Torsional Vibration Analysis of Pre-Twisted Cantilever Beam using FEA

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Abstract- Structures having the shape of blades are often found in several practical engineering applications such as turbines and aircraft rotary wings. For reliable and economic designs of the structures, it is necessary to estimate the modal characteristics of those structures accurately. Among the dynamic characteristics of these structures, determining the natural frequencies and associated mode shapes are of fundamental importance in the study of resonant responses. A single free standing blade can be considered as a pretwisted cantilever beam with a rectangular cross-section. The torsional vibration of pre twist cantilever beam of rectangular cross section is done so that this resembles to a blade.
The differential equation for the torsional vibration of pre-twisted cantilever beam of rectangular cross section has been obtained. The beam is considered as Timoshenko beam instead of Euler-Bernoulli Beam or Rayleigh Beam because it will consider shear correction factor, rotary inertia, warping constant. This is again solved by fem software ANSYS and their results are compared.
Keywords- Pre-twisted Cantilever beam, Finite Element Analysis (FEA).

I. INTRODUCTION

The torsional vibration of a rotating structure can occur in many engineering applications such as turbo-machinery blades, slewing robot arms, aircraft propellers, helicopter rotors, and spinning spacecraft. To design these components, the dynamic characteristic, especially near resonant condition, need to be well examined to assure a safe operation. Among the dynamic characteristics of these structures, determining the natural frequencies and associated mode shapes are of fundamental importance in the study of resonant responses. It is very important for manufacturers of turbo machinery components to know the natural frequencies of the rotor blades, because they have to make sure that the turbine on which the blade is to be mounted does not have some of the same natural frequencies as the rotor blade. Otherwise, a resonance may occur in the whole structure of the turbine, leading to undammed vibrations, which may eventually wreck the whole turbine.

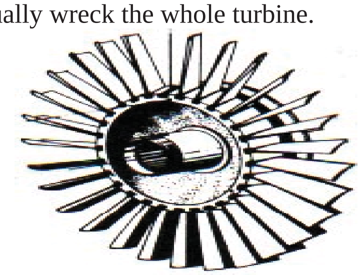


Figure 1. Schematic view of a part of a steam turbine

A single free standing blade can be considered as a pre-twisted cantilever beam with a rectangular cross-section. Vibration characteristics of such a blade are always coupled between the two bending modes in the flap wise and chord wise directions and the torsion mode. The problem is also complicated by several second order effects such as shear deformations, rotary inertia, and fibre bending in torsion, warping of the cross-section, root fixing and Coriolis accelerations.

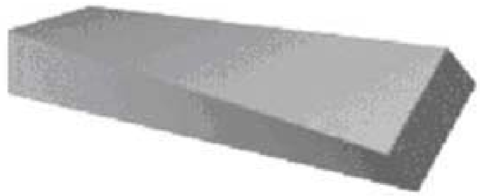


Figure 2. Pre-Twisted Beam Models

The torsional vibration occurs when the centroid and the shear centre of the cross section of the beam do not coincide. This lack of coincidence between the centroid and the shear centre occurs when the beam has less than two axis of symmetry or has anisotropy in the material. This makes the torsional axis different from the elastic axis and thus causes torsional vibration when flexural vibration occurs. When the beam is isotropic and the cross-section of it has two axes of symmetry, centroid and shear centre coincides and flexural vibrations and torsional vibration become independent. The flexural-torsional coupled vibration can be analyzed by combining one of the beam theories for bending with a torsional theory and a consideration of the various warping effects. The simplest model for the analysis of coupled bending and torsional vibration is combining the classical Bernoulli- Euler theory for bending and St. Venant theory for torsion .Inclusion of a warping effect, Bishop and Miao results in a better approximation, especially for higher modes. Also, for non slender beams, applying the Timoshenko Beam theory instead of the Bernoulli-Euler theory along with the inclusion of a warping effect can improve the accuracy for higher modes.

1.1 Types of Vibrations-

Vibration can be defined as regularly repeated movement of a physical object about a fixed point. Vibrations can be classified based on various factors like

a) Nature of excitation (usually the excitation will be periodic).
b) Nature of displacement.

1.2 Nature of Vibration-

Here the vibration depends on the nature of deformation of the beam, when the external forces act on the system. These are of two types namely:-

a) Flexural or transverse vibration.
b) Torsional vibration.

1.3 Flexural Vibration or Transverse Vibration-

These vibrations are only due to the bending of the beam. When the centroid and shear centre both coincides and the load also falls at the same location, then the beam or the physical system is said to have only bending. There are two components of this bending vibration namely:

a) Transverse vibration or Flexural vibration
b) Longitudinal vibration.

1.4 Torsional Vibration-

In the engineering applications it is always not possible to have the cross section which is symmetric. There can be components with various cross sections, with different axis of symmetry.

a) Double axis symmetry
b) Single axis symmetry
c) Zero axis symmetry or unsymmetric bodies

1.5 Beam Theory-

The study of the torsional vibration start from the basic beam theories, it is important to review and study the derivation of the various basic beam theories prior to the study of torsional vibration of beams. Here, basic beam theories of Euler-Bernoulli, Rayleigh, shear and Timoshenko beam theories are reviewed from their derivation. The assumptions made by all models are as follows.

1. One dimension (the axial direction) is considerably larger than the other two.
2. The material is linear elastic (Hookean).

3. The Poisson effect is neglected.
4. The cross-sectional area is symmetric so that the neutral and centroidal axes coincide.
5. The angle of rotation is small so that the small angle assumption can be used.

Euler beam theory discussed was considered, i.e., shear centre and warping are considered to determine the natural frequencies of a cantilever beam in this paper.

1.6 The Timoshenko Beam Model-

As mentioned earlier that the Euler Bernoulli beam theory can approximate the natural frequency in case of higher frequency modes and slender beam, hence Timoshenko beam theory can be implemented in such situations. In this theory deformation due to transverse shear and kinetic energy due to rotation of the cross-section become important. Energy expressions include both shear deformation and rotary inertia.

The assumption made in the previous theory that the plane sections which are normal to the undeformed centroidal axis remain plane after bending, will be retained. However, it will no longer be assumed that these sections remain normal to the deformed axis

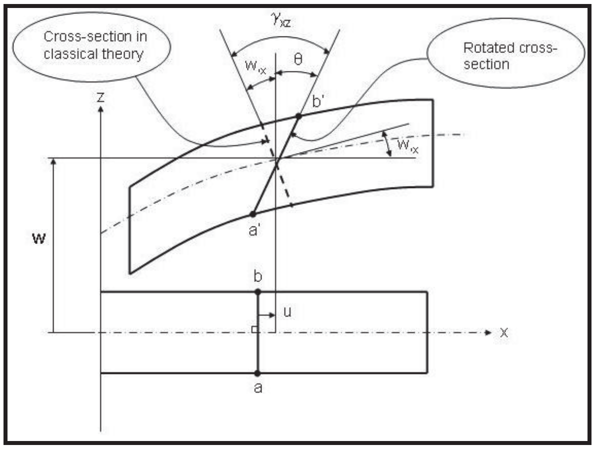


Figure 3. Timoshenko Beam

Assumptions made in Timoshenko Beam theory:-

a) Plane sections such as 'ab', originally normal to the centreline of the beam in the undeformed geometry, remain plane but not necessarily normal to the centreline in the deformed state.
b) The cross-sections do not stretch or shorten, i.e., they are assumed to act
   Like rigid surfaces.

The Timoshenko beam considers few factors which Euler Bernoulli beam theory does not consider. Hence it can be predicted that Timoshenko beam can give accurate results while considering higher frequencies and slender beams. Hence Timoshenko beam is considered in the present project.

II. FINITE ELEMENT SOLUTION

The Finite Element Method (FEM) is a numerical procedure that can be used to obtain solutions to a large class of engineering problems involving stress analysis, heat transfer, fluid flow and vibration and acoustics.

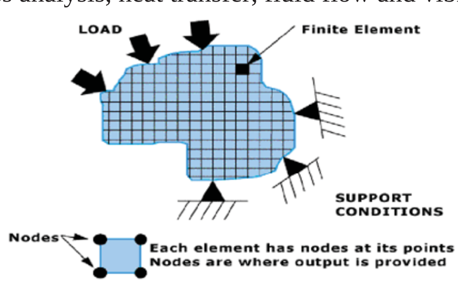


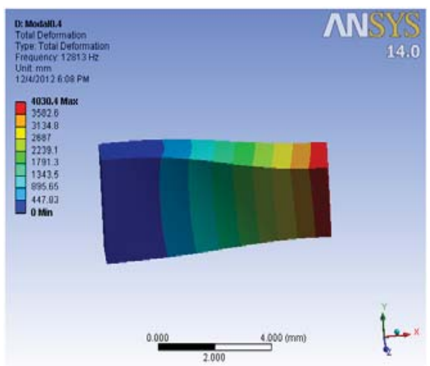
Figure 4. Description of the "finite element"

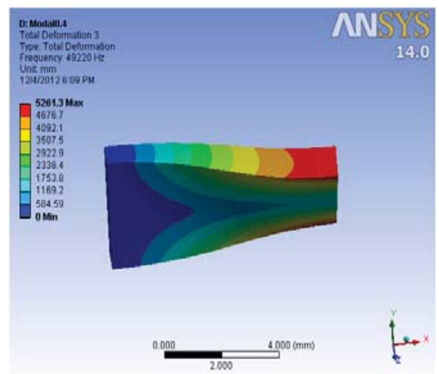
Basic ideas of the FEM originated from advances in aircraft structural analysis. The origin of the modern FEM may be traced back to the early 20th century, when some investigators approximated and modelled elastic continua using discrete equivalent elastic bars. However, Courant has been credited with being the first person to develop the FEM. He used piecewise polynomial interpolation over triangular sub regions to investigate torsion problems in a paper published in 1943. The next significant step in the utilization of Finite Element Method was taken by Boeing. In the 1950's Boeing, followed by others, used triangular stress elements to model airplane wings. But the term finite element was first coined and used by Clough in 1960. And since its inception, the literature on finite element applications has grown exponentially, and today there are numerous journals that are primarily devoted to the theory and application of the method.

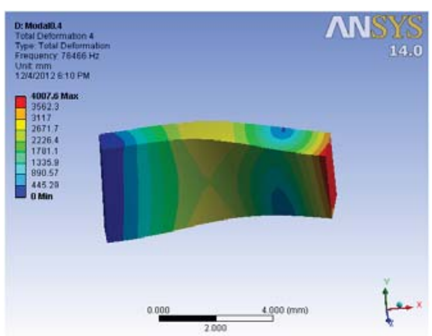
III. RESULTS AND DISCUSSION

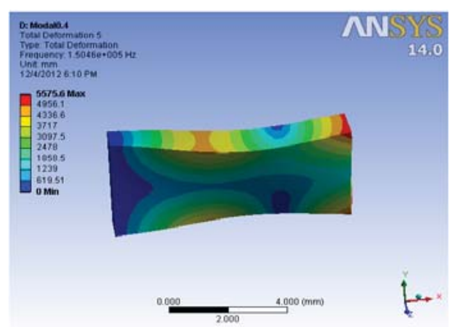
In order to validate the proposed finite element model for the vibration analysis of pre twisted Timoshenko beam, various numerical results are obtained and compared with available solutions in the published literature.

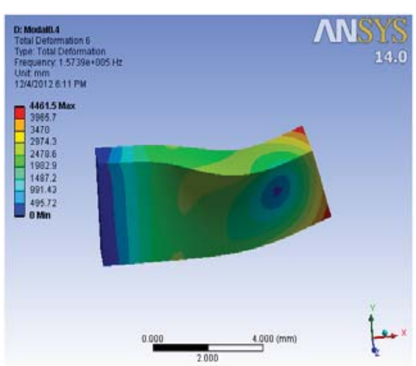
Mode shape of 0.4 radian twist with b/h =1/4 is shown below











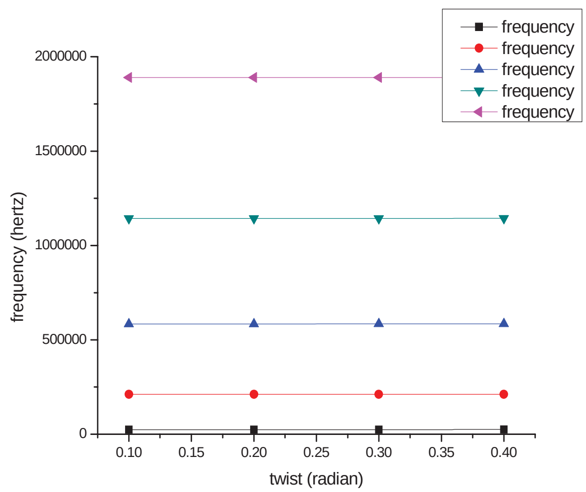
In order to validate the proposed finite element model for the vibration analysis of pre twisted Timoshenko beam, various numerical results are obtained and compared with available solutions in the published literature.

Comparison between ANSYS results and mathematical modelling results

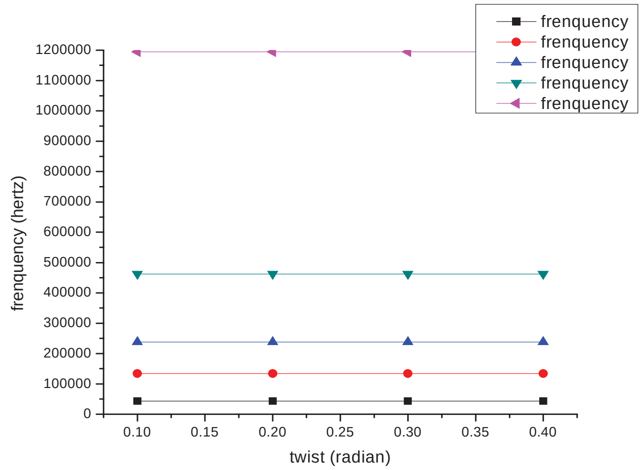
Tab 1. Values of Frequencies for b/h = 1/4 and for 0.4 radian twist

Mode no	Fem solution	Mathematical solution	Difference between the solutions in %
0	2.4097 E+04	2.43970E+04	1.23E+00
1	2.09927E+05	2.10927E+05	4.74E-01
2	5.70624E+05	5.77624E+05	1.21E+00
3	1.11913E+05	1.14113E+05	1.93E+00
4	1.865250E+06	1.88725E+06	1.17E+00

The various examples are considered to evaluate the present finite element formulation for the effects of related parameters (e.g. twist angle, length, breadth to depth ratio) on the natural frequencies of the pre-twisted cantilever Timoshenko beams. The natural frequency ratios for the first five modes of vibration are obtained for different breadth to height ratio and in each case with different twist angles. Graphs are plotted with frequency and twist and it can easily be checked out from the Figures that the natural frequencies increase as the twist angle increases.

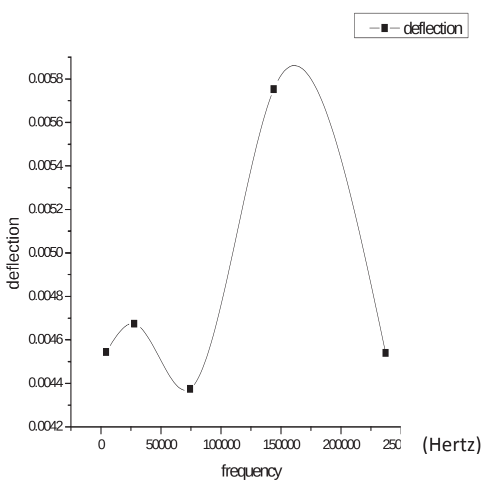


Graph 1. Combined graph for different mode between Frequency Vs twist for b/h ratio = 1/4

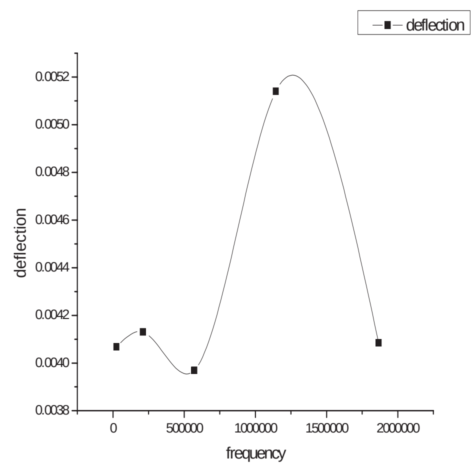


Graph 2. Combined graph for different mode between Frequency Vs twist for b/h ratio = 1/5

Graphs between deflection and frequency for different breadth to height ratio and twist angles are shown below



Graph 3. Deflection Vs frequency for b/h=1/5 and for a twist of 0.4 radians



Graph 4. Deflection Vs frequency for b/h=1/4 and for a twist of 0.4 radians

IV. CONCLUSION

- The equations of motion for the torsional vibration analysis of blades, which have a pre-twisted cross-section, arbitrary orientation are derived.
- The equations of motion are transformed into a dimensionless form by employing dimensionless variables and several dimensionless parameters representing area moment of inertia ratio, the pre-twist angle, are identified.
- The Garlekin's method used here to solve the equation gives an upper bound of frequencies. The resultant obtained for various frequencies of torsional vibrations shows that it increases with the amount of pre-twisted and the thinness of the beam.
- Numerical results of frequencies in different cases are validated the applicability of the proposed method for solving such an engineering problem. The pre-twisted angles influence the natural frequencies of the beams.

- The natural frequencies found were compared with the simulated FEM results. The simulation was carried in software ANSYS
- The simulation software provides the investigator with different mode shape and frequency for all the beam geometry and the resonant data obtained is compared with the mathematical models.
- It has been noted form graphs that for all the cases twist with 0.5 radians has maximum natural frequency.

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