling of continous impulse response.

C. *Bilinear Transformation Technique*

In the impulse invariant method, aliasing occurs when the prototype analog filter is transformed back into the digital filter. To reduce the distortion introduced by aliasing, we start off by tightening the specifications on the digital filter. This is somewhat cumbersome and may lead to several iterations before the "optimal" filter is found.

Aliasing occurs because points in the W axis separated by $2\pi / Td$ are mapped into the same digital frequency w . In the Bilinear transformation method, there is a one-to-one correspondence between Ω and w . So aliasing is avoided in transforming the prototype analog filter back into the digital filter[5].

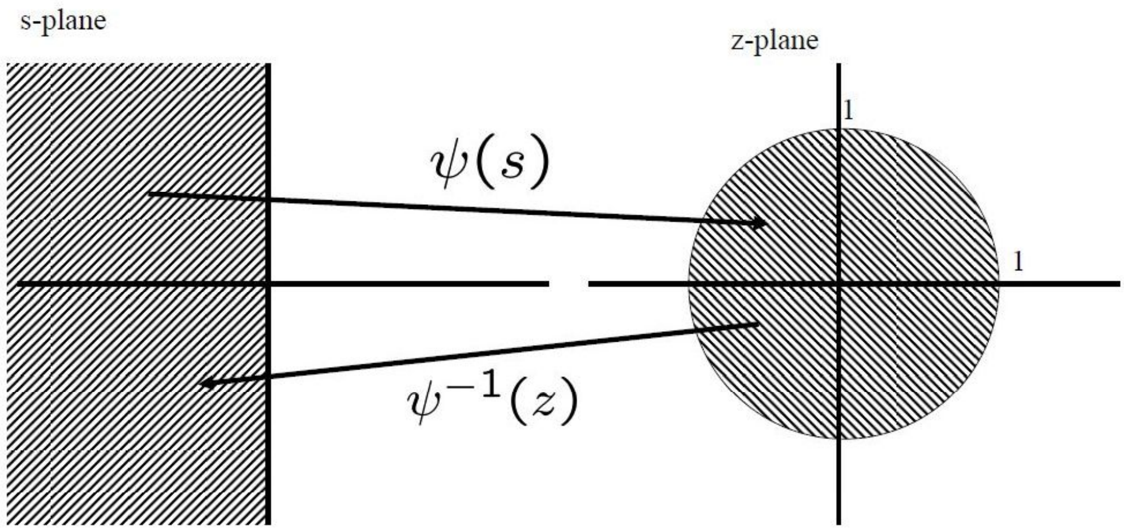

Fig.2 Mapping of S plane to Z plane

It can be shown that the transition from discrete to continous time domain makes Filter design much easier because of the well known methods operating with Bode diagram. The transformation is necessary because H(z) = $H(e^{jwT_d})$ cannot be treated as usual[8]. Applying 11 enables these methods. Only a frequency scaling occurs and has to be corrected. In this case q is approximately equal to s, known from laplace transformation. 2 shows the relationship between s and z. The transformation maps the entire z unit circle to the jΩ axis in the q domain and the inner of the unit circle tho the left q half-plane. The relationship between z and q is

$$z = \frac{1 + \frac{T_d}{2} q}{1 - \frac{T_d}{2} q}, \tag{10}$$

For the exact derivation of the bilinear transformation see [2]. The influence of T_d will cancel when transforming the Specifications and afterwards back transforming the system. The back transformation from q to z is

$$q = \frac{2}{T_d}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right), \tag{11}$$

The relationship between discrete and continous frequency are

$$\Omega = \frac{2}{T_d}\tan\left(\frac{\omega}{2}\right), \tag{12}$$

And

$$\omega = 2arc\tan\left(\frac{T_d \Omega}{2}\right), \tag{13}$$

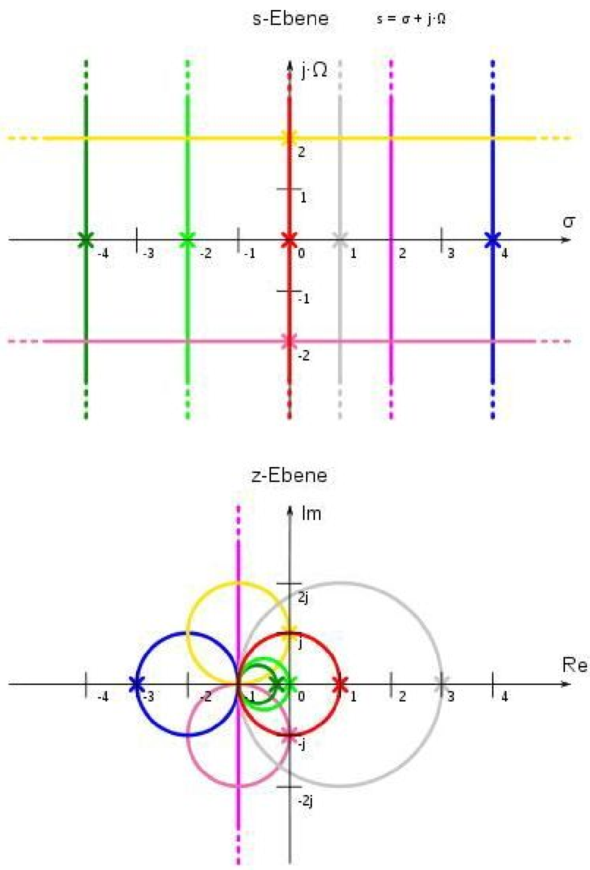

Fig 3 Bilinear Transformation for small frequencies

III. RESULTS

Impulse invariant filter design neglects aliasing and therefore can be seen only as an approximation of the continuous transfer function. In contrast the Bilinear Transformation avoids this aliasing problem but therefore there is a non-linear relationship between discrete and continuous frequency[7]. On stability issues, both of them are good. It can be shown, that by design with impulse invariance, the left-sided poles are transformed into the unit-circle in z domain which means that a stable contin-uous filter will result in a stable discrete filter. Bilinear Transformation maps the imaginary axis onto the unit-circle and the left half-plane into the circle which also causes stable system to remain stable. When designing an impulse invariant system poles and zeros are not being mapped the same way. Poles are corresponding with $z_k = e^{s_k T_d}$ and zeros are dependent on $T_d A_k$. Whereas when using Bilinear Transformation, the whole q-plane is mapped to the z-plane in the way mentioned above, there will be no difference between poles and zeros. Another interesting fact is, that Bilinear Transformation applied to a continuous differentiator will not lead to a discrete differentiator but when designing with impulse invariance and for a sufficiently band limited input signal the discrete system will be a differentiator [3]

IV. CONCLUSION

We cannot say that one of the methods described above is better than the other. It is depending on the filter needed. Bilinear Transformation can only be applied when the non linear relationship between the frequencies is not relevant or can be compensated. An example for this can be filters with piece-wise constant magnitude. Another problem is, when twisting the imaginary axis onto the z-unit-circle, frequency distortion occurs when w reaches $(\pi/T_d\Omega)$. Precautions have to be taken to avoid this [2].

Both methods can be used for filter design but the results are different. With Bilinear Transformation it is not possible to maintain the continuous frequency response whereas with impulse invariance for band limited signals the frequency response can be approximated very well.If it is important to preserve frequency characteristics im-pulse invariance design is preferred. The drawback of this design method is that only the impulse response of the system can be controlled. This concept can be extended to step response invariant design to control only the step response of a filter. The costs of more flexibility with Bilinear Transformation are the non lin-ear relationship between frequencies and the different frequency response of the discrete system.

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