Uncertainty and Disturbance Estimation based Control of Three-axis Stabilized Platform

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Abstract- **This work aims at modelling a stabilized platform system for the ground based applications and design a control system to achieve stable operation of the system in presence of external disturbances. Three member gimbal-frame structure for system model is considered in this article, lagrangian approach is adopted to find out the equations of motion of the stabilized platform system, as this approach presents a simple method to model complex systems. The external disturbances, system non-linearities and the parametric uncertainties are treated as a composite disturbance acting on the system. An Uncertainty and Disturbance Estimator (UDE) is used to estimate the composite disturbance. The estimated disturbance is used to robustify Feedback Linearization based controller designed for nominal system. The closed-loop stability of the system is proved. Simulation results are presented to demonstrate the efficacy of the UDE in estimation of the uncertainties and in meeting the objectives of the design. Lastly, the proposed design is compared with some well-known designs reported in the literature.**

Keywords – Platform Stabilization, Robust Control, Uncertainty and Disturbance Estimator (UDE).

I. INTRODUCTION

In many military applications such as in Search on Move, the payloads are required to be held in stable position throughout their operation. The stabilized platform is an object which can isolate motion of the vehicle, and can measure the change of the platform's motion and position as desired, so that it can make the payload which is fixed on the platform aim at a desired position. In the stabilized platform systems, the basic requirements are to regulate the payload angles even when there are changes in the system dynamics and to have a satisfactory disturbance rejection capability. Inertially stabilized platforms (ISP's) serve the purpose of stabilizing and pointing a broad array of sensors, cameras, telescopes and weapon systems. The concepts and principles of inertially stabilized platforms presented in literature [1] provide information on various techniques of designing stabilized platforms for a versatile applications. The model of Three-Axis Gyro Stabilized Platform discussed in [2] gives a basis for designing ground based stabilization systems to stabilize the motion of the system in three degrees of freedom. On the similar lines the modelling for satellite dish antenna stabilized platforms [3] is discussed where Eulerian approach of modelling is adopted. Owing to the ease of approach that Lagrangian method offers in modelling the complex systems, the modelling of Double-Gimbal Control-Moment Gyro is discussed in [4].

Most of the industrial control systems employ PID control. The PID controller has several important functions: it provides feedback; it has the ability to eliminate steady state errors through integral action; it can anticipate the future though derivative action and thus PID controllers are sufficient for many control problems [5]. Feedback Linearization (FL) is a non-linear control design technique which has attracted research interest in recent years [6]. FL provides satisfactory control of systems under ideal conditions, but in real time scenario in presence of disturbances FL control may not give desired performance. Thus to compensate for disturbances acting on the system many techniques such as Time Delay Control (TDC), Uncertainty and Disturbance Estimator (UDE) approach have appeared in literature. The performance of Input-Output Linearization (IOL) controller has been robustified using TDC approach as discussed by [7]. Techniques like Unknown Input Observer (UIO) [9], Disturbance Observer (DO) [10] are active area of research, to estimate the effects of uncertainties and disturbances acting on the system, following the line of TDC and addressing issues associated with it, a novel Uncertainty and Disturbance Estimation (UDE) technique is proposed in [8]. An application of the UDE in robustifying a feedback linearizing control law for a robot having joint flexibility is presented in [11] wherein the effect of joint flexibility is treated as a disturbance. In [12], the UDE based robust control designs for uncertain non-linear systems with state delays are presented and the authors have shown that, UDE based designs over excellent tracking and disturbance rejection performance. Application of the UDE technique for robustification of the IOL controller is presented in [14], an application of the UDE in robustification of the Input-Output Linearization (IOL) controller is presented wherein the UDE estimated uncertainties are used in robustifying an IOL controller.

In this work, an UDE based robust controller for regulation of 3-axis gimbal platform is proposed. FL controller is designed by considering system non-linearities, external disturbances as a composite quantity. The FL controller is robustified by augmenting it with UDE estimated uncertainty. The paper is organized as follows. In Sec. II, mathematical modelling of the gimbal system is discussed and equations of Motion governing the motion of the stabilized platform are presented, Feedback Linearization based controller design for the present problem is also discussed. In Sec. III, the UDE based controller design for the platform stabilization is discussed whereas closed loop stability of the overall system is presented in Sec. IV. Simulation results demonstrating the effectiveness of the proposed design in presence of external disturbances and parametric uncertainties are given in Sec. V. The results of a comparative study of the proposed design with some of the available designs in literature are presented in Sec. VI, and finally Sec. VII concludes the work.

II. PROBLEM FORMULATION

A. Mathematical Model of the System

The platform stabilization system is a gimbal supported platform as shown in the Fig. 1, it consists of a structure with four nested bodies namely the case, outer gimbal frame, inner gimbal frame and finally the innermost platform. Each body is connected to the subsequent nested frame and free to rotate with one degree of freedom. The case is fixed to the vehicle on which this system is mounted. Case is also connected to the outer gimbal through actuators on either side of the joint and thus the outer gimbal is free to rotate with respect to the case along x-axis. The outer gimbal is connected to the inner gimbal again though actuators and is free to rotate about z-axis. And finally the platform is connected to the inner gimbal by the two ends of the joint through actuators. With external disturbance acting on the gimbal, frame angle deviate from desired accordingly in their respective degrees of freedom.

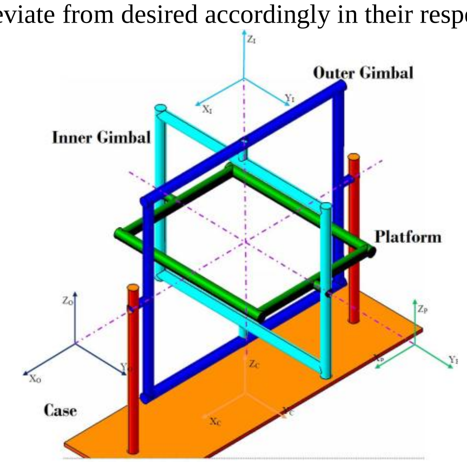


Figure 1. Platform Stabilization System Structure

While developing the mathematical model of the platform stabilization system, certain assumptions are made to simplify the mathematical modelling as below

- All members in the arrangement are rigid.
- The body axes are chosen such that, they are the principal axes at the center of mass of each member. Thus, moment of inertia matrix for each member is a diagonal matrix.
- The effect of gravity not being significant the rate change of potential energy of the system is negligible.

B. System Kinematics

The system under study is as described in Fig.1, with the three gimbal members orthogonal to each other. The relative orientations of the members of the platform stabilization system are represented by the Euler angles and their respective relative angular rates as defined in [2].

- θ - Relative angle between the inner gimbal and the platform, measured about the platform *Y* axis *(Yp)*
- ψ - Relative angle between the outer gimbal and the inner gimbal, measured about the inner gimbal *Z* axis *(Zi)*
- ϕ - Relative angle between the case and the outer gimbal, measured about the outer gimbal *X* axis *(Xo)*

The angular rate of the case is taken as,

$$\omega_v = [p \quad q \quad r]^T \tag{1}$$

The angular rates of the outer frame ω_o, inner frame ω_I and subsequently of the platform ω_p are obtained as,

$$\omega_o = [p + \dot{\phi} \quad qcos\phi + rsin\phi \quad -qsin\phi + rcos\phi]^T \tag{2}$$

$$\omega_I = \begin{bmatrix} (p+\dot{\phi})cos\psi + (qcos\phi + rsin\phi)sin\psi \\ -(p+\dot{\phi})sin\psi + (qcos\phi + rsin\phi)cos\psi \\ -qsin\phi + rcos\phi + \dot{\psi} \end{bmatrix} \tag{3}$$

$$\omega_p = \begin{bmatrix} [(p+\dot{\phi})cos\psi + (qcos\phi + rsin\phi)sin\psi]cos\theta - [-qsin\phi + rcos\phi + \dot{\psi}]sin\theta \\ -(p+\dot{\phi})sin\psi + (qcos\phi + rsin\phi)cos\psi + \dot{\theta} \\ [(p+\dot{\phi})cos\psi + (qcos\phi + rsin\phi)sin\psi]sin\theta + [-qsin\phi + rcos\phi + \dot{\psi}]cos\theta \end{bmatrix} \tag{4}$$

C. System Dynamics

The equations of motion governing the motion of the gimbals system are developed using the Euler-Lagrangian approach. To describe the dynamics of the gimbal system the generalized coordinates are chosen as the roll , pitch and yaw angles *(ϕ, ψ ,θ)*. The Lagrange's Equation for non-conservative systems given by (5) is applied for each generalized coordinate ϕ, ψ and θ as

$$\frac{d}{dt}\left(\frac{\partial K_G}{\dot{q}_i}\right) - \left(\frac{\partial K_G}{q_i}\right) = Q_i \tag{5}$$

Where, K_G is the total kinetic energy of the system. Q_i is the Generalized force arising due to non-conservative forces. The differential equations in [2] representing model of gimbal platform are simultaneous equations in terms of the angular accelerations. After algebraic simplifications mathematical model describing the motion of the gimbals system is obtained as,

$$\ddot{\phi} = \frac{B}{b_2} - \left[\frac{b_3}{b_2}\left(\frac{b_2(Ca_1 - Ac_1 - B(a_1c_2 - c_1a_2))}{b_2^2a_1 - b_3(a_1c_2 - c_1a_2)}\right)\right]; \qquad \ddot{\psi} = \left(\frac{b_2(Ca_1 - Ac_1 - B(a_1c_2 - c_1a_2))}{b_2^2a_1 - b_3(a_1c_2 - c_1a_2)}\right);$$

$$\ddot{\theta} = \frac{A}{a_1} - \frac{a_2}{a_1}\left[\frac{B}{b_2} - \frac{b_3}{b_2}\left(\frac{b_2(Ca_1 - Ac_1 - B(a_1c_2 - c_1a_2))}{b_2^2a_1 - b_3(a_1c_2 - c_1a_2)}\right)\right] \tag{6}$$

where

$a_1 = I_{Py}$

$a_2 = -I_{Py}\, sin(\psi)$; $A = Q_\theta - Q_{Py}$; $b_2 = (I_{PZ} - I_{PX})\, cos(\psi)\, cos(\theta)\, sin(\theta)$;

$b_3 = I_{Iz} + I_{PX}\, sin^2(\theta) + I_{pz}\, cos^2(\theta)$; $B = Q_\psi - QIz$; $c_1 = -I_{Py}\, sin(\psi)$; $C = Q_\phi - Q_{ox}$

$c_2 = I_{ox} + I_{PX}\, cos^2(\psi)\, cos^2(\theta) + I_{Py}\, sin^2(\psi) + I_{pz}\, cos^2(\psi)\, sin^2(\theta) + I_{Ix}\, cos^2(\psi) + I_{Iy}\, sin^2(\psi)$

Here Q_ϕ, Q_ψ , Q_θ are the generalized forces that are external to the system or not derivable from a scalar potential function and defined as,

$$Q_\phi = -D_\phi \dot{\phi} + T_\phi, \quad Q_\psi = -D_\psi \dot{\phi}\psi + T_\psi, \quad Q_\theta = -D_\theta \dot{\theta} + T_\theta$$

D_ϕ, D_ψ , D_θ are the coefficients of viscous friction between the adjacent gimbals system members and T_ϕ, T_ψ , T_θ are the torques inputs acting on the outer gimbal, inner gimbal and platform respectively. Here, k_1- k_9 can be found out by simple algebraic manipulation. Further simplifying (6)

$$\begin{bmatrix} \ddot{\phi} \\ \ddot{\psi} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} k_1(Q_\psi - Q_{Iz}) + k_2(Q_\theta - Q_{Py}) - k_3(Q_\phi - Q_{ox}) \\ k_4(Q_\phi - Q_{ox}) - k_5(Q_\theta - Q_{Py}) - k_6(Q_\psi - Q_{Iz}) \\ k_7(Q_\theta - Q_{Py}) - k_8(Q_\psi - Q_{Iz}) - k_9(Q_\phi - Q_{ox}) \end{bmatrix} + \begin{bmatrix} -k_3 & k_1 & k_2 \\ k_4 & -k_6 & -k_5 \\ -k_9 & -k_8 & k_7 \end{bmatrix} \begin{bmatrix} T_\phi \\ T_\psi \\ T_\theta \end{bmatrix} \tag{7}$$

Now defining state variables as,

$$X = [x_1 \quad x_2 \quad x_3]^T = [\phi \quad \psi \quad \theta]^T \tag{8}$$

Writing (7) in terms of newly defined variables,

$$\ddot{X} = M(X, \dot{X}) + gT \tag{9}$$

where

$$M(X, \dot{X}) = \begin{bmatrix} k_1(Q_\psi - Q_{Iz}) + k_2(Q_\theta - Q_{Py}) - k_3(Q_\phi - Q_{ox}) \\ k_4(Q_\phi - Q_{ox}) - k_5(Q_\theta - Q_{Py}) - k_6(Q_\psi - Q_{Iz}) \\ k_7(Q_\theta - Q_{Py}) - k_8(Q_\psi - Q_{Iz}) - k_9(Q_\phi - Q_{ox}) \end{bmatrix}; g = \begin{bmatrix} -k_3 & k_1 & k_2 \\ k_4 & -k_6 & -k_5 \\ -k_9 & -k_8 & k_7 \end{bmatrix}; T = [T_{x_1} \quad T_{x_2} \quad T_{x_3}]^T \tag{10}$$

where,

$$\mathrm{Q}_{11} = (\mathrm{Q}_{\mathrm{x2}} - \mathrm{Q}_{\mathrm{Iz}}); \ \mathrm{Q12} = (Q_{\mathrm{x3}} - \mathrm{Q}_{\mathrm{py}}); \ \mathrm{Q13} = (\mathrm{Q}_{\mathrm{x1}} - \mathrm{Q}_{\mathrm{ox}})$$

The output vector to be controlled is selected as

$$y = [y_1 \quad y_2 \quad y_3]^T = [\phi \quad \psi \quad \theta]^T \tag{11}$$

The control objective is to design a robust controller using roll, pitch, and yaw angle feedback and their derivatives such that the gimbal platform remains stable as per the specifications imposed.

D. Feedback Linearization based Control

The Feedback Linearization (FL) [6] is one of the widely used approach in non-linear control systems design. Advantage of FL design is that it provides a systematic framework for designing controllers for non-linear systems. In this work, the FL approach is employed for designing controller stabilizing gimbal axes. Consider the dynamics given by (9)-(11), the matrix g of the gimbal system is non-singular in whole state-space. For the system of (9), non-linear coordinate transformation is not required and the control which achieves feedback linearization with the angular positions ϕ, ψ and θ as outputs can be obtained as [6]

$$T = g^{-1}(-M(X, \dot{X}) + \vartheta) \tag{12}$$

where $\vartheta = [\vartheta_{x_1} \quad \vartheta_{x_2} \quad \vartheta_{x_3}]$ is the outer loop control. Substituting the FL control (15) in (12) results into a linear and decoupled input-output relationship as

$$\ddot{X} = \vartheta \tag{13}$$

Now defining the outer-loop control, ϑ_i, as

$$\vartheta_i = -k_{i2}\dot{x}_i - k_{i1}x_i; \ i = 1; 2; 3 \tag{14}$$

The controller gains, k_{ij}, are the design parameters to be chosen to achieve desired regulation. FL control requires exact cancellation of the non-linearities, it offers asymptotic regulation only when the models are known exactly and the fed back states are measured without any error. In practice, these conditions are hard to meet and so the performance of FL control degrades. Since modelling uncertainties and external disturbances are always present, there is a need to robustify the FL based controller. As robustness is an important concern with the FL control, application of the UDE technique for robustification of the FL controller is presented in next section.

III. UDE BASED CONTROLLER

In this work, the UDE technique of [8] is used for designing a robust controller for gimbal stabilization. Reader is referred to [8] for a detailed development of UDE technique.

A. *FL+UDE Controller*

Consider the dynamics given by (9), since in practice system model is known precisely, it becomes necessary to compensate effects of modelling errors and inaccuracies. To this end, matrix, g is taken as $g = g_0 + \Delta g$ where g_o is a chosen constant diagonal matrix and Δg is its associated uncertainty. In view of this, the dynamics of (12) is rewritten as

$$\ddot{X} = \left[M\left(X, \dot{X}\right) + (g - g_o)T\right] + g_o T + d' \tag{15}$$

where d' represents the effect of external disturbances, if any. Now denoting the total uncertainty as $d = M\left(X, \dot{X}\right) + (g - g_o)T + d'$, the dynamics of (15) takes the form

$$\ddot{X} = d + g_o T \tag{16}$$

where $d = [d_1 \quad d_2 \quad d_3]^T = [d_\phi \quad d_\psi \quad d_\theta]^T$. With g_0 diagonal, it is straightforward to verify that the dynamics of (16) is decoupled. In view of this, the dynamics for *i*th axis can be written as

$$\ddot{x}_i = d_i + b_{ii} T_i \tag{17}$$

where b_{ii} are the diagonal elements of g_o. To address the issue of the uncertainty, the FL control takes the form as

$$T_i = \frac{1}{b_{ii}}\left(-\hat{d}_i + \vartheta_i\right) \tag{18}$$

where $\hat{d}_i$ is estimate of effect of uncertainties i.e. d_i. We designate the controller of (18) as FL+UDE controller. Substituting (18) in (17) leads to

$$\ddot{x}_i = \vartheta_i + \ d_i - \hat{d}_i \tag{19}$$

from where one gets

$$d_i = \ddot{x}_i - \vartheta_i + \ \hat{d}_i \tag{20}$$

Now, as proven in [8] and [13], uncertainty can be estimated by passing the function of (20) through a filter $G_{if}(s)$ of adequate bandwidth, specifically, if the filter $G_{if}(s)$ is of the form of

$$G_{if}(s) = \frac{1}{1+\tau_{if}s}; \mathrm{i} = 1;\ 2;\ 3 \tag{21}$$

here, $G_{if}(s)$ is a first order low pass filter with a time constant of τ_{if}. Then the estimate of uncertainty d_i denoted as $\hat{d}_i$ can be obtained by solving

$$\tau_{if}\dot{\hat{d}}_i(t) + \hat{d}_i(t) = d_i(t) \tag{22}$$

Now from (20) and (22)

$$\tau_{if}\dot{\hat{d}}_i(t) + \hat{d}_i(t) = \ddot{x}_i - \vartheta_i + \ \hat{d}_i$$

Now, solving above relation, we get

$$\tau_{if}\dot{\hat{d}}_i(t) = \ddot{x}_i - \vartheta_i \tag{23}$$

Integrating both sides of (23)

$$\hat{d}_i(t) = \frac{1}{\tau_{if}}\dot{x}_i(t) - \frac{1}{\tau_{if}}\int \vartheta_i \, dt \tag{24}$$

Substituting (24) into (18) gives the FL+UDE control law as

$$T_i = \frac{1}{b_{ii}}\left[-\frac{1}{\tau_{if}}\dot{x}_i(t) + \vartheta_i + \frac{1}{\tau_{if}}\int \vartheta_i \, dt\right] \tag{25}$$

The robustified FL control (25) has been designated as the FL+UDE controller. From (25), while the controller achieves the objective of robustification of the FL control, the implementation of the same requires measurement of angular positions as well as velocities.

IV. CLOSED LOOP STABILITY

From plant in (16), the dynamics are decoupled and hence stability for roll-axis (or for any *i*th axis) is established. To this end, defining $x_{i1} = \phi$ and $x_{i2} = \dot{\phi}$, the dynamics of (16) can be rewritten in a phase variable form as

$$\begin{aligned} \dot{x}_{i1} &= x_{i2} \\ \dot{x}_{i2} &= b_{ii}T_i + d_i \\ y &= x_{i1} \end{aligned} \tag{26}$$

Defining the state vector as $x_{ip} = [x_{i1} \quad x_{i2}]^T = [\phi \quad \dot{\phi}]$, the system of (26) can be written as

$$\dot{x}_{ip} = A_{ip}x_{ip} + B_{ip}T_i + B_{id}d_i \tag{27}$$

where

$$A_{ip} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix};\ B_{ip} = \begin{bmatrix} 0 \\ b_{ii} \end{bmatrix};\ B_{id} = \begin{bmatrix} 0 \\ 1 \end{bmatrix};\ C_{ip} = [1 \quad 0]$$

The FL+UDE control (18) using ϑ_i of (14) can be written as

$$T_i = \frac{1}{b_{ii}}\left(-k_{i2}\dot{x}_i - k_{i1}x_i - \hat{d}_i\right) \tag{28}$$

Now substituting control (28) into (27) and defining the state feedback gain vector, K_{ip} as $K_{ip} = \begin{bmatrix} \frac{k_{i1}}{b_{ii}} & \frac{k_{i2}}{b_{ii}} \end{bmatrix}$, one can work out the following error dynamics

$$\dot{x}_{ip} = \left(A_{ip} - B_{ip}K_{ip}\right)x_{ip} + B_{id}\tilde{d}_i \tag{29}$$

Lastly, the uncertainty estimation error dynamics is obtained. From (23) and (25), the estimate of the uncertainty, d_i, is given as

$$\hat{d}_i = G_{if}(s)d_i \tag{30}$$

From (30) one gets

$$d_i = \frac{\hat{d}_i}{G_{if}(s)} \tag{31}$$

With the uncertainty estimation error defined as $\tilde{d}_i = d_i - \hat{d}_i$ and using (24) and carrying some simplifications gives

$$\dot{\tilde{d}}_i = -\frac{1}{\tau_{if}}\tilde{d}_i + \dot{d}_i \tag{32}$$

Combining (29), and (32) yields the following error dynamics for the closed loop system

$$\begin{bmatrix} \dot{x}_{ip} \\ \dot{\tilde{d}} \end{bmatrix} = \begin{bmatrix} \left(A_{ip} - B_{ip}K_{ip}\right) & -B_{id} \\ 0 & -\frac{1}{\tau_{if}} \end{bmatrix} \begin{bmatrix} x_{ip} \\ \tilde{d}_i \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \dot{d}_i \tag{33}$$

From (33), the system matrix being in a block triangular form, now as proved in [14], it can be easily verified that the eigenvalues of the system matrix are given by

$$\left|sI - \left(A_{ip} - B_{ip}K_{ip}\right)\right| \left|s - \left(-\frac{1}{\tau_{if}}\right)\right| = 0 \tag{34}$$

Since pair $\left(A_{ip}, B_{ip}\right)$ is controllable, the controller gain, K_{ip} can be chosen appropriately along with $\tau_{if} > 0$ to ensure stability for the error dynamics. As the error dynamics is driven by $\dot{d}_i$, it is obvious that for bounded $\left|\dot{d}_i\right|$, bounded input-bounded output stability is assured. Additionally, if the rate of change of uncertainty is negligible, i.e., if $\dot{d}_i \approx 0$, then the error dynamics is asymptotically stable [14]. From (32) one can observe that smaller the value of filter time constant, τ_{if}, smaller will be the estimation error. It can also be noted that estimation does not depend on the magnitude of the uncertainty but does depend on its rate of change. As stated earlier, the error dynamics (33) is asymptotically stable if $\dot{d}_i = 0$.

V. SIMULATIONS AND RESULTS

The system model parameters and the control specifications for simulation are defined below, the time constant τ_{if} of low-pass filter of (21) is taken as $\tau_{if} = 0.005\ sec$. The various parameters of the system model are taken as follows:

The coefficients of viscous friction are assumed to be *3.454 N.m* for all three axes. Initial angular position mismatch in roll, yaw and pitch are taken to be *3^o* respectively. Control specifications for design are taken as, $t_s = 0.4\ sec,\ \zeta = 0.8$ and $e_{ss} = 0.05^o$. Uncertainty of *10%* is considered in all elements of inertia matrix of (35) and in coefficient of viscous friction, external torque disturbances of magnitude of $d_\phi = 12250\sin(2\pi t)\ N.m,\ d_\psi = 11060\sin(2\pi t)\ N.m$, and $d_\theta = 10080\sin(2\pi t)\ N.m$ are considered to be acting on the system. Corresponding angular acceleration disturbances are obtained by dividing torque by inertia in roll, yaw and pitch axes respectively. The inertia values in roll, yaw and pitch axes are $[6124.1 \quad 5530.2 \quad 5040.2]kg.m^2$ respectively. The Inertia matrix *I* obtained from corresponding *Solid-Works*® modelling is,

$$I=\begin{bmatrix} I_{ox} & I_{oy} & I_{oz} \\ I_{Px} & I_{Py} & I_{Py} \\ I_{Ix} & I_{Iy} & I_{Iz} \end{bmatrix}=\begin{bmatrix} 1244.372 & 865.858 & 1484.444 \\ 1037.950 & 673.160 & 1931.123 \\ 1115.625 & 725.507 & 1428.554 \end{bmatrix} kg.m^2 \tag{35}$$

The constant diagonal matrix g_o is chosen as

$$g_o=\begin{bmatrix} -1/I_{ox} & 0 & 0 \\ 0 & -1/I_{Iz} & 0 \\ 0 & 0 & -1/I_{Py} \end{bmatrix}$$

Simulations are carried out, from Figs. 2(a)-(c), the roll yaw and pitch angles are plotted and it can be seen that the UDE based controller offered a satisfactory regulation performance. It is important to note that function $M(X,\dot{X})$ i.e. dynamics of the system is considered unknown, additionally external torque disturbance is also acting on the system. Figs. 2(d)-(f) shows actual and estimated disturbance, UDE is capable of estimating disturbance accurately. Figs. 2(g)-(i) gives the control torques of roll, yaw and pitch axes respectively.

VI. COMPARISON WITH EXISTING DESIGNS

Simulations are carried out to compare the performance of the proposed design with some well-known existing controllers. The controllers considered for comparison purpose are (a) PD control (b) FL control and (c) the proposed UDE based controller. A brief of the controllers is given for the sake of completeness as follows:

Design-1: PD control Design

The PD control is widely used control technique [5] and is used as Design-1 in this comparison. The control law for the PID control can written as,

$$T_i=-k_{id}\dot{x}_i-k_{ip}x_i;\ i\ =\ 1;\ 2;\ 3 \tag{36}$$

Now that the system under consideration is nonlinear, for control design purpose system is first linearized using Jacobian linearization [6] to find controller gains. Controller gains are chosen to satisfy performance specifications as given in Sec. 5. Subsequently these gains are used to control the nonlinear dynamics of the stabilization system.

Design-2: Feedback Linearization (FL) Control

Feedback Linearization based control as presented in [6] is used as Design-2 in this comparison. The controller takes the form of (37). The controller gains for the design of the FL controller are same as that used in Sec. 5.

$$T=g^{-1}(-M(X,\dot{X})+\vartheta) \tag{37}$$

Design-3: FL+UDE Control

The proposed UDE based design is designated as Design-3 for the purpose of comparison. Simulation parameters and other design parameters are same as given in Sec. 5.

To compare the performance, simulations are carried out by considering the data given in Sec. 5 for all designs. A disturbance torque as discussed in Sec. 5 is also considered. From Fig. 3 (a)-(c), it can be observed that while the Designs-1 and 2 have resulted steady state error in position regulation, the FL+UDE controller has offered better regulation. One important point to be considered that, while for design-2, M and g are considered known whereas for Design-3, M is considered as unknown and only nominal value of g i.e. g_o is used for controller design. Fig-3(d)-(e) gives control torque history of various designs. Peak steady-state errors of various designs have been shown in Table-1, from where one can observe that UDE based controller minimizes e_{ss} compared to other two designs.

Table -1 Peak e_{ss} of Various Designs

Design	e_ϕ (deg)	e_ψ (deg)	e_θ (deg)
Design-1	unstable	unstable	unstable
Design-2	0.3085	0.2963	0.2563
Design-3	0.0069	0.0070	0.0075

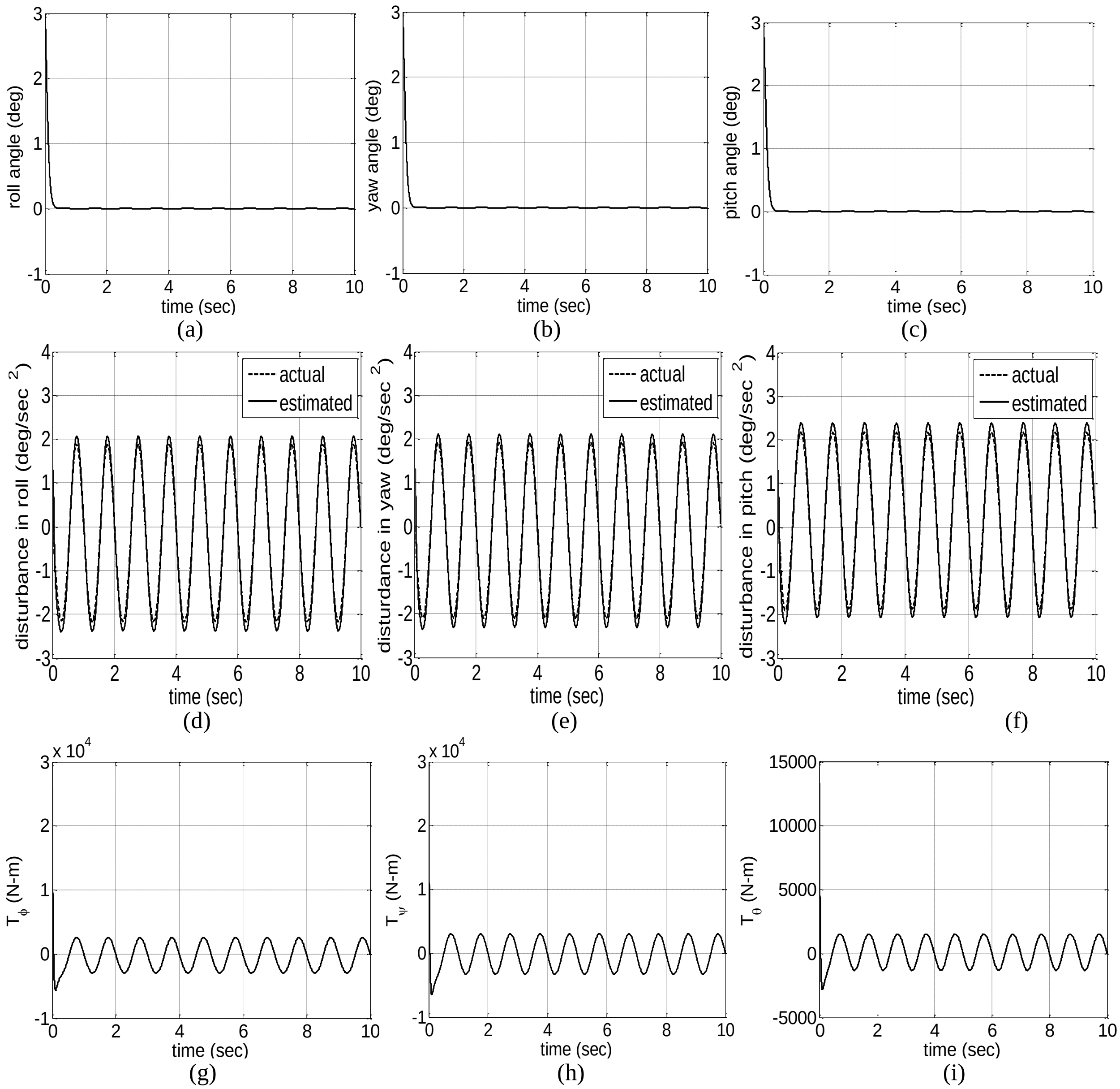
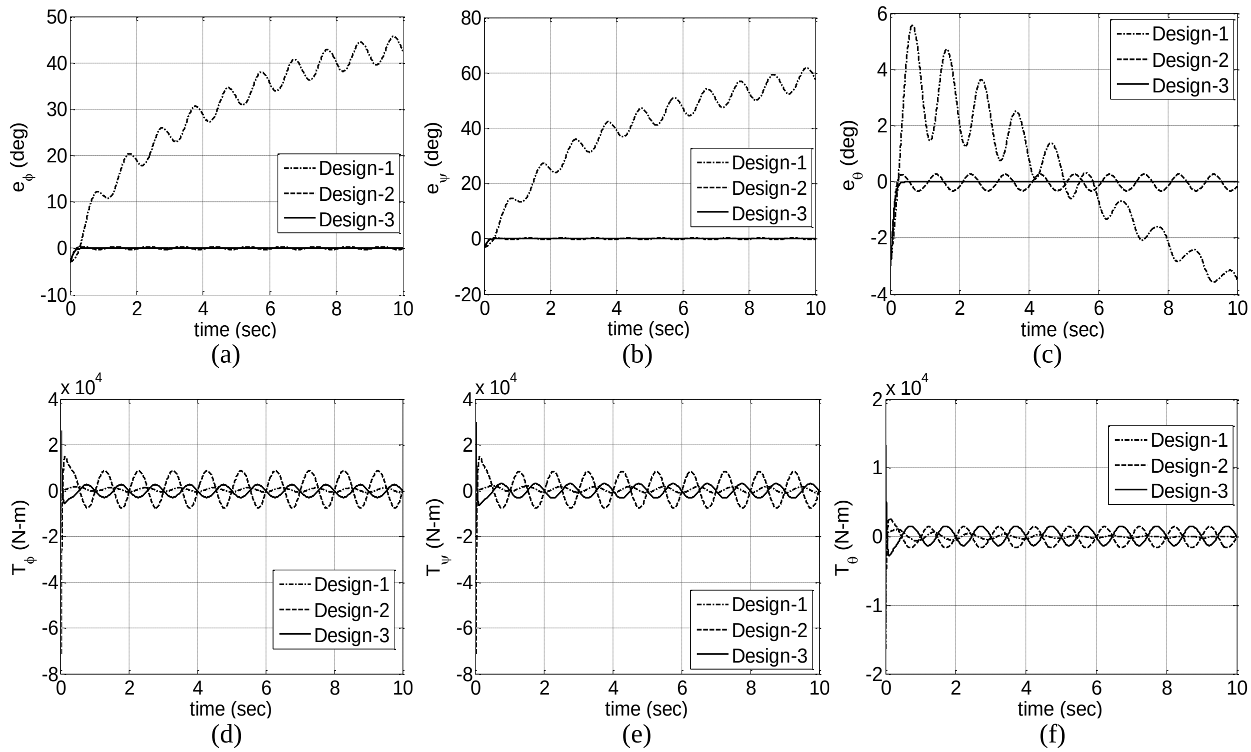


Figure 2. Performance of UDE based controller in presence of disturbances

VII. CONCLUSION

In this paper, a new approach based on the UDE technique is proposed for designing a robust controller for stabilization of three-axis gimbal platform. The FL controller is robustified with UDE estimated uncertainties. In doing so, system nonlinearities, external disturbances and modeling inaccuracies have been considered as to be estimated uncertainty. Hardware realization of the stabilization system is in progress, proposed control strategy will be tested on hardware platform and forms future task.



Performance Comparison of UDE based Controller with various Designs

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