

# Mathematical Modeling and Simulation of Direct Acting Pressure Relief Valve with the Effects of Compressibility of Oil Using MATLAB SIMULINK

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**Abstract - the trend in hydraulic power applications is to improve efficiency and performance of the hydraulic system parts. This paper examines the performance of direct operated relief valve type(DPR06). A mathematical model is derived that describes the dynamics of a single stage relief valve embedded within a simple hydraulic circuit. The aim is to capture the mechanisms of instability of such valves, taking into account of oil compressibility. The model is simulated using MATLAB / SIMULINK model (R2007). This report contains the assumptions, variables, mathematical modeling and analysis involved in modeling and simulating pressure relief valve.**

**Key words: - hydraulic system, DPR Pressure relief valve, oil compressibility, MATLAB / SIMULINK etc.**

## I.INTRODUCTION

Hydraulic circuits are known sometimes to show undesirable behavior that is peculiar to nonlinear dynamical systems and. The source of the nonlinearity is often backlash, dry friction, on-off switches or impacting components whose behavior is not well modeled by smooth evolution equations. There are by now countless examples of such non smooth nonlinear problems that engineers face when they design mechanical systems, see e.g.[2,3,16] and references therein.

Relief valves are widely used to limit pressure in hydraulic power transmission and control systems. There is a rich literature that describes their usage in hydraulic circuits and gives information on their design and application; see [1] for a brief overview, or [11, 14] for a more industrial perspective. There have been numerous documented industrial examples where these kinds of valves have been found to vibrate when their equilibrium loses stability, and many researchers have been interested in the investigation of this phenomenon. As far back as the 1960s researchers suspected that the piping to and from the relief valve cannot be neglected. Kasai [10] carried out a detailed investigation of a simple poppet valve and he deduced a stability criterion analytically. He also showed that effects other than fluid nonlinearity can lead to stability loss, such as changes in the poppet geometry or the oil temperature. Moreover he performed experiments and found good coincidence with his analytical results. Thomann [15] was also interested in the analysis of a pipe-valve system. He used a simple poppet type valve but analyzed how different poppet geometries affect the stability. He investigated a conical and a cylindrical poppet together with conical or cylindrical seats, and their combination. Hayashi et al.[8, 9], built up a model with a constant supply pressure and investigated the valve's response and stability, finding that the point of instability can be characterized as a Hopf bifurcation.

The pressure relief valve is used in almost every hydraulic system (Figure. 1). The function of the relief valve is to limit the maximum pressure that can exist in a system. Under ideal condition, the relief valve should provide alternative flow path to the tank for the system oil while keeping the system pressure constant.

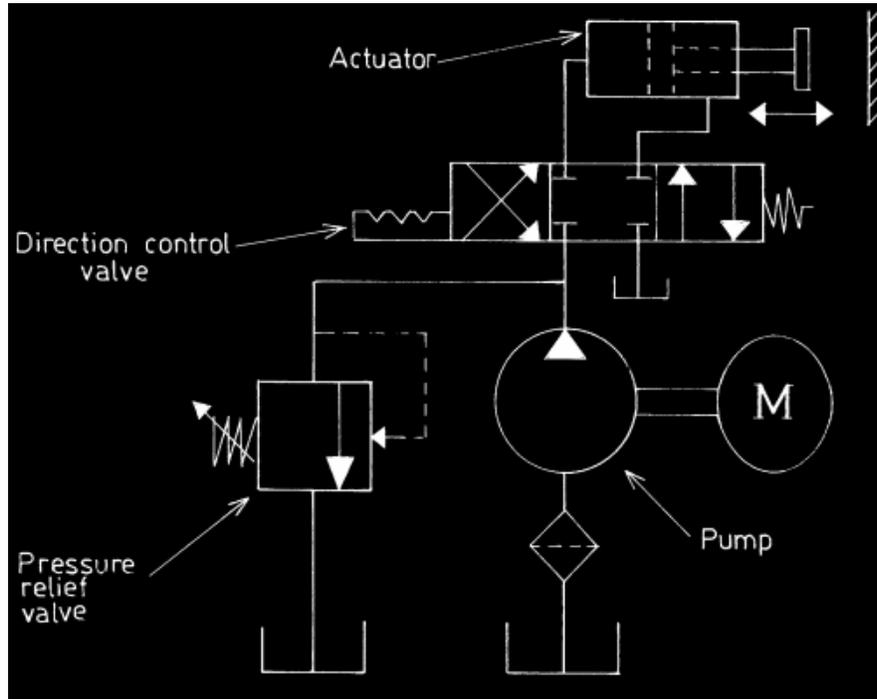


Figure 1. The Physical system.

## II. MATHEMATICAL MODEL

Although the dynamic behavior of a PRV is strongly influenced by its geometric configuration and dimensions, a simplified geometry, as shown in Fig. 3, was considered to the development of a mathematical model. The simplified system is composed of a spring, a cap or disc and a input flow pipe (valve wall). For the flow analysis through the PRV, the fluid was considered as incompressible and isothermal. Due to the cylindrical shape of the geometry, the flow was considered axi-symmetric.

### A. Dynamic characteristics

The PRV starts opening when the operation pressure  $P_a$  exceeds the set point pressure  $P_{sp}$ . During the disc displacement, the Newton's second law can be applied to the system illustrated in Fig. 4, resulting in the spring-disc dynamic system equation, Eq. (1).

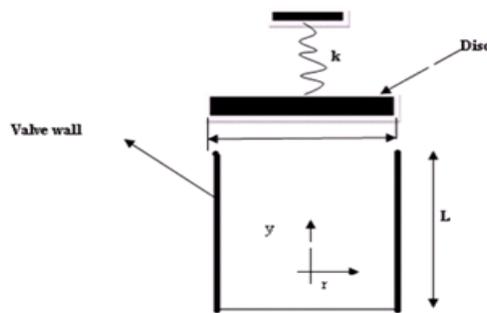


Figure 2. Simplified System.[3]

$$F_f - k(Y_D + Y_0) - c \frac{dY_D}{dt} - m_D g - P_0 A = m_D \frac{d^2 Y_D}{dt^2} \quad (1)$$

where  $F_f$  is the force applied by fluid to disc,  $k$  is the spring constant,  $Y_D$  is the disc displacement,  $Y_0$  is the spring initial deformation,  $c$  is the spring viscous damping coefficient,  $m_D$  is the disc mass,  $g$  is the gravity acceleration,  $P_0$  is the external pressure (atmospheric pressure) and  $A$  is the cross section of the little pipe / disc area. Applying the principle of conservation of linear momentum in the  $y$  direction, to control volume inside the PRV illustrated in Fig. 3, neglecting the time  $y$  momentum variation inside the control volume, since it can be considered small in relation to the others quantities, results in[3]

$$\sum F_y = \frac{\partial}{\partial t} \int_{cv} \rho u_p dV + \frac{\partial}{\partial t} \int_{cs} \rho u_p \dot{u} \cdot d\bar{A} \rightarrow -\rho g A(L + Y_D) - F_f + P_a A = -u_e \rho u_e A \quad (2)$$

Where  $\rho$  is the fluid density,  $L$  is the length of the valve wall,  $F_f$  is the reaction applied by the disc to fluid,  $u_e$  is the average velocity coming into the control volume and  $P_a$  is the operation pressure or the PRV input pressure. Combining the spring-disc dynamic equation, Eq. (1), with the fluid momentum conservation equation, Eq. (2), the following expression is obtained

$$0 = m_D \frac{d^2 Y_D}{dt^2} + c \frac{dY_D}{dt} + (k + \rho g A) Y_D + k Y_0 + \rho g A L + m_D g - (P_a - P_0) A - Q^2 \frac{\rho}{A} \quad (3)$$

Where  $Q = \rho u_e A$  is the average flow rate coming into the control volume.

The initial spring deformation  $Y_0$  can be determined as a function of the set point pressure  $P_{sp}$  to open the relief valve, by applying Eq. (3) to the instant immediately before the valve opening, i.e.,  $Y_D = 0$  and  $Q=0$ ,  $P_a = P_{sp}$ .

$$Y_0 = \frac{1}{k} [(P_{sp} - P_0) A - \rho g A L - m_D g] \quad (4)$$

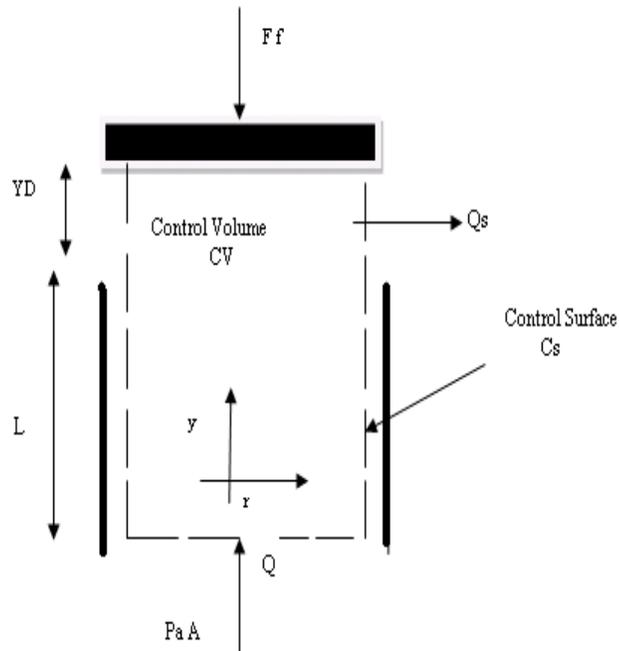
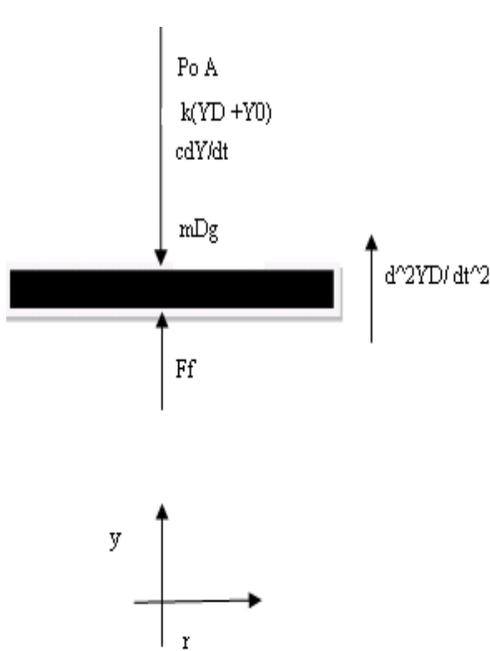


Figure 3, Free Body Diagram      Figure 4. Control Volume inside of the PRV during its actuation

Equating (3) can be simplified with Eq. (4) as

$$0 = m_D \frac{d^2 Y_D}{dt^2} + c \frac{dY_D}{dt} + (k + \rho g A) Y_D - (P_a - P_{sp}) A - Q^2 \frac{\rho}{A} \quad (5)$$

Applying the principle of mass conservation into the PRV control volume during its actuation, Fig. 5, the average flow rate that exits from the control volume  $Q_s$  can be related with the inflow rate  $Q$  and the disc displacement  $Y_D$  as

$$0 = \frac{\partial}{\partial t} \int_{cv} u \rho \, dV + \frac{\partial}{\partial t} \int_{cs} u \rho \, d\bar{A} \rightarrow 0 = A \frac{dY_D}{dt} - Q + Q_s \quad (6)$$

Further, the valve outflow rate  $Q_s$  can be defined by the valve equation as [10]

$$Q_s = C_d A \sqrt{2 \frac{(P_a - P_0)}{\rho}} \quad (7)$$

Where  $C_d$  is the valve coefficient and  $A$  is a reference area, in this work it was considered as the disc area.

Finally, the equation that governs the dynamic behaviour of the PRV during its actuation can be obtained by combining Eqs. (7), (6) and (5) as

$$0 = m_D \frac{d^2 Y_D}{dt^2} + (c - 2C_d A \sqrt{2\rho(P_a - P_0)}) \frac{dY_D}{dt} + (k + \rho g A) Y_D - (P_a - P_{sp}) A - \left(\frac{dY_D}{dt}\right)^2 \rho A - 2C_d^2 A (P_a - P_0) \quad (8)$$

### B. Compressibility factor.

However due to compressibility of the oil and elasticity of the tubes, the flow rate at the test valve can be different from that at the exit of the pump (indicated in Fig 1.). So we consider a hypothetical chamber whose volume is equal to the total volume of the system when filled with oil, within which we allow a flow rate difference between the inlet and the outlet. This chamber will represent the stiffness of the system.

The pressure rate with time can be given by

$$\frac{dP_a}{dt} = \frac{\beta}{V} (Q - Q_s) \quad (9)$$

Where  $V$  represents the total volume of the system,  $\beta$  denotes the bulk modulus of the oil,  $P_a$  is the operating pressure,  $Q$  is the oil flow rate at the inlet of the valve and  $Q_s$  is the flow rate of the oil leaving the valve through the orifice.

Where

$$Q_s = C_d A \sqrt{2 \frac{(P_a - P_0)}{\rho}}$$

In above equation  $A$  is the cross sectional area of the valve inlet. In general, an accurate expression for the orifice cross sectional area as a function of the valve displacement,  $A$  will be nonlinear and will depend on the precise valve geometry. Nevertheless, since during most operations, the valve displacements will be small it is reasonable to linearise.

### III. DPR06 VALVE SPECIFICATIONS

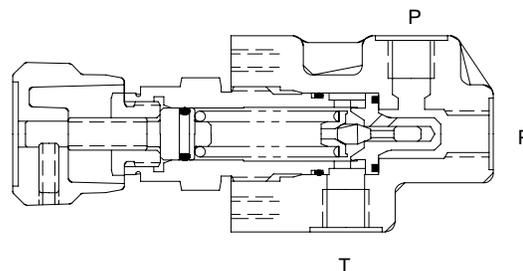


Figure 5. DPR Valve

The valve chosen was DPR06, Direct acting pressure relief valve from the Polyhydron Pvt. Ltd., Belgaum.

Construction: - Direct acting poppet type.

Flow direction: - From P to T.

Operating pressure for port P: - 400 bar and 700 bar

Operating pressure for port T:- 315 bar.

Pressure setting ranges:- Upto 25, 50, 100, 200, 315, 400 and 700 bar max.

Hydraulic Medium:- mineral oil

Viscosity range:- 10CST to 380CST

Bulk modulus:- 1.6Gpa.

#### IV. SIMULINK ®MODELS

The equations (1), (6) and (9) are represented in SIMULINK ®MODEL to analyze the displacement of the poppet, Outlet flow rate and pressure rate with considering the effects of bulk modulus with the time rate. The MATLAB SIMULINK models are given below.

Mass of the poppet and spring:-  $M = 43.186 \cdot 10^{-3} \text{ kg}$

Damping coefficient:-  $B = 10 \text{ Ns/m}$

Operating pressure:-  $P = 50 \text{ bar} (F = P \cdot A = 62.83 \text{ N})$

Stiffness of Spring  $K = 9.96 \cdot 10^3 \text{ N/M}$

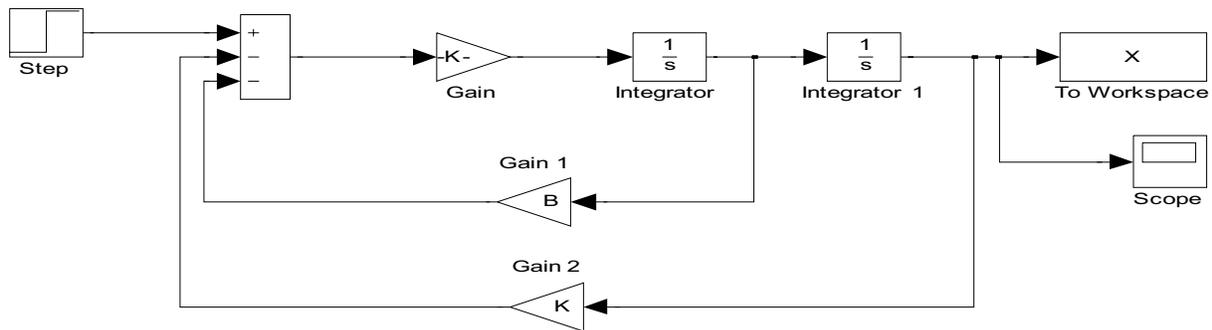


Figure 6. Poppet displacement model

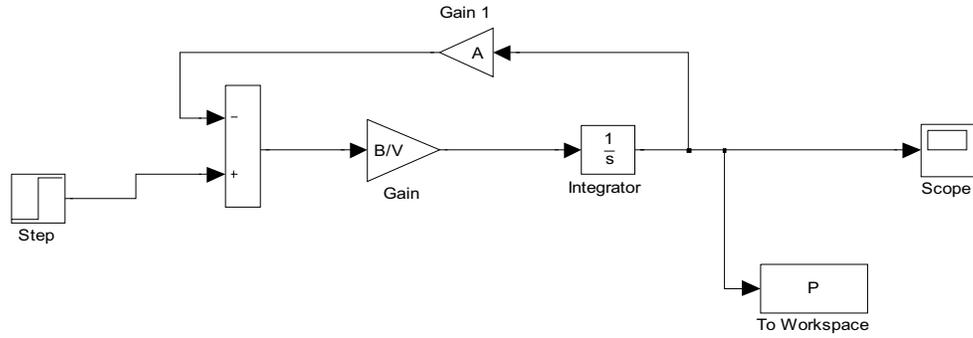


Figure 7. Outlet flow ratet model

### V. ANALYSIS

The valve model was ready for analysis and simulations. Certain parameters like stiffness of spring, volume, inlet flow rate, bulk modulus and area were kept constant. The analysis was considered with time as given in Fig. 9.10.11.

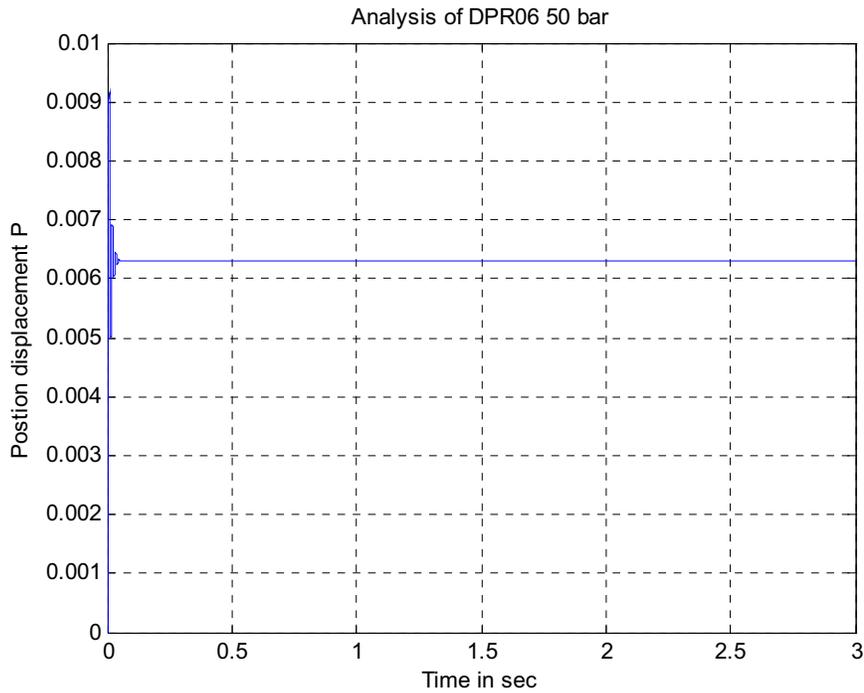


Figure 9. Position displacement V/s time

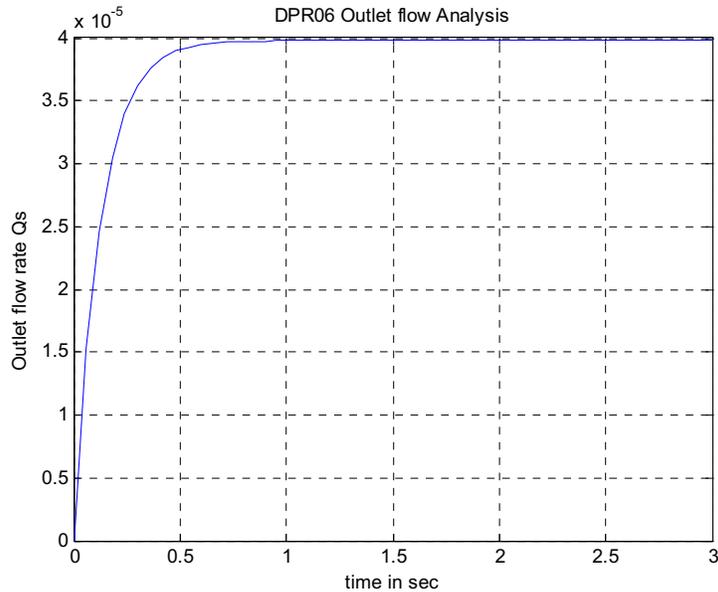


Figure 9. Outlet flow rate V/s time

## VI. CONCLUSION

The present work has derived a mathematical model for a direct acting spring loaded pressure relief valve. The developed mathematical model is represented in MATLAB SIMULINK which predicts the disc behavior and the input – output flow rate of the relief valve during its transient (dynamic characteristics) and equilibrium state. Although a simplified geometry was considered, the methodology can be applied to more complex geometries

## VII. ACKNOWLEDGEMENT

I would like to take this opportunity to thanks Mr. Anand Samant, Joint Managing Director, PSPL, Belgaum.. and Late shri Suresh Hundre, Polyhydron Pvt. Ltd., Belgaum and Siddhartha Hundre, Corporate Manager, Polyhydron Pvt. Ltd, Belgaum.who allowed me to analyse the valve from their organisations.

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