A Multiple Attribute Decision Making Methodology for Process Optimization in Small Scale Industries

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Abstract – This paper focused on systematic attributes identification which are basically the key performance contributors. Scientific approach of MADM will be adopted to find out the best alternative amongst various choices. Decision making is the study of identifying and choosing alternatives based on the values and preferences of the decision maker. Making a decision implies that there are alternative choices to be considered, and in such a case we want not only to identify as many of these alternatives as possible but to choose the one that best fits with our goals, objectives, desires, values, and so on..” Decision maker (DM) is often faced with the problem of selecting alternatives that are associated with conflicting attributes. Multiple Attribute Decision Making (MADM) involves “making preference decisions (such as evaluation, prioritization, and selection) over the available alternatives that are characterized by multiple, usually conflicting, attributes”. The problems of MADM are diverse, and can be found in virtually any topic. Even Ben Franklin, over 200 years ago recognized the presence of multiple attributes in everyday decisions, and suggested a workable solution. Effective MADM methods such as AHP, TOPSIS, and PROMETHEE will be used to analyse the problem and to find out best alternative.

Keywords – Multiple Attribute Decision Making (MADM), Optimization, Small Scale Industries, Attributes, AHP, TOPSIS, PROMETHEE

I. Introduction

Manufacturing is the backbone of any industrialized nation. Its importance is emphasized by the fact that, as an economic activity, it comprises approximately 20 to 30% of the value of all goods and services produced. A country’s level of manufacturing activity is directly related to its economic health. In general, the higher the level of manufacturing activity in a country, the higher will be the standard of living of its people. [8] .The selection decisions are complex, as decision making is more challenging today. Necessary conditions for achieving efficient decision making consist in understanding the current and upcoming events and factors influencing the whole manufacturing environment, in exploring the nature of decision-making processes and the reach of different typologies of methods and techniques, and finally in structuring appropriately the decision-making approach based on a wide range of issues related to manufacturing systems design, planning, and management. [10] Decision makers in the manufacturing sector frequently face the problem of assessing a wide range of alternative options, and selecting one based on a set of conflicting criteria. It must be noted that in choosing the right alternative, there is not always a single definite criterion of selection, and decision makers have to take into account a large number of criteria including technological, economic, ethical, political, legal, and social factors. [8] Multi-attribute decision making method, which was taking as the base for the decision making model, is one of the decision-making support methods. The decision making model is based on a chosen list of criteria, parameters, variables or factors, which we wish to monitor in the decision making process. [7] This paper focuses on simple procedures which lead the decision maker to select the best alternative from the set of alternatives available.

II. Multi-Attribute Decision Making

Consider a multi-attribute decision making problem with "*m*" criteria and "*n*" alternatives. Let $C_1,\ldots,C_m$ and $A_1,\ldots,A_n$ denote the criteria and alternatives, respectively. A standard feature of multi-attribute decision making methodology is the *decision table* as shown below. In the table each row belongs to a criterion and each column describes the performance of an alternative. As shown in decision table, weights $w_1,\ldots,w_m$ are assigned to the criteria. Weight w_i reflects the relative importance of criteria C_i to the decision, and is assumed to be positive. The weights of the criteria are usually determined on subjective basis. They represent the opinion of a single decision maker or synthesize the opinions of a group of experts using a group decision technique, as well.

The values $x_1,\ldots,x_n$ associated with the alternatives in the decision table are the final ranking values of the alternatives.

Table -1 The Decision Table

		x_1	.	.	x_n
		$\mathbf{A}_1$	.	.	$\mathbf{A}_n$
w_1	$\mathbf{C}_1$	a_{11}	.	.	a_{m1}
.	.	.	.	.	.
.	.	.	.	.	.
w_m	$\mathbf{C}_m$	a_{m1}	.	.	a_{mn}

2.1 Analytical Hierarchy Process (AHP)–

AHP is a widely used multi-criteria decision making tool. Unlike the conventional methods, In AHP the pair-wise comparisons are used to derive accurate ratio and scale priorities. Developed by Thomas Saaty [9], AHP provides a proven, effective means to deal with complex decision making and can assist in identifying and weighing criteria, analyzing the data collected and expediting the decision-making process. AHP helps capture both subjective and objective evaluation measures, providing a useful mechanism for checking the consistency of the evaluations thus reducing bias in decision making, [3].

The relevant attributes of the decision problem such as the selection criteria and objectives reside between the top and bottom levels. Next, relative weights to each item in the corresponding level are assigned. Scores are then synthesized through the model, yielding a composite score for each choice at every layer, as well as an overall score. This relative scoring within each level will result in a matrix of scores, say *a (i, j)*. The matrix holds the expert judgment of the pair-wise comparisons. However, the judgment should be consistent.

In general, the inconsistency ratio should be less than 0.1 or so to be considered reasonably consistent. Particularly, a matrix *a (i, j)* is said to be consistent if all its elements follow the transitivity and reciprocity rules below:

$$a_{i,j} = a_{i,k} * a_{k,j} \qquad (1)$$

$$a_{i,j} = \frac{1}{a_{j,i}} \qquad (2)$$

where i, j and k are any alternatives of the matrix, [3]. The relational scale used in ranking is presented in Table 2.

Table 2: AHP importance scale [6].

Intensity of Importance	Relative Importance
1	No Importance
2	Weak or slight
3	Moderate importance
4	Moderate plus
5	Strong importance
6	Strong plus
7	Very strong or demonstrated importance
8	Very, very strong
9	Extreme importance

The pair-wise comparison matrices can also be represented as:

$$A = \begin{bmatrix} a11 & \cdots & a1n \\ \vdots & \ddots & \vdots \\ an1 & \cdots & ann \end{bmatrix} = \begin{bmatrix} W1/W1 & \cdots & W1/Wn \\ \vdots & \ddots & \vdots \\ Wn/W1 & \cdots & Wn/Wn \end{bmatrix} \quad (3)$$

For a consistent matrix, we can demonstrate that:

$$A = \begin{bmatrix} W1/W1 & \cdots & W1/Wn \\ \vdots & \ddots & \vdots \\ Wn/W1 & \cdots & Wn/Wn \end{bmatrix} X \begin{bmatrix} w1 \\ \vdots \\ wn \end{bmatrix} = n \begin{bmatrix} w1 \\ \vdots \\ wn \end{bmatrix} \quad (4)$$

Or in a matrix form:

$$A*w = n*w \quad (5)$$

where A is the comparison matrix , w is the eigenvector and *n* is the dimension of the matrix. The equation above can be treated as an eigenvalue problem. For a slightly inconsistent matrix, the eigenvalue and the eigenvector are only slightly modified. Proved in [9], Saaty demonstrated that for consistent reciprocal matrix, the largest eigenvalue is equal to the number of comparisons, or $\lambda_{max} = n$. Then he gave a measure of consistency, called Consistency Index as a deviation or a degree of consistency using the following formula:

$$CI = \frac{\lambda max - n}{n-1} \quad (6)$$

Knowing the Consistency Index, the next question is how do we use this index? Again, the research in [3, 9] proposed to use the index by comparing it with the appropriate random consistency index through picking randomly generated reciprocal matrix using the scale: 1/9, 1/8… 1… 8, 9 and then get the random Consistency Index. The Average Random Consistency Index of a sample size of 500 matrices is shown in the table below:

Table 3: Random index (RI) for the factors used in the decision making process.

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49	1.51	1.48	1.56	1.57	1.59

Proposed by [9], a Consistency Ratio is a comparison between Consistency Index and Random Consistency Index, or in formula:

$$CR = \frac{CI}{RI} \quad (7)$$

If the value of Consistency Ratio is smaller or equal to 10%, the inconsistency is acceptable. Alternately, if the Consistency Ratio is greater than 10%, the subjective judgment should be revised.

2.2. The PROMETHEE method –

The decision table is the starting point of the PROMETHEE methodology introduced by Brans and Vincke (1985) and Brans et al. (1986). The scores *aij* need not necessarily be normalized or transformed into a common dimensionless scale. We only assume that, for the sake of simplicity, a higher score value means a better performance. It is also assumed that the weights *wi* of the criteria have been determined by an appropriate method (this is not a part of the PROMETHEE methods), furthermore, $\sum_{i=1}^{m} Wi = 1$. Here, following Brans and Mareschal (1994), we give a brief review of the PROMETHEE methods.

In order to take the deviations and the scales of the criteria into account, a preference function is associated to each criterion. For this purpose, a preference function $P_i(A_j,A_k)$ is defined, representing the degree of the preference of alternative A_j over A_k for criterion C_i. We consider a degree in normalized form, so that

$0 \le P_i(A_j,A_k) \le 1$ and

$P_i(A_j,A_k) = 0$ means no preference or indifference,

$P_i(A_j,A_k) \approx 0$ means weak preference,

$P_i(A_j,A_k) \approx 1$ means strong preference, and

$P_i(A_j,A_k) = 1$ means strict preference.

In most practical cases $Pi(Aj,Ak)$ is function of the deviation $d = a_{ij} - a_{ik}$, i.e. $P_i(A_j,A_k)= p_i\,(a_{ij} - a_{ik})$, where p_i is a nondecreasing function, $p_i(d)=0$ for $d \leq 0$, and $0 \leq p_i\,(d) \leq 1$ for $d > 0$. A set of six typical preference functions was proposed by Brans and Vincke (1985) and Brans et al. (1986). The simplicity is the main advantage of these preferences functions: no more than two parameters in each case, each having a clear economical significance.

A multicriteria preference index $\pi\,(A_j,A_k)$ of A_j over A_k can then be defined considering all the criteria:

$$\Pi(A_j, A_k) = \sum_{i=1}^{m} W_i P_i(Aj, Ak)$$

This index also takes values between 0 and 1, and represents the global intensity of preference between the couples of alternatives.

In order to rank the alternatives, the following precedence flows are defined:

Positive outranking flow:

$$\Phi^{+}(A_j) = \frac{1}{n-1} \sum_{k=1}^{n} \pi(Aj, Ak)$$

Negative outranking flow:

$$\Phi^{-}(A_j) = \frac{1}{n-1} \sum_{k=1}^{n} \pi(Aj, Ak)$$

The positive outranking flow expresses how much each alternative is outranking all the others. The higher $\varphi + (Aj)$, the better the alternative. $\varphi + (Aj)$ represents the *power* of Aj, its *outranking* character.

The negative outranking flow expresses how much each alternative is outranked by all the others.

The smaller $\varphi - (Aj)$, the better the alternative. $\varphi - (Aj)$ represents the *weakness* of Aj, its *outranked* character.

The PROMETHEE I partial ranking

Aj is *preferred to* Ak when $\varphi + (Aj) \geq \varphi + (Ak)$, $\varphi - (Aj) \leq \varphi - (Ak)$, and at least one of the inequalities holds as a strict inequality.

Aj and Ak are *indifferent* when $\varphi + (Aj)=\varphi + (Ak)$ and $\varphi - (Aj)=\varphi - (Ak)$.

Aj and Ak are *incomparable* otherwise.

In this partial ranking some couples of alternatives are comparable, some others are not. This information can be useful in concrete applications for decision making.

The PROMETHEE II complete partial ranking

If a complete ranking of the alternatives is requested by the decision maker, avoiding any incomparabilities, the *net outranking flow* can be considered:

$\varphi\,(Aj) = \varphi + (Aj) - \varphi - (Aj)$.

The PROMETHEE II complete ranking is then defined:

Aj is *preferred to* Ak when $\varphi\,(Aj) > \varphi\,(Ak)$, and

Aj and Ak are *indifferent* when $\varphi\,(Aj)=\varphi\,(Ak)$.

All alternatives are now comparable, the alternative with the highest $\varphi\,(Aj)$ can be considered as best one. Of course, a considerable part of information gets lost by taking the difference of the positive and negative outranking flows.

PROMETHEE V: Optimization under constraints

Optimization under constraints is a typical problem of operations research. The problem of finding an optimal selection of several alternatives, given a set of constraints, belongs to this field.

PROMETHEE V extends PROMEHEE II to this selection problem. The objective is to maximize the total net outranking flow value of the selected alternatives meanwhile they are feasible to the constraints. Binary variables are introduced to represent whether an alternative is selected or not, and integer programming techniques are applied to solve the optimization problem.

2.3. *TOPSIS method* –

This method not only considers DM's preference and experience for selection criteria, but also includes various tangible constraints, for example, the buyer's budget, suppliers' capacity and delivery time. On the other hand, fuzzy TOPSIS approach helps to convert DMs' preference and experience to meaningful results by applying linguistic values to assess each criterion and alternative.

Linguistic variables shown in figure below are used to assess the importance weight of each criterion.

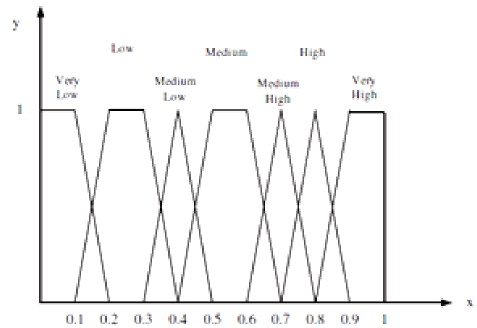

Figure 1. Linguistic variables for importance weight of each criterion.

Linguistic variables shown in figure below are used to rate each manufacturing line with respect to each criterion.

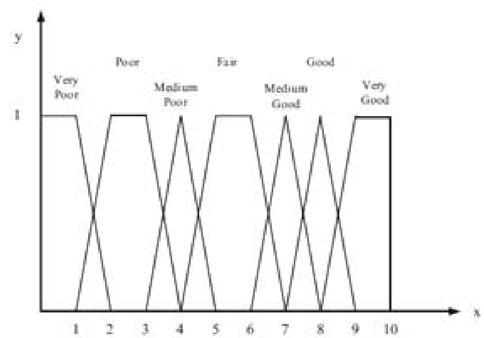

Figure 2. Linguistic variables for ratings

The linguistic evaluations are then converted into trapezoidal fuzzy numbers to construct a fuzzy-decision matrix and determine the fuzzy weight of each criterion, as shown in Table 4.

Table 4. Fuzzy decision-matrix and fuzzy weights of three manufacturing lines

	C_1	C_2	C_3
S_1	(7,8,8,9)	(5,7,8,10)	(5,8,9,10)
S_2	(5,6.7,7.3,9)	(7,8,8,9)	(7,8,8,9)
S_3	(8,8.7,9.3,10)	(8,9,10,10)	(5,6,7,8)
S_4	(7,8,8,9)	(5,7.7,8.3,10)	(5,6.7,7.3,9)
Weights	**(0.7,0.8,0.8,0.9)**	**(0.7,0.8,0.8,0.9)**	**(0.7,0.9,1.0,1.0)**

Table 4 is used to construct a normalized fuzzy-decision matrix. Using the normalized fuzzy decision matrix in Table 5, a weighted normalized fuzzy decision matrix is constructed as shown in Table 6.

Table 5. Normalized fuzzy decision-matrix

	C_1	C_2	C_3
1	(0.7,0.8,0.8,0.9)	(0.5,0.7,0.8,1)	(0.5,0.8,0.9,1)
2	(0.5,0.67,0.73,0.9)	(0.7,0.8,0.8,0.9)	(0.7,0.8,0.8,0.9)
3	(0.8,0.87,0.93,1)	(0.8,0.9,1,1)	(0.5,0.6,0.7,0.8)
4	(0.7,0.8,0.8,0.9)	(0.5,0.77,0.83,1)	(0.5,0.67,0.73,0.9)

Table 6. Weighted normalized fuzzy decision-matrix

	C_1	C_2	C_3
1	(0.49,0.64,0.64,0.81)	(0.35,0.56,0.64,0.9)	(0.35,0.72,0.9,1)
2	(0.35,0.53,0.59,0.81)	(0.49,0.64,0.64,0.81)	(0.49,0.72,0.8,0.9)
3	(0.56,0.69,0.75,0.9)	(0.56,0.72,0.8,0.9)	(0.35,0.54,0.7,0.8)
4	(0.49,0.64,0.64,0.81)	(0.35,0.61,0.67,0.9)	(0.35,0.6,0.73,0.9)

III. EXPERIMENT AND RESULT

For the purpose of research work, production lines of valve body are considered. There are two production lines available for the production of valve body. These lines consist of a combination of conventional and non-conventional machineries.

Attributes applicable for production of VALVE BODY are given in Table 7. These attributes affect the rate of production to a great extent. Therefore the thorough study of these attributes is necessary and decision regarding the selection of production lines should be made depending on these attributes.

Table 7: Applicable Attributes

S. N.	Attributes	Line-1	Line-2
1	Production Quantity	270/Day	266/Day
2	Production Cycle Time	5066 Sec.	5074 Sec.
3	Tool Wear Rate 1	Carbide Tool- 80/M	Carbide Tool- 74/M
4	Tool Wear Rate 2	H.S.S.- 30/M	H.S.S.- 28/M
5	Average Tool Life	24-30 Hrs.	20-24 Hrs.
6	Work In Progress	60	65
7	Temperature	34°C	30°C
8	Humidity	64%	60%
9	Sound	98 Db Spl	105 Db Spl
10	Light	800 (Lux)	700 (Lux)
11	Number Of Buffer Storages	JIT	2
12	Raw Material & Tools Availability	Very Good	Good
13	Skill Level Of Workers & Training Schedule	Adequate	Good

When AHP method was first applied to the applicable attributes for both the production lines, the following result was obtained.

Decision Matrix

		Attributes												
		PQ	PCT	TWR1	TWR2	ATL	WIP	TEMP	HMDT	SO	LT	NBS	RMTA	SLTS
Production Line	Line-1	270	5066	80	30	27	60	34	64	98	800	0.8950	0.8950	0.4950
	Line-2	266	5074	74	28	22	65	30	60	105	700	0.4950	0.6950	0.6950

Normalized Matrix

		Attributes												
		PQ	PCT	TWR1	TWR2	ATL	WIP	TEMP	HMDT	SO	LT	NBS	RMTA	SLTS
Production Line	Line-1	1	1	1	1	1	1	0.8823	0.9375	1	1	1	1	0.7122
	Line-2	0.9851	0.9984	0.925	0.9333	0.8148	0.9230	1	1	0.9333	0.875	0.5530	0.7765	1

Pairwise Comparison Matrix

		Attributes												
		PQ	PCT	TWR1	TWR2	ATL	WIP	TEMP	HMDT	SO	LT	NBS	RMTA	SLTS
Attributes	PQ	1.000	2.000	4.000	4.000	3.000	3.000	6.000	7.000	7.000	8.000	3.000	4.000	4.000
	PCT	0.500	1.000	2.000	2.000	1.500	1.500	3.000	3.500	3.500	4.000	1.500	2.000	2.000
	TWR1	0.250	0.500	1.000	1.000	0.750	0.750	1.500	1.750	1.750	2.000	0.750	1.000	1.000
	TWR2	0.250	0.500	1.000	1.000	0.750	0.750	1.500	1.750	1.750	2.000	0.750	1.000	1.000
	ATL	0.333	0.667	1.333	1.333	1.000	1.000	2.000	2.333	2.333	2.667	1.000	1.333	1.333
	WIP	0.333	0.667	1.333	1.333	1.000	1.000	2.000	2.333	2.333	2.667	1.000	1.333	1.333
	TEMP	0.167	0.333	0.667	0.667	0.500	0.500	1.000	1.167	1.167	1.333	0.500	0.667	0.667
	HMDT	0.143	0.286	0.571	0.571	0.429	0.429	0.857	1.000	1.000	1.143	0.429	0.571	0.571
	SO	0.143	0.286	0.571	0.571	0.429	0.429	0.857	1.000	1.000	1.143	0.429	0.571	0.571
	LT	0.125	0.250	0.500	0.500	0.375	0.375	0.750	0.875	0.875	1.000	0.375	0.500	0.500
	NBS	0.333	0.667	1.333	1.333	1.000	1.000	2.000	2.333	2.333	2.667	1.000	1.333	1.333
	RMTA	0.250	0.500	1.000	1.000	0.750	0.750	1.500	1.750	1.750	2.000	0.750	1.000	1.000
	SLTS	0.250	0.500	1.000	1.000	0.750	0.750	1.500	1.750	1.750	2.000	0.750	1.000	1.000

Weights of the Attributes												
PQ	PCT	TWR1	TWR2	ATL	WIP	TEMP	HMDT	SO	LT	NBS	RMTA	SLTS
0.245	0.123	0.061	0.061	0.082	0.082	0.041	0.035	0.035	0.031	0.082	0.061	0.061

Production Line	Rank
Line-1	0.975435
Line-2	0.909553

When PROMETHEE method was applied to the applicable attributes for both the production lines, the following result was obtained.

Aggregate preference Function Matrix

	Line-1	Line-2	Ø+
Line-1	0	0.863	0.863
Line-2	0.137	0	0.137
Ø-	0.137	0.863	

Production Line	Ø+	Ø-	Net Ø
Line-1	0.863	0.137	0.726
Line-2	0.137	0.863	-0.726

When TOPSIS method was applied to the applicable attributes for both the production lines, the following result was obtained.

	Ideal Best	Ideal Worst
PQ	0.1745	0.1719
PCT	0.0870	0.0869
TWR1	0.0448	0.0414
TWR2	0.0446	0.0416
ATL	0.0636	0.0518
WIP	0.0603	0.0556
TEMP	0.0307	0.0271
HMDT	0.0255	0.0239
SO	0.0256	0.0239
LT	0.0233	0.0204
NBS	0.0718	0.0397
RMTA	0.0482	0.0374
SLTS	0.0497	0.0354

Separation Measure			
Production Line	sa+	sa-	p(r)
Line-1	0.0152	0.0365	0.7067
Line-2	0.0366	0.0151	0.2925

IV.CONCLUSION

When the MADM was applied to the production lines of Valve Body Manufacturing, it was observed that the production line 1 is having the highest rank when calculated using all the three methods.

REFERENCES

[1] Adel Hatami-Marbini, Saber Saati (2009) An application of fuzzy TOPSIS method in an SWOT analysis *Mathematical Science Vol.3, No. 2,* 173-190

[2] B. Malakooti, Y.Q. Zhou, (1994) "Feed forward artificial neural networks for solving discrete multiple criteria decision making problems", *Management Science* 401542-1561

[3] Ishizaka Alessio and Lusti Markus, (2002) An Intelligent Tutoring System for AHP, *Proceedings of the 9th International Conference on Operational Research KOI*, 215-223

[4] J.H.P. Paelinck, (1976) "Qualitative multiple criteria analysis", environmental protection and multiregional development, *Papers of the Regional Science Association* 36, 59-74

[5] J.Ma, Z.P. Fan and L.H. Huang, (1998) "A subjective and objective integrated approach to determine attribute weights", *European Journal of Operational Research*

[6] János Fülöp (2000) "Introduction to Decision Making Methods" *Hungarian Academy of Sciences Journal*

[7] M. Bohanec, V. Rajkovič, (1999) "Multi-Attribute Decision Modeling: Industrial Applications of DEX", *Informatica*, vol. 23, 487–491

[8] Dr. R. Venkata Rao (2007) "Decision Making in the Manufacturing Environment." *Springer Series in Advanced Manufacturing*

[9] Saaty, T.L. (1980) The Analytical Hierarchy Process, *McGraw-Hill, New York*

[10] W.D. Cook and M. Kress, (1994) "A multiple-criteria composite index model for quantitative and qualitative data", *European Journal of Operational Research* 78, 367-379