Displacement Response of a Mass against Step Input under Seismic Excitation

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Abstract- This paper deals with the base excitation problem. In this case a single degree of freedom spring-mass-damper system with vibrating base is considered for study. The base is excited by a step displacement input and displacement response of the mass is found out. As a case study numerical values of mass, stiffness and damping coefficient are taken. This case study problem is solved by both analytical method and by using multi body dynamics software. In analytical method the solving of the problem is started from the equation of motion. The displacement response of the mass consists of both transient response up to 2 seconds and steady state response after 2 seconds. Both methods are compared and solution convergence is achieved.

Keywords – Mass, seismic excitation, step displacement input, transient response, steady state response.

I. INTRODUCTION

Many mechanical systems are mounted on a moving support or base. In these cases base is given a displacement either in the form of step or sinusoidal. Examples of base excitation can be of automobile suspension system which is excited by the road surface. The term seismic excitation refers to the sort of excitation acting on buildings during earthquakes. In analytical method the equation of motion for base excitation problem is written by using Newton's second law of motion. The solution of this equation is assumed. The solution is assumed based on the most common situation occurring in spring-mass-damper system i.e underdamped system in which ξ (damping factor) is less than one. The solution consists of constants which are found out by using initial conditions. The base is subjected to step displacement input in the form of step(time, initial time, initial function value, final time, final function value). This paper also presents a solving a problem by using multibody dynamics software.

II. SYSTEM UNDER INVESTIGATION

A single degree of freedom spring-mass-damper system is shown in figure 1[2]. It is required to investigate the displacement response of the mass x(t) for a step input to the base which is in the form of step(time, 0, 0, 0.05, 50).

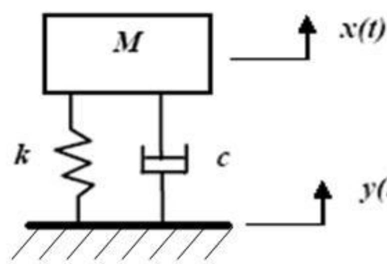


Figure 1. Seismically excited spring-mass-damper system

Nomenclature	Description of the parameter	Numerical value
M	Mass	137 kg
k	Stiffness	8.5 N/mm
c	Damping coefficient	0.65 N-s/mm
t_i	Initial time	0 sec
t_f	Final time	0.05 sec
y_i	Initial displacement	0 mm
y_f	Final displacement	50 mm

Table 1. Parameters [2]

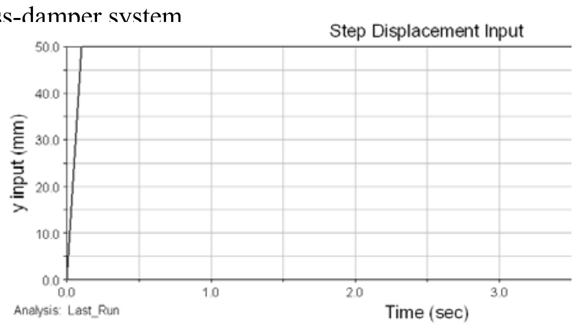


Figure 2. Step displacement input

III ANALYTICAL METHOD

The equation of motion for system in figure 1 is given below.

$$M\ddot{x} = -k(x-y) - c(\dot{x} - \dot{y})$$
$$M\ddot{x} + c\dot{x} + kx = c\dot{y} + ky$$

For the above problem we can assume the solution of the form,

$$x(t)= c+ e^{-\xi w_n t} [a \cos (w_d t)+ b \sin(w_d t)]$$

c= steady state displacement of the mass due to the base movement= 50mm
a and b are account for initial conditions [3].
If the system was initially at rest, then $x(0)= \dot{x}(0) = 0$, c= 50 mm

The solution becomes,

$$x(t)= 50+ e^{-\xi w_n t} [a \cos (w_d t)+ b \sin(w_d t)] \;.....(1)$$

From initial conditions, we can write,

$$x(0)= 50+ e^{-w_n.0}[a*1+0]= 0$$
$$x(0)= 50+ 1*a =0$$
$$a= -50$$

Diferentaiting,

$$\dot{x}(t)= e^{-\xi w_n t}\, a.-\sin(w_d t).w_d+a \cos (w_d t).\, e^{-\xi w_n t}(-\xi w_n)+ e^{-\xi w_n t}\, b.\cos (w_d t).w_d+ b \sin (w_d t).\, e^{-\xi w_n t}(-\xi wn)$$

$$\dot{x}(t)=-50.-\sin(w_d t).w_n \sqrt{(1-\xi^2)}+ (-50).\cos (w_d t)(- \xi w_n)+b \cos (w_d t).w_n \sqrt{(1-\xi^2)}+ b \sin (w_d t).(- \xi w_n)=0$$

$$\dot{x}(0)= 50\xi+ b.\sqrt{(1-\xi^2)}= 0$$

$$50\xi= -b.\sqrt{(1-\xi^2)}$$

$$b= - 50\xi/\sqrt{(1-\xi^2)}$$

substituting a and b in equation (1)

$$x(t)= 50+ e^{-\xi.w_n.t}[-50. \cos(w_d t) + (-50\xi/\sqrt{(1-\xi^2)}). \mathrm{Sin}(w_d t)]$$

Natural frequency,

$w_n=\sqrt{k/M} =\sqrt{(8500/137)} = 7.88$ rad/sec and $f_n= w_n/2\pi = 7.88/2\pi =1.254$Hz

Damping factor $\xi= c/c_c$

Crtitical damping coefficient $c_c= 2Mw_n= 2*137*7.88 = 2159.12$ N-s/m

$\xi= 650/2159.12 = 0.301$

Damped natural frequency $w_d= w_n\sqrt{(1-\xi^2)} = 7.88\sqrt{(1-(0.301)^2} =7.515$ rad/sec and $f_d= w_d/2\pi = 7.515/2\pi= 1.196$Hz

Dispalcement response of the mass is,

$$x(t)= 50+ e^{-0.301*7.88*0.05}[-50.\cos (7.515*0.05) + (-50*0.301/\sqrt{(1-(0.301)^2)} \sin (7.515*0.05)]$$
$$= 50+0.8882[-50*0.9999- 15.78*6.56*10^{-3}]$$
$$=50+ 0.8882[-49.995-0.1036]$$

$x(t)= 50- 44.5 = 5.5$mm.

IV Solution by using multibody dynamics software

In this solution, ADAMS software [1] is used to study the displacement response of the mass under base excitation.

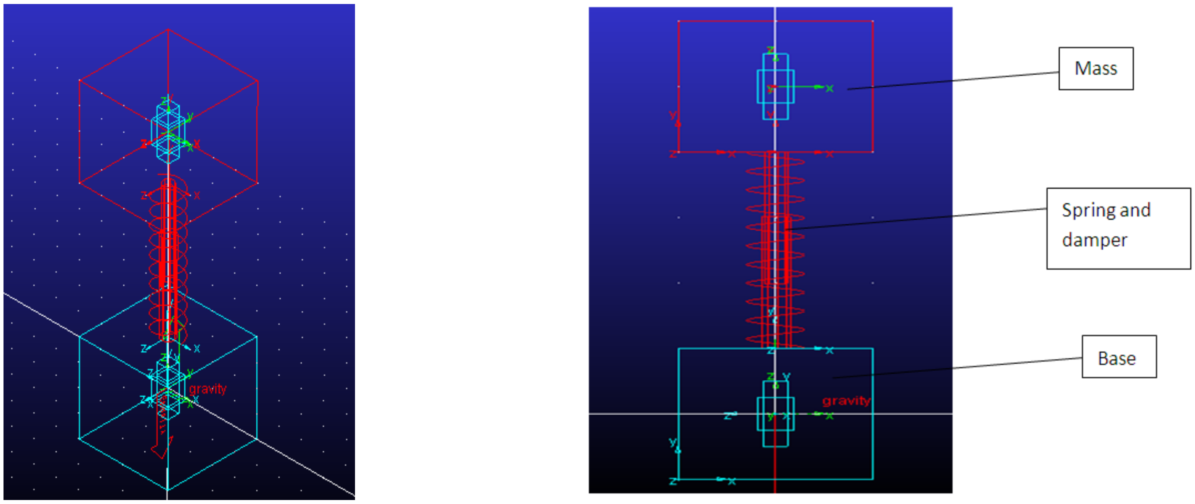


Figure 3. Three dimensional and two dimensional view of the spring-mass-damper system (simulation model)

The simulation model of the spring-mass-damper system is constructed by using the main tool box of ADAMS software [1]. The model has the following parts,

- Mass
- Base
- Spring
- Damper

Mass ans base are connected by spring and damper. The model has two translational joints. First joint is between ground and mass (joint 1), second between ground and base (joint 2).

A. Defining Step Input

Step input is given to the translational joint 2. Motion is imposed on this joint in Z direction by specifying disp(time) = STEP(time, 0, 0, 0.05, 50). This creates a function that starts at zero at time = 0, and ramps to 50 mm at time = 0.05 seconds. It remains at this final value after the final time [2].

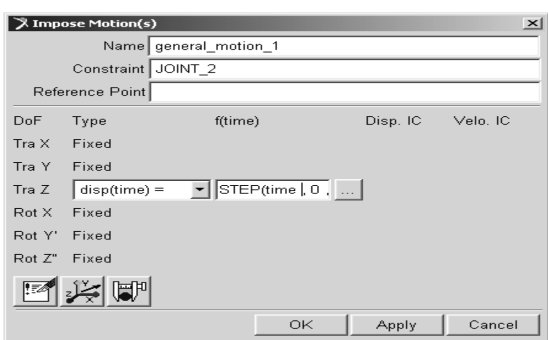


Figure 4. Impose Motion Dialogue Box

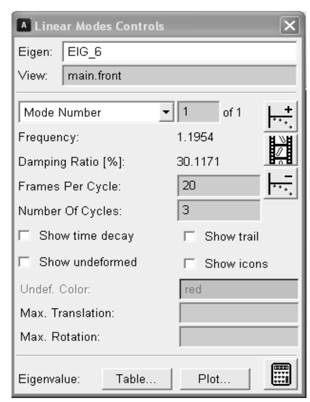


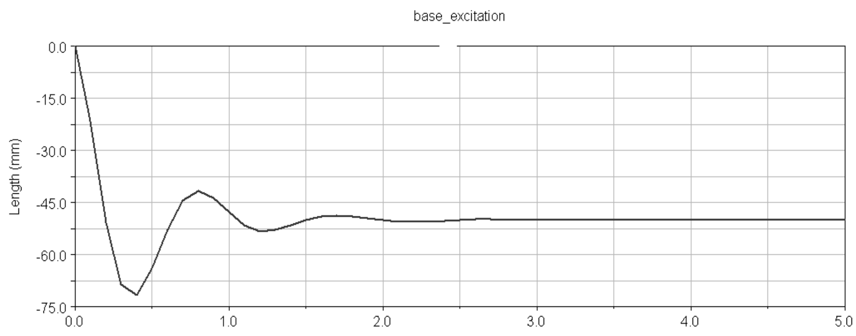
Figure 5. Linear modes

Spring and damper are created as a connection between mass and base. Preload M*g is also entered. After this, static equilibrium is found out and linear system analysis is performed to compute the linear modes resulting in the value of natural frequency and damping factor.

B. RUNNING A TIME DOMAIN SIMULATION [4]

Simulation is carried out with end time of 2 seconds and in steps of 100. Start the simulation at equilibrium. For this analysis it is important that the model be in its equilibrium position at time = 0. After the simulation is completed results are viewed.

The graph that appears in figure 6 represents the vertical displacement of the mass from its equilibrium condition (as a function of time).



Time (sec)

Figure 6. Displacement response of the mass x(t)

V.CONCLUSION

Initially upto 2 seconds the displacement shows a transient response and after 2 seconds the displacement shows a steady state response. At time = 0.05 sec. the displacement value of the mass is 5.5 mm from analytical method and -7.5 from software.

Parameter	Analytical method	Software method
Natural frequency (f_n)	1.254 Hz	1.254 Hz
Damped frequency (f_d)	1.196 Hz	1.1954 Hz
Displacement of mass at 0.05 sec.	5.5 mm	- 7.5 mm (Negative sign indicates that mass moves in downward direction when simulation is carried out. But actually the given value of displacement to the base is in upward direction)

Table 2. Comparison of results

Though the error between analytical method and software method comes out be 26.7%, the difference in the value of displacement of the mass between these two methods is 2 mm which is very negligible. Hence convergence is achieved.

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