A Robust Digital Image Watermarking Using Wavelet Filters

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Abstract - Wavelets are mathematical functions that cut up data into different frequency components, and then study each component with a resolution matched to its scale. They have advantages over traditional Fourier methods in analysing physical situations where the signal contains discontinuities and sharp spikes. Wavelets were developed independently in the fields of mathematics, quantum physics, electrical engineering, and seismic geology. Interchanges between these fields during the last ten years have led to many new wavelet applications such as image compression, turbulence, human vision, radar, and earthquake prediction. This paper introduces wavelets to the interested technical person outside of the digital signal processing field. I describe the history of wavelets beginning with Fourier, compare wavelet transforms with Fourier transforms, state properties and other special aspects of wavelets, and finish with some interesting applications such as image compression, musical tones, and de-noising noisy data.

I. WAVELETS OVERVIEW

The fundamental idea behind wavelets is to analyse according to scale. Indeed, some researchers in the wavelet field feel that, by using wavelets, one is adopting a whole new mind-set or perspective in processing data. Wavelets are functions that satisfy certain mathematical requirements and are used in representing data or other functions. This idea is not new. Approximation using superposition of functions has existed since the early 1800’s, when Joseph Fourier discovered that he could superpose sine’s and cosines to represent other functions. However, in wavelet analysis, the scale that we use to look at data plays a special role. Wavelet algorithms process data at different scales or resolutions. If we look at a signal with a large “window,” we would notice gross features. Similarly, if we look at a signal with a small “window,” we would notice small features. The result in wavelet analysis is to see both the forest and the trees, so to speak.

II. SIMILARITIES BETWEEN FOURIER AND WAVELET TRANSFORMS

The fast Fourier transform (FFT) and the discrete wavelet transform (DWT) are both linear operations that generate a data structure that contains \( \log_2 n \) segments of various lengths, usually filling and transforming it into a different data vector of length \( 2^n \).

The mathematical properties of the matrices involved in the transforms are similar as well. The inverse transform matrix for both the FFT and the DWT is the transpose of the original. As a result, both transforms can be viewed as a rotation in function space to a different domain. For the FFT, this new domain contains basis functions that are sine’s and cosines. For the wavelet transform, this new domain contains more complicated basis functions called wavelets, mother wavelets, or analysing wavelets. Both transforms have another similarity. The basic functions are localized in frequency, making mathematical tools such as power spectra (how much power is contained in a frequency interval) and scale grams (to be defined later) useful at picking out frequencies and calculating power distributions.

III. DISSIMILARITIES BETWEEN FOURIER AND WAVELET TRANSFORMS

The most interesting dissimilarity between these two kinds of transforms is that individual wavelet functions are localized in space. Fourier sine and cosine functions are not. This localization feature, along with wavelets’ localization of frequency, makes many functions and operators using wavelets “sparse” when transformed into the wavelet domain. This sparseness, in turn, results in a number of useful applications such as data compression, detecting features in images, and removing noise from time series.
One way to see the time-frequency resolution differences between the Fourier transform and the wavelet transform is to look at the basis function coverage of the time-frequency plane. Figure 1 shows a windowed Fourier transform, where the window is simply a square wave. The square wave window truncates the sine or cosine function to fit a window of a particular width. Because a single window is used for all frequencies in the WFT, the resolution of the analysis is the same at all locations in the time-frequency plane.

An advantage of wavelet transforms is that the windows vary. In order to isolate signal discontinuities, one would like to have some very short basis functions. At the same time, in order to obtain detailed frequency analysis, one would like to have some very long basis functions. A way to achieve this is to have short high-frequency basis functions and long low-frequency ones. This happy medium is exactly what you get with wavelet transforms.

One thing to remember is that wavelet transforms do not have a single set of basis functions like the Fourier transform, which utilizes just the sine and cosine functions. Instead, wavelet transforms have an infinite set of possible basis functions. Thus wavelet analysis provides immediate access to information that can be obscured by other time-frequency methods such as Fourier analysis.

IV. WAVELET ANALYSIS

Now we begin our tour of wavelet theory, when we analyse our signal in time for its frequency content. Unlike Fourier analysis, in which we analyse signals using sines and cosines, now we use wavelet functions.

V. THE FAST WAVELET TRANSFORM

The DWT matrix is not sparse in general, so we face the same complexity issues that we had previously faced for the discrete Fourier transform. We solve it as we did for the FFT, by factoring the DWT into a product of a few sparse matrices using self-similarity properties. The result is an algorithm that requires only order operations to transform an \( n \)-sample vector. This is the “fast” DWT of Mallat and Daubechies.
VI. WAVELET PACKETS

The wavelet transform is actually a subset of a far more versatile transform, the wavelet packet transform. Wavelet packets are particular linear combinations of wavelets. They form bases which retain many of the orthogonally, smoothness, and localization properties of their parent wavelets. The coefficients in the linear combinations are computed by a recursive algorithm making each newly computed wavelet packet coefficient sequence the root of its own analysis tree.

VII. ADAPTED WAVEFORMS

Because we have a choice among an infinite set of basis functions, we may wish to find the best basis function for a given representation of a signal. A basis of adapted waveform is the best basis function for a given signal representation. The chosen basis carries substantial information about the signal, and if the basis description is efficient (that is, very few terms in the expansion are needed to represent the signal), then that signal information has been compressed.

According to Wicker Hauser, some desirable properties for adapted wavelet bases are

1. speedy computation of inner products with the other basis functions;
2. speedy superposition of the basis functions;

For adapted waveform analysis, researchers seek a basis in which the coefficients, when rearranged in decreasing order, decrease as rapidly as possible. To measure rates of decrease, they use tools from classical harmonic analysis including calculation of information cost functions. This is defined as the expense of storing the chosen representation. Examples of such functions include the number above a threshold, concentration, entropy, and logarithm of energy, Gauss-Markov calculations, and the theoretical dimension of a sequence.

VIII. WAVELET APPLICATIONS

The following applications show just a small sample of what researchers can do with wavelets.

IX. FBI FINGERPRINT COMPRESSION

Between 1924 and today, the US Federal Bureau of Investigation has collected about 30 million sets of fingerprints (7). The archive consists mainly of inked impressions on paper cards. Facsimile scans of the impressions are distributed among law enforcement agencies, but the digitization quality is often low. Because a number of jurisdictions are experimenting with digital storage of the prints, incompatibilities between data formats have recently become a problem. This problem led to a demand in the criminal justice community for a digitization and a compression standard.

In 1993, the FBI’s Criminal Justice Information Services Division developed standards for fingerprint digitization and compression in cooperation with the National Institute of Standards and Technology, Los Alamos National Laboratory, commercial vendors, and criminal justice communities.

Let’s put the data storage problem in perspective. Fingerprint images are digitized at a resolution of 500 pixels per inch with 256 levels of gray-scale information per pixel. A single fingerprint is about 700,000 pixels and needs about 0.6 Mbytes to store. A pair of hands, then, requires about 6 Mbytes of storage. So digitizing the FBI’s current archive would result in about 200 terabytes of data. (Notice that at today’s prices of about $900 per Gbyte for hard-disk storage, the cost of storing these uncompressed images would be about a 200 million dollars.) Obviously, data compression is important to bring these numbers down.
X. MUSICAL TONES

Victor Wicker Hauser has suggested that wavelet packets could be useful in sound synthesis. His idea is that a single wavelet packet generator could replace a large number of oscillators. Through experimentation, a musician could determine combinations of wave packets that produce especially interesting sounds.

Wicker Hauser feels that sound synthesis is a natural use of wavelets. Say one wishes to approximate the sound of a musical instrument. A sample of the notes produced by the instrument could be decomposed into its wavelet packet coefficients. Reproducing the note would then require reloading those coefficients into a wavelet packet generator and playing back the result. Transient characteristics such as attack and decay—roughly, the intensity variations of how the sound starts and ends—could be controlled separately (for example, with envelope generators), or by using longer wave packets and encoding those properties as well into each note. Any of these processes could be controlled in real time, for example, by a keyboard.

Notice that the musical instrument could just as well be a human voice, and the notes words or phonemes. A wavelet-packet-based music synthesizer could store many complex sounds efficiently because similarly, a wave packet-based speech synthesizer could be used to reconstruct highly compressed speech signals. Figure 8 illustrates a wavelet musical tone or tone burst.

![Wavelets for music: a graphical representation of a Wicker Hauser tone burst. This screenshot of a tone burst was taken while it was playing in the Macintosh commercial sound program Kaboom! Factory. (Tone burst courtesy Victor Wicker Hauser)](image)

XI. WAVELETS ENDNOTE

Most of basic wavelet theory has been done. The mathematics have been worked out in excruciating detail and wavelet theory is now in the refinement stage. The refinement stage involves generalizations and extensions of wavelets, such as extending wavelet packet techniques.

The future of wavelets lies in the as-yet uncharted territory of applications. Wavelet techniques have not been thoroughly worked out in applications such as practical data analysis, where for example discretely sampled time-series data might need to be analysed. Such applications offer exciting avenues for exploration.

XII. BIOGRAPHY

Amara Grapes is a computational physicist, astronomer, and consultant working on numerical analysis, scientific research, technical writing, and WWW site projects for companies as well as government laboratories (NASA-Ames), and universities (Stanford). Her work experience, primarily in astronomy, astrophysics, and planetary science research, was gained from her current cooperative agreement at NASA-Ames, where she has been associated for eleven years, from Stanford University, and from previous jobs at the University of Colorado and the Jet Propulsion Laboratory. She earned her B.S. in Physics in 1984 from the University of California, Irvine and her M.S. in Physics (w/computational physics option) in 1991 from San Jose State University. When she is not working at her fulltime job (Stanford), or preparing for her move to Germany to begin an astrophysics PhD project (April 1998), she works on the IDL version (called Wavelet Workbench) of a freeware Mat lab wavelet algorithm package called Wave Lab, a joint Stanford/NASA Ames project.

REFERENCES