Chemically Reacting, Radiating and Rotating MHD Convective Flow of Visco-Elastic Fluid through Porous Medium in Vertical Channel

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Abstract: An analytical study of radiating and chemically reacting magnetohydrodynamic (MHD) convective flow of a viscoelastic, incompressible and electrically conducting fluid through a porous medium filled in a vertical channel is carried out. The fluid and the channel rotate in unison with an angular velocity about the axis normal to the planes of the plates of the channel. A magnetic field of uniform strength is applied along the axis of rotation. The fluid is acted upon by periodic time variation of the pressure gradient in the vertically upward direction. The temperature of one of the plates is non-uniform and the temperature difference of the walls of the channel is high enough to induce heat transfer due to radiation. An exact analytical solution of the problem is obtained. Two cases of small and large rotations have been considered to assess the effects of different parameters involved in the flow problem. The velocity, temperature, species concentration, the respective amplitudes and the phase angles of the skin friction, Nusselt number and Sherwood number are shown graphically and discussed in detail.

Keywords: Visco-elastic, reacting, convection, magnetohydrodynamic (MHD), rotating, heat radiation.

I. INTRODUCTION

The magnetohydrodynamic (MHD) rotating flow of electrically conducting viscoelastic incompressible fluids have gained considerable attention because of its numerous applications in cosmical and geophysical fluid dynamics. The subject of geophysical dynamics nowadays has become an important branch of fluid dynamics due to the increasing interest to study environment. In geophysics it is applied to measure and study the positions and velocities with respect to a fixed frame of reference on the surface of earth which rotate with respect to an inertial frame in the presence of its magnetic field. In astrophysics it is applied to study the stellar and solar structure, inter planetary and inter stellar matter, solar storms and flares etc. During the last few decades it also finds its application in engineering. Among the applications of rotating flow in porous media to engineering disciplines, one can find the food processing industry, chemical process industry, centrifugation and filtration processes and rotating machinery. In recent years a number of studies have also appeared in the literature on the fluid phenomena on earth involving rotation to a greater or lesser extent viz. Vidyanidhu and Nigam [1] Gupta [2] Jana and Datta [3]. Mazumder [4] obtained an exact solution of an oscillatory Couette flow in a rotating system. Thereafter Ganapathy [5] presented the solution for rotating Couette flow. Singh [6] analyzed the oscillatory magnetohydrodynamic (MHD) Couette flow in a rotating system in the presence of transverse magnetic field. Singh [7] also obtained an exact solution of magnetohydrodynamic (MHD) mixed convection flow in a rotating vertical channel with heat radiation.The study of the flows of visco-elastic fluids is important in the fields of petroleum technology and in the purification of crude oils. In recent years, flows of visco-elastic fluids attracted the attention of several scholars in view of their practical and fundamental importance associated with many industrial applications. Literature is replete with the various flow problems considering variety of geometries such as Rajgopal [8-9], Rargopal and Gupta [10-11], Ariel [12], Pop and Gorla [13]. Hayat et al [14] discussed periodic unsteady flows of a non-Newtonian fluid. Choudhury and Das [15] studied the oscillatory viscelastic flow in a channel filled with porous medium in the presence of radiative heat transfer. Singh [16] analyzed viscoelastic mixed convection MHD oscillatory flow through a porous medium filled in a vertical channel. Taking the rotating frame of reference into account Puri [17] investigated rotating flow of an elastic-viscous fluid on an oscillating plate. Puri and Kulshrestha [18] analyzed rotating flow of non-Newtonian fluids. Rajgopal [19] investigated flow of viscoelastic fluids between rotating disks. Applying quasilinearization to the problem Verma et al [20] analyzed steady laminar flow of a second grade fluid between two rotating porous disks. Hayat et al [21] studied fluctuating flow of a third order fluid on a porous plate in a rotating medium. Hayat et al [22] investigated the unsteady hydromagnetic rotating flow of a conducting second grade fluid.

Chemical reactions usually accompany a large amount of exothermic and endothermic reactions. These characteristics can be easily seen in a lot of industrial processes. Recently, it has been realized that it is not always permissible to neglect the convection effects in porous constructed chemical reactors. The reaction produced in a porous medium was extraordinarily in common, such as the topic of PEM fuel cells modules and the polluted underground water because of discharging the toxic substance, etc. Muthucumaraswamy and Ganesan [23] studied effect of the chemical reaction and injection on flow characteristics in an unsteady upward motion of an isothermal plate. Deka *et al.* [24] studied the effect of the first order homogeneous chemical reaction on the process of an unsteady flow past an infinite vertical plate with a constant heat and mass transfer. Chamkha [25] assumed that the plate is embedded in a uniform porous medium and moves with a constant velocity in the flow direction in the presence of a transverse magnetic field. Kesavaiah ***et.al*** [26] studied the effects of the chemical reaction and radiation absorption on an unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in a porous medium with heat source and suction. Rajput and Kumar [27] Radiation effects on MHD flow past an impulsively started vertical plate with variable heat and mass transfer. Kandasamy et al. [28] studied the nonlinear MHD flow, with heat and mass transfer characteristics, of an incompressible, viscous, electrically conducting, Boussinesq fluid on a vertical stretching surface with chemical reaction and thermal stratification effects. Muthucumaraswamy et al. [29] studied effects on first order chemical reaction on flow past an accelerated isothermal vertical plate in a rotating fluid with variable mass diffusion. Devika et al. [30] analyzed MHD oscillatory flow of a visco elastic fluid in a porous channel with chemical reaction. Singh and Kumar [31] investigate fluctuating heat and mass transfer on unsteady MHD free convection flow of radiating and reacting fluid past a vertical porous plate in slip-flow regime. Chand et al. [32] analyzed Hall effect on radiating and chemically reacting MHD oscillatory flow in a rotating porous vertical channel in slip flow regime.

The aim of the present analysis is to study MHD convective flow of an electrically conducting viscoelastic incompressible fluid flow through a porous medium in a vertical channel in the presence of chemical reaction and thermal radiation. The entire system rotates about an axis perpendicular to the planes of the plates of the channel and a uniform magnetic field is also applied along this axis of rotation. The magnetic Reynolds number is assumed to be small enough so that the induced magnetic field is neglected. The non-uniform temperature and species concentration at two different plates of the channel varies periodically with time. A closed form solution of the heat and mass transfer flow is obtained and the results are discussed with the help of graphs.

II. BASIC EQUATIONS

In order to derive basic equations for the problem under consideration following assumptions are made:

(i) The flow considered is unsteady and laminar.
(ii) The fluid is finitely conducting and with constant physical properties.
(iii) A magnetic field of uniform strength is applied normal to the flow.
(iv) The magnetic Reynolds number is taken to be small enough so that the induced magnetic field is neglected.
(v) Hall effect, electrical and polarization effects are neglected.
(vi) It is assumed that the fluid is optically thin with relatively low density.
(vii) The entire system (consisting of channel plates and the fluid) rotates about an axis perpendicular to the plates.
(viii) Since plates are infinite so all physical quantities except pressure depend only on z^* and t^*.

Under these assumptions, we write hydromagnetic equations of continuity, motion and energy in a rotating frame of reference as:

$$\nabla . V = 0, \tag{1}$$

$$\frac{\partial V}{\partial t} + (V.\nabla V) + 2\Omega \times V = \nabla . \exists - \frac{\vartheta_1}{K^*} V + \frac{1}{\rho} (J \times B) + F, \tag{2}$$

$$\frac{\partial T^*}{\partial t^*} + (V.\nabla)T^* = \frac{k}{\rho c_p}\nabla^2 T^* - \frac{\nabla q}{\rho c_p}, \tag{3}$$

$$\frac{\partial C^*}{\partial t^*} + (V.\nabla)C^* = D\nabla^2 C^* - K_r(C^* - C_1). \tag{4}$$

In equation (2) the last term on the left hand side is the Coriolis force. On the right hand side of (2) the last term $\boldsymbol{F} = g\beta(T^* - T_1) + g\beta^*(C^* - C_1)$ accounts for the force due to buoyancy and the second last term is the Lorentz force due to magnetic field **B** and is given by

$$\boldsymbol{J} \times \boldsymbol{B} = \sigma(\boldsymbol{V} \times \boldsymbol{B}) \times \boldsymbol{B}\,. \tag{5}$$

In the first term on the R. H. S. of equation (2), $\exists$ is the Cauchy stress tensor and the constitutive equation derived by Coleman and Noll [33] for an incompressible homogeneous fluid of second order is

$$\exists = -p^* I + \mu_1 A_1 + \mu_2 A_2 + \mu_3 A_1^2. \tag{6}$$

Here $-p^* I$ is the interdeterminate part of the stress due to constraint of incompressibility, μ_1, μ_2 and μ_3 are the material constants describing viscosity, elasticity and cross-viscosity respectively. The kinematic A_1 and A_2 are the Rivelen Ericson constants defined as

$$A_1 = (\nabla \bar{V}) + (\nabla \bar{V})^T, \quad A_2 = \frac{dA_1}{dt} + (\nabla \bar{V})^T A_1 + A_1(\nabla \bar{V}),$$

where ∇ denotes the gradient operator and d/dt the material time derivative. According to Markovitz and Coleman [34] the material constants μ_1, μ_3 are taken as positive and μ_2 as negative. The modified pressure $p^* = p' - \frac{\rho}{2}|\Omega \times \boldsymbol{R}|^2$, where **R** denotes the position vector from the axis of rotation, p' denotes the fluid pressure, ***J*** is the current density and all other quantities have their usual meanings and have been defined in the text time to time.

The last term in equation (3) stands for heat due to radiation and is given by

$$\frac{\partial q^*}{\partial z^*} = 4a^*\sigma^*(T^{*4} - T_1^4), \tag{7}$$

for the case of an optically thin gray gas. Here a^* is the mean absorption coefficient and σ^* is Stefan- Boltzmann constant. We assume that the temperature differences within the flow are sufficiently small such that T^{*4} may be expressed as a linear function of the temperature. This is accomplished by expanding T^{*4} in a Taylor series about T_1 and neglecting higher order terms, thus

$$T^{*4} \cong 4T_1^{*3}T^* - 3T_1^4. \tag{8}$$

Substituting (8) into (7) and simplifying, we obtain

$$\frac{\partial q^*}{\partial z^*} = 16a^*\sigma^* T_1^3(T^* - T_1). \tag{9}$$

The substitution of equation (9) into the energy equation (3) for the heat due to radiation, we get

$$\frac{\partial T^*}{\partial t^*} + (V.\nabla)T^* = \frac{k}{\rho c_p}\nabla^2 T^* - \frac{16a^*\sigma^* T_1^3}{\rho c_p}(T^* - T_1). \tag{10}$$

III. FORMULATION OF THE PROBLEM

We consider an unsteady flow of a viscoelastic (second order) incompressible and electrically conducting fluid bounded by two infinite insulated vertical plates distance 'd' apart as shown in Fig.1. A coordinate system is chosen such that the X^* -axis is oriented upward along the centerline of the channel and Z^*-axis taken perpendicular to the planes of the plates lying in $z^* = \pm\frac{d}{2}$ planes. The non-uniform temperature of the plate at $z^* = +\frac{d}{2}$ and the species concentration at the plate $z^* = -\frac{d}{2}$ are respectively assumed to be varying periodically with time. The Z^*- axis is considered to be the axis of rotation about which the fluid and the plates are assumed to be rotating as a solid body with a constant angular velocity $\mathbf{\Omega}$ (0. 0, Ω^*). A transverse magnetic field of uniform strength $\mathbf{B}$ (0, 0, B_0) is also applied along the axis of rotation. The velocity may reasonably be assumed with its components along x^*, y^*, z^* directions as $\mathbf{V}$ (u^*, v^*, 0). Since the plates are infinite in X*-direction so all physical quantities except pressure depend only on $z^*$and t^*. The equation of continuity (1) is then satisfied identically for non-porous plates. A schematic diagram of the physical problem considered is shown in Figure 1.

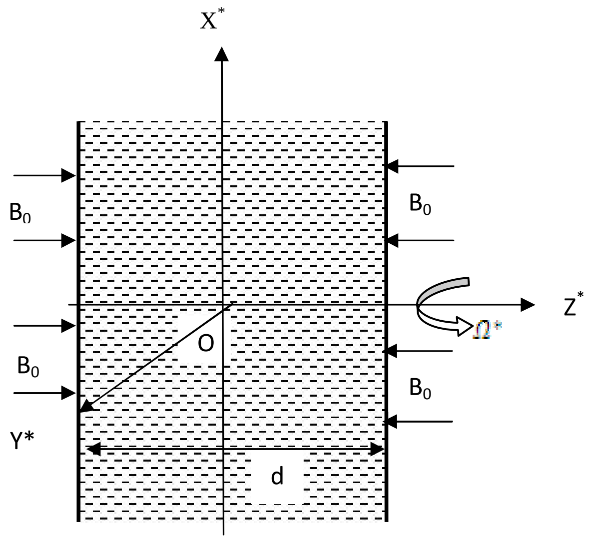

Fig.1. Physical configuration of the physical problem.

Using the velocity and the magnetic field distribution as stated above the magnetohydrodynamic (MHD) flow in the rotating channel is governed by the following Cartesian equations:

$$\frac{\partial u^*}{\partial t^*} - 2\Omega^* v^* = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \vartheta_1 \frac{\partial^2 u^*}{\partial z^{*2}} + \vartheta_2 \frac{\partial^3 u^*}{\partial z^{*2}\partial t^*} - \frac{\sigma B_0^2}{\rho} u^* - \frac{\vartheta_1 u^*}{K^*} + g\beta(T^* - T_1) + g\beta^*(C^* - C_1), \qquad (11)$$

$$\frac{\partial v^*}{\partial t^*} + 2\Omega^* u^* = -\frac{1}{\rho}\frac{\partial p^*}{\partial y^*} + \vartheta_1 \frac{\partial^2 v^*}{\partial z^{*2}} + \vartheta_2 \frac{\partial^3 v^*}{\partial z^{*2}\partial t^*} - \frac{\sigma B_0^2}{\rho} v^* - \frac{\vartheta_1 v^*}{K^*}, \qquad (12)$$

$$0 = -\frac{1}{\rho}\frac{\partial p^*}{\partial z^*}, \qquad (13)$$

$$\frac{\partial T^*}{\partial t^*} = \frac{k}{\rho c_p}\frac{\partial^2 T^*}{\partial z^{*2}} - \frac{16a^*\sigma^* T_1^3}{\rho c_p}(T^* - T_1), \qquad (14)$$

$$\frac{\partial C^*}{\partial t^*} = D\frac{\partial^2 C^*}{\partial z^{*2}} - K_r(C^* - C_1), \qquad (15)$$

where ρ is the density, ϑ_1 is the kinematic viscosity, ϑ_2 is the viscoelasticity, p^* is the modified pressure, t^* is the time, σ is the electric conductivity, g is the acceleration due to gravity, β is the volumetric coefficient of thermal expansion, β^* is the volumetric coefficient of expansion with concentration, k is the thermal conductivity, c_p is the specific heat at constant pressure, D is the molecular diffusivity and K_r is the chemical reaction. Equation (13) shows the constancy of the hydrodynamic pressure along the axis of rotation.

The boundary conditions for the problem are

$$z^* = -\frac{d}{2}: \quad u^* = v^* = 0, \; T^* = T_1, C^* = C_1 + (C_2 - C_1)\cos\omega^* t^*, \tag{16}$$
$$z^* = \frac{d}{2}: \quad u^* = v^* = 0, \; T^* = T_1 + (T_2 - T_1)\cos\omega^* t^*, C^* = C_1, \tag{17}$$

where ω^* is the frequency of oscillations.

Introducing the following non-dimensional quantities into equations (11) and (15)

$$x = \frac{x^*}{d}, \; y = \frac{y^*}{d}, z = \frac{z^*}{d} \; t = \omega^* t^*, u = \frac{u^*}{U}, \; v = \frac{v^*}{U}, \; \theta = \frac{T^* - T_1}{T_2 - T_1}, C = \frac{C^* - C_1}{C_2 - C_1}, \omega = \frac{\omega^* d^2}{\vartheta_1}, p = \frac{dp^*}{\rho \vartheta_1 U}, \tag{18}$$

we get

$$\omega\frac{\partial u}{\partial t} - 2\Omega v = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial z^2} + \gamma\omega\frac{\partial^3 u}{\partial z^2 \partial t} - (M^2 + K^{-1})u + Gr\,\theta + Gm\,C, \tag{19}$$
$$\omega\frac{\partial v}{\partial t} + 2\Omega u = -\frac{\partial p}{\partial y} + \frac{\partial^2 v}{\partial z^2} + \gamma\omega\frac{\partial^3 v}{\partial z^2 \partial t} - (M^2 + K^{-1})v, \tag{20}$$
$$\omega Pr\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial z^2} - N^2\theta, \tag{21}$$
$$\omega Sc\frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial z^2} - K_r Sc C, \tag{22}$$

where U is the mean axial velocity, '*' represents the dimensional physical quantities,

$\Omega = \frac{\Omega^* d^2}{\vartheta_1}$ is the rotation parameter,

$\gamma = \frac{\upsilon_2}{d^2}$ is the visco-elastic parameter,

$Gr = \frac{g\beta d^2 (T_2 - T_1)}{\vartheta_1 U}$ is the Grashof number,

$Gm = \frac{g\beta^* d^2 (C_2 - C_1)}{\vartheta_1 U}$ is the modified Grashof number,

$M = B_0 d\sqrt{\frac{\sigma}{\rho\vartheta_1}}$ is the Hartmann number,

$K = \frac{K^*}{d^2}$ is the permeability of the porous medium,

$Pr = \frac{\rho\vartheta_1 c_p}{k}$ is the Prandtl number,

$N = 4d\sqrt{\frac{a^*\sigma^* T_1^3}{k}}$ is the radiation parameter,

$Sc = \frac{\vartheta_1}{D}$ is the Schmidt number,

$K_r = \frac{K_r^* d^2}{\vartheta_1}$ is the chemical reaction parameter.

The boundary conditions in the dimensionless form become

$$z = -\frac{1}{2}: \quad u = v = 0, \; \theta = 0, C = \cos t, \tag{23}$$
$$z = \frac{1}{2}: \quad u = v = 0, \; \theta = \cos t, C = 0. \tag{24}$$

We shall assume now that the fluid flows under the influence of pressure gradient varying periodically with time in the X^*-axis only is of the form

$$-\frac{\partial p}{\partial x} = A\cos t \quad \text{and} \quad -\frac{\partial p}{\partial y} = 0, \tag{25}$$

where A is a constant.

IV. SOLUTION OF THE PROBLEM

Now combine equations (19) and (20) into single equation by introducing a complex function F = u + iv, we get

$$\omega\frac{\partial F}{\partial t} + 2i\Omega F = A\cos t + \frac{\partial^2 F}{\partial z^2} + \gamma\omega\frac{\partial^3 F}{\partial z^2 \partial t} - (M^2 + K^{-1})F + Gr\,\theta + Gm\,C. \tag{26}$$

In order to solve the problem it is convenient to adopt complex notations and assume the solution of the problem as

$$F(z,t) = F_0(z)\, e^{it}, \quad \theta(z,t) = \theta_0(z)\, e^{it}, C(z,t) = C_0(z) e^{it} \quad -\frac{\partial y}{\partial x} = A \cos t = Ae^{it}. \tag{27}$$

with corresponding boundary conditions as

$$z = -\tfrac{1}{2}: \quad F = 0, \quad \theta = 0, C = \cos t, \tag{28}$$

$$z = \tfrac{1}{2}: \quad F = 0, \quad \theta = \cos t, C = 0. \tag{29}$$

The boundary conditions (28) and (29) in complex notations can also be written as

$$z = -\tfrac{1}{2}: \quad F = 0, \quad \theta = 0, C = e^{it}, \tag{30}$$

$$z = \tfrac{1}{2}: \quad F = 0, \quad \theta = e^{it}, C = 0. \tag{31}$$

Substituting expressions (27) in equations (26), (21) and (22), we get

$$a^2 \frac{d^2 F_0}{dz^2} - m^2 F_0 = -A - Gr\, \theta_0 - Gm\, C_0\,, \tag{32}$$

$$\frac{d^2\theta_0}{dz^2} - n^2\theta_0 = 0, \tag{33}$$

$$\frac{d^2 C_0}{dz^2} - l^2\theta_0 = 0\,, \tag{34}$$

where $\quad a = \sqrt{1 + i\omega\gamma}\,, \; l = \sqrt{Sc\,(K_r + i\omega)}, \quad m = \sqrt{M^2 + K^{-1} + 2i\Omega + i\omega}\,, \quad n = \sqrt{N^2 + i\omega Pr}\,.$

The transformed boundary conditions reduce to

$$z = -\tfrac{1}{2}: \quad F_0 = 0, \quad \theta_0 = 0, C_0 = 1, \tag{35}$$

$$z = \tfrac{1}{2}: \quad F_0 = 0, \quad \theta_0 = 1, C_0 = 0. \tag{36}$$

The ordinary differential equations (32), (33) and (34) are solved under the boundary conditions (35) and (36) for the velocity, temperature and species concentration fields. The solution of the problem is obtained as

$$F(z,t) = \left[\frac{A}{m^2}\left(1 - \frac{\cosh \frac{m}{a}z}{\cosh \frac{m}{2a}}\right) + \frac{Gr}{(a^2n^2 - m^2)}\left\{\frac{\sinh \frac{m}{a}\left(\frac{1}{2}+z\right)}{\sinh \frac{m}{a}} - \frac{\sinh n\left(\frac{1}{2}+z\right)}{\sinh n}\right\} + \frac{Gm}{(a^2 l^2 - m^2)}\left\{\frac{\sinh \frac{m}{a}\left(\frac{1}{2}-z\right)}{\sinh \frac{m}{a}} - \frac{\sinh l\left(\frac{1}{2}-z\right)}{\sinh l}\right\}\right] e^{it}\,, \tag{37}$$

$$\theta(z,t) = \frac{\sinh n\left(\frac{1}{2}+z\right)}{\sinh n} e^{it}\,, \tag{38}$$

$$C(z,t) = \frac{\sinh l\left(\frac{1}{2}-z\right)}{\sinh l} e^{it}. \tag{39}$$

The primary velocity is given by the real part of complex function F(z,t).

From the velocity field we can now obtain the skin-friction τ_L at the left plate in terms of its amplitude and phase angle as

$$\tau_L = \left(\frac{\partial F}{\partial z}\right)_{z=-\frac{1}{2}} = |F| \;\cos(t + \varphi)\;, \tag{40}$$

with $F_r + i\, F_i = \frac{A}{ma}\tanh \frac{m}{2a} + \frac{Gr}{(a^2n^2 - m^2)}\left\{\frac{\frac{m}{a}}{\sinh\frac{m}{a}} - \frac{n}{\sinh n}\right\} - \frac{Gm}{(a^2 l^2 - m^2)}\left\{\frac{m}{a}\coth\frac{m}{a} - l\coth l\right\}.$ (41)

The amplitude is $|F| = \sqrt{F_r^2 + F_i^2}$ and the phase angle $\varphi = \tan^{-1}\frac{F_i}{F_r}$. (42)

From the temperature field given in equation (38) the heat transfer coefficient Nu (Nusselt number) in terms of its amplitude and the phase angle can be obtained as

$$Nu = \left(\frac{\partial\theta}{\partial z}\right)_{z=-\frac{1}{2}} = |H| \cos(t + \psi), \tag{43}$$

where $H_r + i\, H_i = \frac{n}{\sinh n}$. (44)

The amplitude $|H|$ and the phase angle ψ of the heat transfer coefficient Nu (Nusselt number) are given by

$|H| = \sqrt{H_r^2 + H_i^2}$ and $\psi = \tan^{-1}\left(\frac{H_i}{H_r}\right)$ respectively. (45)

Similarly, The amplitude and the phase angle at the left plate (z=-0.5) of the Sherwood number can be obtained from equation (39) for the species concentration as

$$Sh = \left(\frac{\partial C}{\partial z}\right)_{z=-\frac{1}{2}} = |C| \cos(t + \zeta), \quad (46)$$

where $C_r + iC_i = -l \coth l$. (47)

The amplitude $|C|$ and the phase angle ζ of the heat transfer coefficient Nu (Nusselt number) are given by

$|C| = \sqrt{C_r^2 + C_i^2}$ and $\psi = \tan^{-1}\left(\frac{C_i}{C_r}\right)$ respectively. (48)

V. DISCUSSION OF THE RESULTS

The MHD convective flow in an infinite vertical channel with transverse magnetic field is analyzed when the entire system rotates about an axis perpendicular to the planes of the plates. In the presence of chemical reaction and thermal radiation an exact solution of the problem is obtained. The velocity, temperature and species concentration field and the shear stress, Nusselt number and Sherwood number in terms of their amplitudes and phase angles are evaluated numerically for different sets of the values of rotation parameter Ω, viscoelastic parameter γ, Reynolds number Re, Hartmann number M, Grashof number Gr, modified Grashof number Gm, Prandtl number Pr, radiation parameter N, Schmidt number Sc, reaction parameter K_r, pressure gradient A and the frequency of oscillations ω. To be more realistic the two values Pr=0.7 and 7of the Prandtl number chosen to represent air and water and that of the Schmidt number Sc=0.22 and 0.94 represent Hydrogen and Carbon dioxide respectively. These numerical values are then shown graphically to assess the effect of each parameter for the two cases of small (Ω = 10) and large (Ω = 20) rotations.

Fig.2 illustrates the variation of the velocity with the increasing rotation of the system. It is quite obvious from this figure that velocity goes on decreasing with increasing rotation Ω of the entire system. The velocity profiles initially remain parabolic with maximum at the centre of the channel for small values of rotation parameter Ω=5 and then as rotation increases i.e., Ω=10, the velocity profiles flatten. For further increase in rotation (Ω= 20, 30) the maximum of velocity profiles no longer occurs at the centre but shift towards the walls of the channel. It means that for large rotation there arise boundary layers on the walls of the channel. The effect of the viscoelastic parameter γ on the velocity profiles are shown in Fig. 3. The figure clearly shows that the velocity decreases tremendously with the increasing values of γ for the small (Ω = 10) and large (Ω= 20) rotation of the system. For given values of other parameters the velocity is maximum in the case of Newtonian fluid i.e., γ=0.

The variations of the velocity profiles with the Grashof number Gr are presented in Fig.4. For small (Ω=5) and large (Ω=20) rotations, the velocity increases with increasing Grashof number. The maximum of the parabolic velocity profiles shifts toward right half of the channel due to the greater buoyancy force in this part of the channel due to the presence of hotter plate. The effects of modified Grashof number Gm are presented in Figure 5 and it is observed that the velocity increases with increasing Gm for both the cases of small (Ω=10) and large (Ω=20) rotations of the channel. In this case the maximum of the parabolic velocity profiles shifts toward left half of the channel.

The effects of the magnetic field on the velocity field are depicted in Fig.6. It is observed that with increasing Hartmann number M velocity decreases for small (Ω=10) rotation but increases for large (Ω=20) rotations. This means that increasing Lorentz force due to increasing magnetic field strength resists the backward flow caused by the large rotation of the system. Fig.7 shows the variations of the velocity with the permeability of the porous medium K. It is observed from the figure that the velocity decrease with the increase of K for both small (Ω=10) and large (Ω=20) rotations of the system. We find from Fig.8 that with the increase of Prandtl number Pr the velocity decreases for small (Ω=10) and large (Ω=20) rotations both. Fig.9 shows that for small (Ω=5) and large (Ω=20) rotations both the velocity decreases with increasing radiation parameter N. It is observed from Figs.10 and

11 that the velocity goes on decreasing with the increasing Schmidt number Sc and chemical reaction parameter K_r. The effect of the frequency of oscillations ω on the velocity is exhibited in Fig.12. It is noticed that velocity decreases with increasing frequency ω for either case of channel rotation large or small.

The temperature profiles are shown in Figure 13. It is quite clear from this figure that the temperature decreases with the increase of each of the parameters involved i.e., Prandtl number Pr, radiation parameter N and the frequency of oscillations ω. Similarly, the species concentration also decreases with either of the Schmidt number Sc, reaction parameter K_r and the frequency of oscillations ω as is depicted in Figure 14. The amplitude $|H|$ and the phase angle of the Nusselt number are presented in Figures 15 and 16 respectively. Figure 15 reveals that $|H|$ decreases sharply with the increase of Prandtl number or radiation parameter as frequency ω increases. The amplitude remains almost constant with increasing frequency of oscillations ω. The phase angle ψ of rate of heat transfer shown in figure 16 oscillates between phase lag and phase lead as ω increases. The wave length due to the increase of Prandtl number Pr and the radiation parameter N increases and decreases respectively. The amplitude $|C|$ and phase angle ζ of the Sherwood number are shown in Figures 17 and 18. The amplitude $|C|$ decreases with the increase of Schmidt number Sc and reaction parameter K_r. However, the phase angle ζ increases with Sc but decreases with K_r.

The skin-friction τ_L in terms of its amplitude $|F|$ and phase angle has been shown in Figures. 19 and 20 respectively for the sets of values listed in Table 1. The effect of each of the parameter on $|F|$ and is assessed by comparing each curve with dotted curves I in these figures. In Figure.19 the comparison of the curves IV, V, VII, X and XII with dotted curve I (---) indicate that the amplitude increases with the increase of Grashof number Gr, modified Grashof number Gm, permeability of the porous medium K, the pressure gradient parameter A and chemical reaction K_r. Similarly the comparison of the curves II, III,VI, VIII, IX and XI with dotted curve I depicts that the skin-friction amplitude decreases with the increase of rotation parameter Ω, viscoelastic parameter γ, Hartmann number M, Prandtl number Pr, radiation parameter N and Schmidt number Sc. It is obvious that $|F|$ goes on decreasing with increasing frequency of oscillations ω. From Figure. 20 showing the variations of the phase angle φ of the skin-friction it is clear that there is always a phase lag because the values of remain negative throughout. Comparing curves II, III, IV, V, VII and XI with dotted curve I (---) it is observed that the phase lag increases with the increase of rotation parameter Ω , viscoelastic parameter γ, Grashof number Gr, modified Grashof number Gm, permeability of the porous medium K and Schmidt number Sc. Also the comparison of curves VI, VIII, IX, X and XII with dotted curve I (---) indicate that the phase lag decreases with the increase of Hartmann number M, Prandtl number Pr, radiation parameter N, pressure gradient A and the chemical reaction parameter K_r. Phase lag goes on increasing with increasing frequency of oscillations ω.

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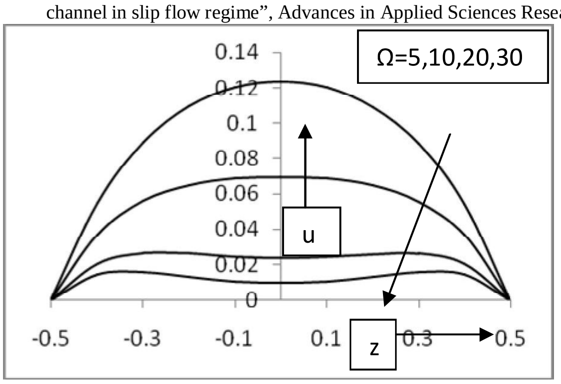

Figure 2. Velocity variation with Ω for γ=0.2, Gr=1, Gm=1, M=2, K=0.2, Pr=0.7, N=1, A=2, Sc=0.22, Kr=1 and t=0.

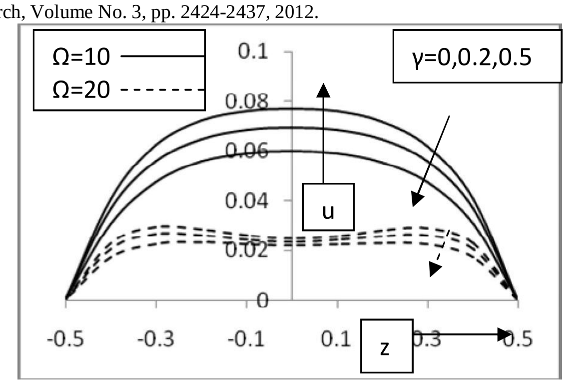

Figure 3. Velocity variation with γ for Ω=10, Gr=1, Gm=1, M=2, K=0.2, Pr=0.7, N=1, A=2, Sc=0.22, Kr=1 and t=0.

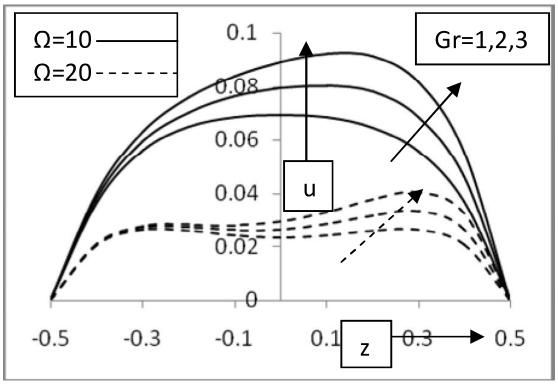

Figure 4. Velocity variation with Gr for Ω=10, γ=0.2, Gm=1, M=2, K=0.2, Pr=0.7, N=1, A=2, Sc=0.22, Kr=1 and t=0.

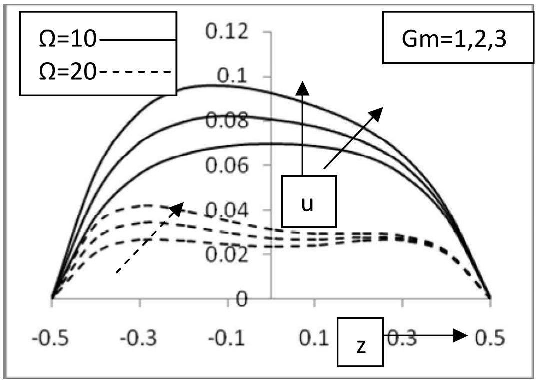

Figure 5. Velocity variation with Gm for Ω=10, γ=0.2, Gr=1, M=2, K=0.2, Pr=0.7, N=1, A=2, Sc=0.22, Kr=1 and t=0.

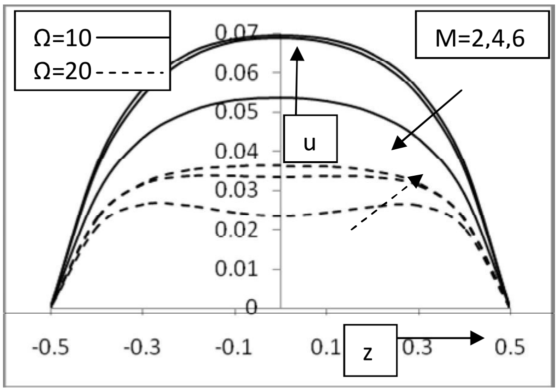

Figure 6. Velocity variation with M for Ω=10, γ=0.2, Gr=1, Gm=1,, K=0.2, Pr=0.7, N=1, A=2, Sc=0.22, Kr=1 and t=0.

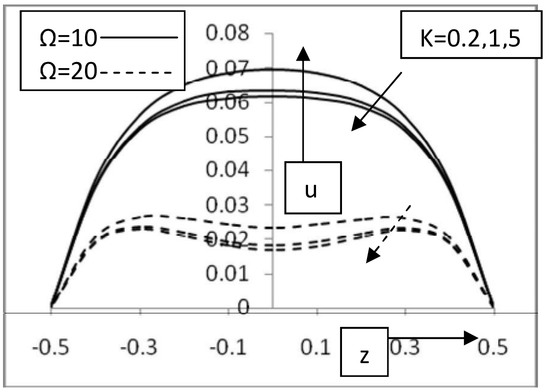

Figure 7. Velocity variation with K for Ω=10, γ=0.2, Gr=1, Gm=1, M=2, Pr=0.7, N=1, A=2, Sc=0.22, Kr=1 and t=0.

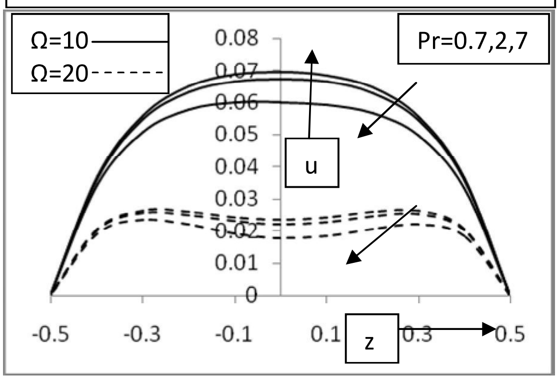

Figure 8. Velocity variation with Pr for Ω=10, γ=0.2, Gr=1, Gm=1, M=2, K=0.2, N=1, A=2, Sc=0.22, Kr=1 and t=0.

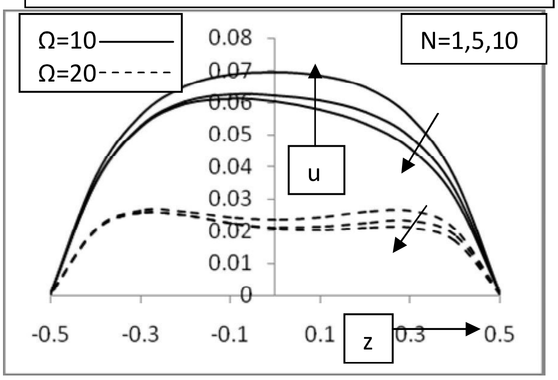

Figure 9. Velocity variation with N for Ω=10, γ=0.2, Gr=1, Gm=1, M=2, K=0.2, Pr=0.7, A=2, Sc=0.22, Kr=1 and t=0.

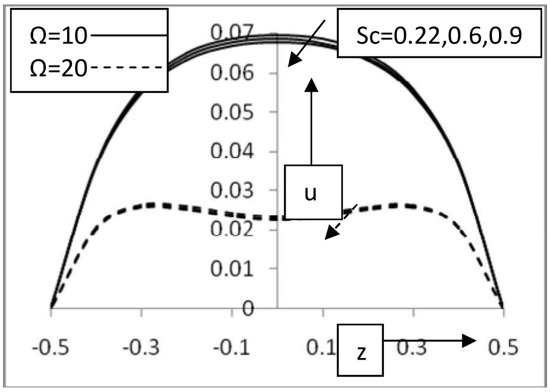

Figure 10. Velocity variation with Sc for Ω=10, γ=0.2, Gr=1, Gm=1, M=2, K=0.2, Pr=0.7, A=2, N=1, Kr=1 and t=0.

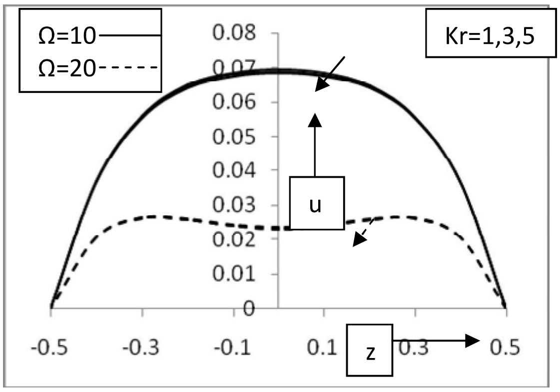

Figure 11. Velocity variation with Kr for Ω=10, γ=0.2, Gr=1, Gm=1, M=2, K=0.2, Pr=0.7, A=2, N=1, Sc=0.22and t=0.

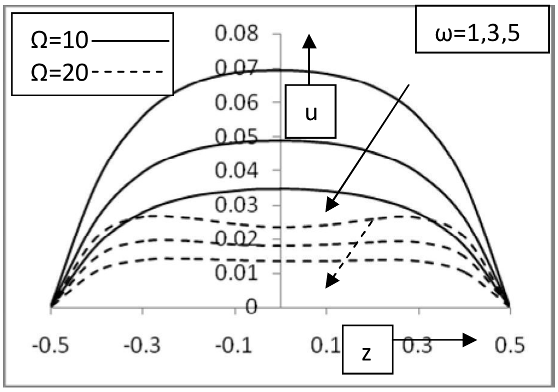

Figure 12. Velocity variation with Kr for Ω=10, γ=0.2, Gr=1, Gm=1, M=2, K=0.2, Pr=0.7, A=2, N=1, Sc=0.22and t=0.

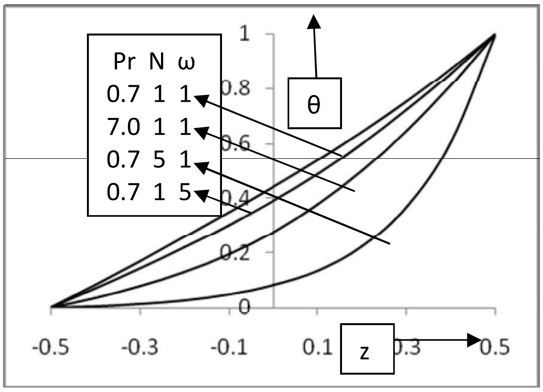

Figure 13. Temperature profiles for t=0.

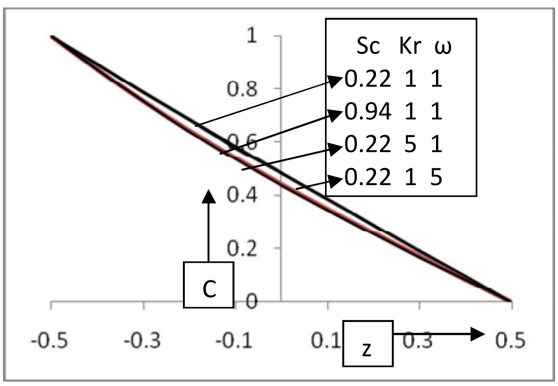

Figure 14. Species concentration for t=0.

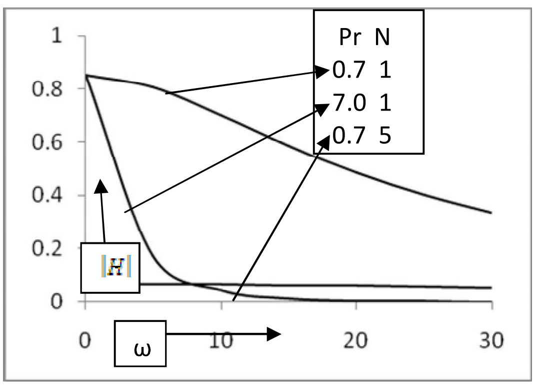

Figure 15. Amplitude of rate of heat transfer.

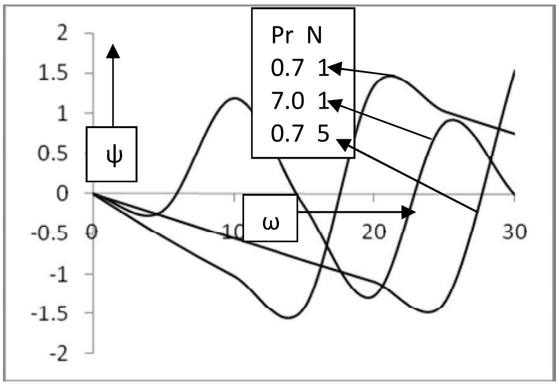

Figure 16. Phase angle of rate of heat transfer.

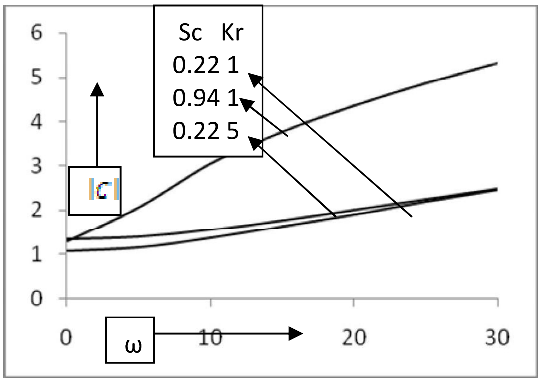

Figure 17. Amplitude of Sherwood number.

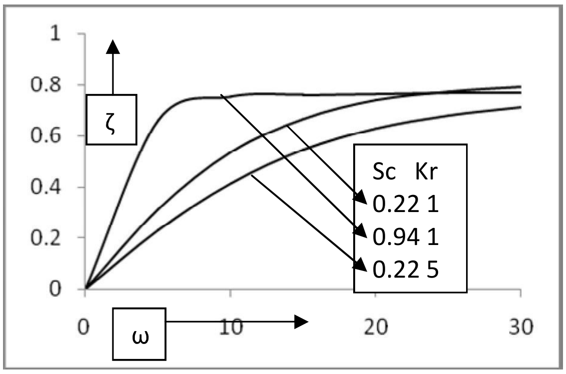

Figure 18. Phase angle of Sherwood number.

Table 1. Sets of parameter values plotted in Figures 19 & 20.

Ω	γ	Gr	Gm	M	K	Pr	N	A	Sc	Kr	Curves
10	0.2	1	1	2	0.2	0.7	1	2	0.22	1	I (---)
20	0.2	1	1	2	0.2	0.7	1	2	0.22	1	II
10	0.3	1	1	2	0.2	0.7	1	2	0.22	1	III
10	0.2	2	1	2	0.2	0.7	1	2	0.22	1	IV
10	0.2	1	2	2	0.2	0.7	1	2	0.22	1	V
10	0.2	1	1	4	0.2	0.7	1	2	0.22	1	VI
10	0.2	1	1	2	1.0	0.7	1	2	0.22	1	VII
10	0.2	1	1	2	0.2	7.0	1	2	0.22	1	VIII
10	0.2	1	1	2	0.2	0.7	5	2	0.22	1	IX
10	0.2	1	1	2	0.2	0.7	1	3	0.22	1	X
10	0.2	1	1	2	0.2	0.7	1	2	0.94	1	XI
10	0.2	1	1	2	0.2	0.7	1	2	0.22	5	XII

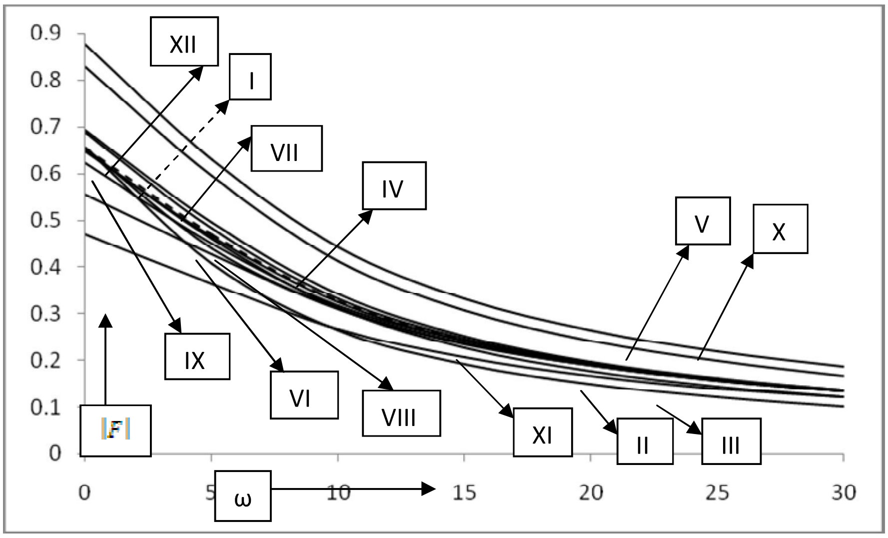

Figure 19. Amplitude of skin-friction.

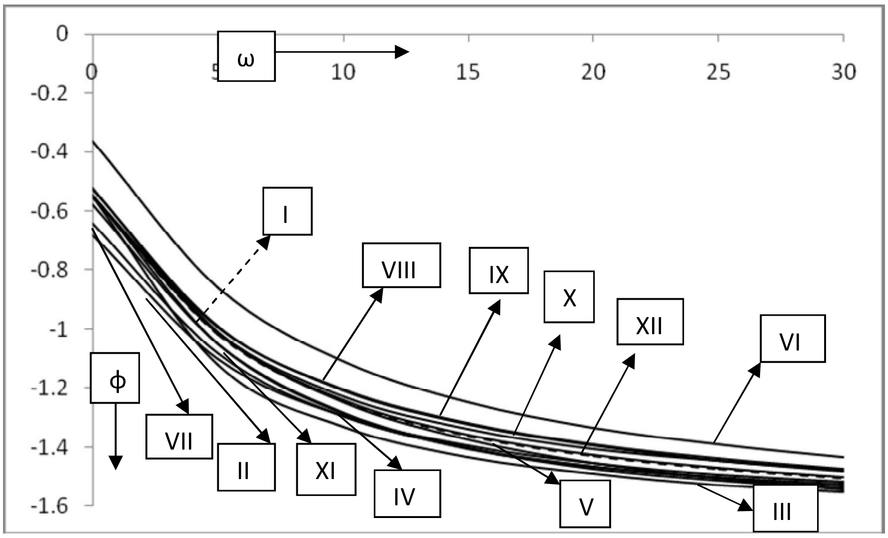

Figure 20. Phase angle of skin friction.