Mathematical Modelling for Refrigerant Flow in Diabatic Capillary Tube

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Abstract- In the present investigation, an attempt is made to develop a mathematical model of diabatic capillary tube. The mathematical model has been developed by using equations of conservation of mass, momentum and energy. We need to calculate both single phase and two phase length for predicting the length of diabatic capillary tube. Flow of refrigerants inside a capillary tube is a complex phenomenon and is further complicated by heat exchange region in case of diabatic capillary tube. The presence of heat exchange region causes a delay of vaporization. When the vaporization starts pressure falls sharply. Friction Factor also plays an important role in the pressure drop. Moody (1944) correlation is used to calculate the friction factor. McAdams et al. (1942) viscosity correlation has been used to evaluate the two phase viscosity of the refrigerant. Input parameters have been taken from the data of Mendoca et al. (1998).

Nomenclature-

d	capillary tube internal diameter, m
D	coil diameter, m
Ds	suction line diameter, m
ΔT_{sub}	degree of sub cooling, K
E	roughness height, m
F	friction factor
G	mass velocity, kg/m²/s
L	capillary tube length, m
m	mass flow rate, kg/s
P	pressure, Pa
T	temperature, °c
Greek Letters-	
μ	viscosity, kg/m/s
ρ	density, kg/m^3
Subscripts	
h_x	heat exchanger
in	capillary tube inlet k
C	condenser
E	Evaporator
Out	outlet

I. INTRODUCTION

In this chapter, the capillary tubes, their classifications and the flow phenomena associated with capillary tube diabatic capillary tubes have also been discussed. The objectives of the proposed work have also been mentioned.

1.1 Capillary tube and its classification-

In a vapour compression refrigeration cycle, the refrigerant vapours emerging from the evaporator after producing the cooling effect enters the compressor. The compressor activates the refrigerant to high temperature and high pressure superheated vapours. These high temperature vapours get condensed inside the condenser. The saturated liquid refrigerant enters the capillary tube where the expansion pressure from high pressure to low pressure takes place. After expansion the refrigerant re-enters the evaporator and the cycle continues.

Capillary is used as an expansion device in a refrigerant in low capacity vapour compression systems like domestic refrigerators, freezers, window type air conditioners etc. A capillary tube is a 1-6 m long tube of drawn copper with an inside diameter generally from 0.5 to 2 mm, which connects the outlet of the condenser to the inlet of the evaporator. The capillary tube has low cost and is simple in construction. The refrigeration system employing capillary tube requires low starting torque motor as the pressure across the capillary tube equalizes.

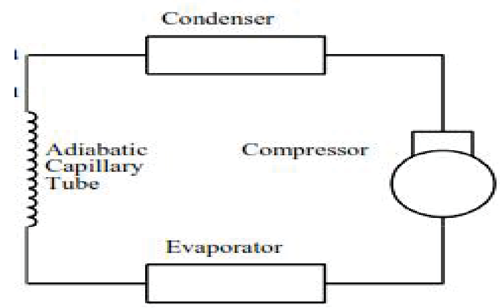


Fig. 1.1 Vapour Compression Refrigeration System

Depending upon the flow, the capillary tubes can be classified as follows:

- adiabatic capillary tube and
- diabatic capillary tube

In adiabatic arrangement, the capillary tube is thermally insulated and the heat exchange with the ambient is negligible.

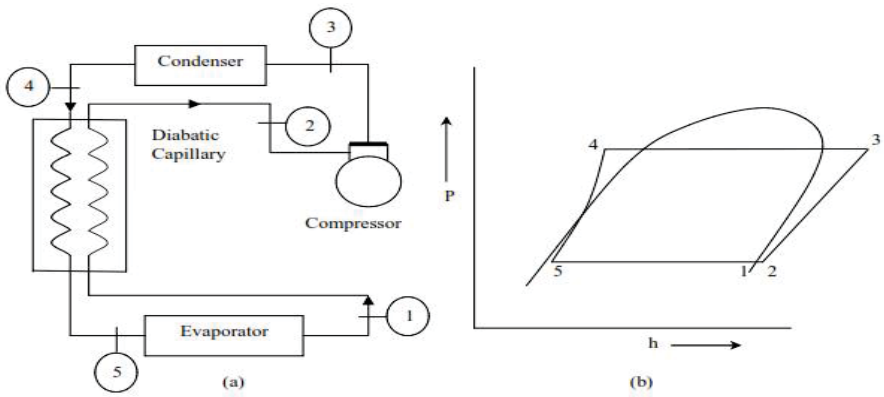


Fig. 1.2 (a) VCRS using diabatic capillary tube (b) p-h diagram

In a diabatic flow arrangement, shown in Fig. 1.2., the capillary tube is bonded with the cold compressor suction line in a counter flow arrangement. Employing diabatic capillary tube in a refrigeration system results in higher refrigerating effect, thus, a better system performance is achieved. As shown in Fig. 1.2, the enthalpy of liquid refrigerant flowing in the capillary tube continuously falls in the single phase region as well due to heat exchange with the cold suction line of compressor. With the result the enthalpy of the refrigerant falls continuously throughout the capillary tube length. The presence of heat exchanger produces considerable sub cooling in the liquid region, and consequently, causes a delay in flash point. Unlike adiabatic capillary tubes where the refrigerant temperature in the single phase region is constant, heat transfer to the suction line causes the temperature of refrigerant and, hence, the saturation pressure of the liquid refrigerant to fall. The reduction in saturation pressure causes a delay in the onset of flashing, and thus results, a longer single phase liquid length.

The thermal contact between the capillary tube and the compressor suction line can be attained in two ways viz., lateral and concentric arrangement.

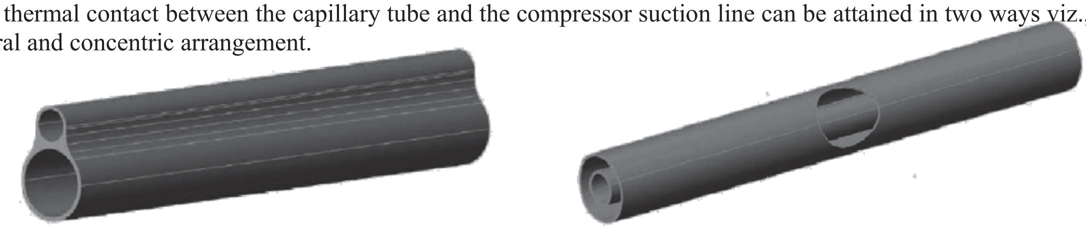


Fig. 1.3 (a) Lateral arrangement (b) Concentric arrangement

The two arrangements are shown in Fig 1.3. In lateral configuration, the capillary tube is bonded with the compressor suction line by means of a solder or brazing joint. Further, the copper tape is wound around the two bonded tubes so that the walls of the tubes could attain thermal equilibrium. In concentric arrangement, the capillary tube occupies the core of suction line compressor as shown in Fig. 1.3 (b). In the concentric arrangement, the capillary tube is surrounded by the cold vapours of the suction line emerging from the evaporator. Thus, the capillary tube is in direct contact with the vapours heat transfer from the capillary tube to the suction line vapours will take placing. Therefore the concentric arrangement is preferred over the lateral arrangement as the contact thermal resistance is smaller and a better system performance is achieved.

II. MODELLING PROCEDURE

In this chapter, the mathematical model has been developed by applying laws of conservation of mass, momentum and energy on capillary tube and compressor suction - line. The mathematical model obtained is a set of differential equation.

2.1 Finite Difference Method-
Finite Difference methods are used to solve differential equations numerically. Physical domain is converted to computational domain by discretizing it into a number of linear elements. Following steps describe the procedure of solving differential equation.

- Governing differential equations are obtained by applying the laws of conservation of energy momentum and mass.
- Using finite difference approximation to transform a given differential equations into difference equations.
- Algebraic Equations in nodal unknowns are thus obtained.
- These simultaneous algebraic equations can be solved by any numerical method.
- The solution of these algebraic is the solution at the nodes.

The finite difference approximations using Taylor Series:

$$f(x_0+\Delta x) = f(x)_0 + \Delta x f'(x) + \frac{\Delta x^2}{2!} f''(x) + \cdots \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$f(x_0-\Delta x) = f(x)_0 - \Delta x f'(x) + \frac{\Delta x^2}{2!} f''(x) + \cdots \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Approximation for first Derivative (forward difference formulation) from equation (1)

$$f'(x_0) = \frac{f(x_0+\Delta x) - f(x)_0}{\Delta x} - f(x)$$

$$f'(x_0) = \frac{f(x_0+\Delta x) - f(x)_0}{\Delta x} - f(\Delta x)$$

Or

$$f'(x_0) \approx f(x_0+\Delta x) - f(x_0)$$

$$f_i = \frac{f_{i+1} - f_i}{\Delta x}$$

Backward difference formulation from equation (2)

$$f'(x_0) = \frac{f(x) - f(x_0-\Delta x)}{\Delta x} + f''^{(x)\frac{\Delta x}{2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$f_i' = \frac{f_i - f_{i-1}}{\Delta x} + f(\Delta x) \quad \textit{where } f(\Delta x) \textit{ containing higher order terms.}$$

$$f_i' \approx \frac{f_i - f_{i-1}}{\Delta x} \qquad \textit{having an order of accuracy } \Delta x$$

Central Difference formulation

$$f(x_0+\Delta x) - f(x_0-\Delta x) = 2\Delta x f'(x_0) + \frac{2\Delta x^3}{3!} f''(x_0)$$

$$f'(x_0) = \frac{f(x_0+\Delta x) - f(x_0-\Delta x)}{2\Delta x} - \frac{\Delta x^2}{3} f''(x_0) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

$$f_i' = \frac{f_{i+1} - f_{i-1}}{2\Delta x} \qquad \textit{having an order of accuracy } \Delta x$$

Approximation for second derivative (Central difference formulation) Adding Equation (1) and (2) we get

$$f(x_0+\Delta x)+f(x_0-\Delta x)=2f(x_0)+\frac{2\Delta x^2}{2!}f(x_0)+\cdots \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

$$f(x_0)\approx\frac{f(x_0+\Delta x)-2f(x_0)+f(x_0-\Delta x)}{\Delta x^2}-f(h^2)$$

$$f_i''=\frac{f_{i+1}-2f_i+f_{i-1}}{\Delta x^2}$$

2.2 Mathematical Modelling-

The diabatic capillary tube of lateral configuration is shown in Fig 2.1. In lateral configuration, the capillary tube is brazed on the suction line and then the assembly is wrapped with copper tape. The capillary tube is insulated to avoid heat transfer to the surroundings. For the purpose of analysis, capillary tube has been divided into three regions: the initial adiabatic length (L_{in}), the intermediary region length or the heat exchange region length (L_{hx}), and the final adiabatic length (L_f).

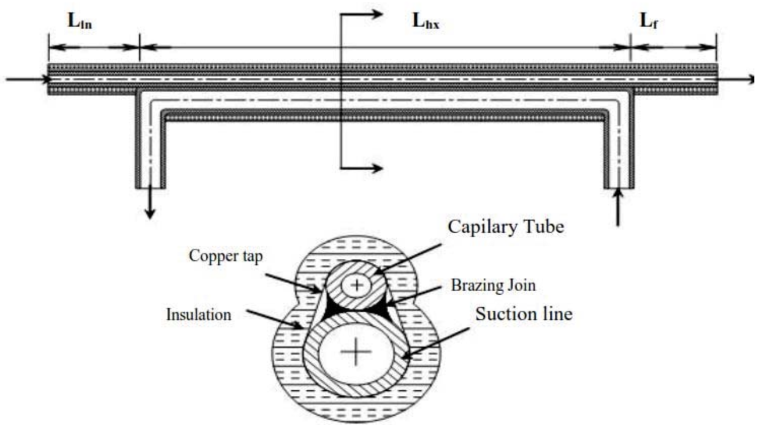


Fig. 2.1 Sectional views of diabatic straight capillary tube

The developed mathematical model is based on the following assumptions:

- Straight, horizontal and constant inner diameter and roughness of the capillary tube
- Negligible thermal resistance at the contact of the capillary tube and suction line
- One-dimensional and steady turbulent flow through the capillary tube
- Homogenous two-phase flow
- No heat exchange with the surroundings

Table 2.1 shows the governing equations for both capillary tube and suction line fluid flow using principle of mass, momentum and energy conservations.

Table 2.1: Governing Equations

Conservation Equations	Capillary Tube	Suction Line
Mass	$G_c=\frac{m}{A_c}$	$G_s=\frac{m}{A_s}$
Momentum	$-\frac{dP}{dZ}=f\frac{G_c^2}{2d_c}+G_c^2\frac{dv}{dz}$	$-\frac{dP}{dZ}=f\frac{G_s^2}{2d_s}+G_s^2\frac{dv}{dz}$
Energy	$\frac{dT_c}{dZ}=-\frac{h_c\pi d_c(T_c-T_W)}{mc_{pc}}-\frac{G_c^2}{2}\frac{d^2v}{dz}$	$\frac{dT_s}{dZ}=-\frac{h_s\pi d_s(T_W-T_s)}{mc_{ps}}-\frac{G_s^2}{2}\frac{d^2v}{dz}$

Heat balance at the tube wall can be expressed by considering the fluid flow in the capillary tube and suction line. Therefore,

$$h_c\pi d_c(T_c-T_W)=h_s\pi d_s(T_W-T_s)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Where, heat transfer coefficient h_c and h_s can be determine from the following equation

$$h=\frac{Nu.k}{d}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Where Nu is the Nusselt number is given by Gnielinski[10] correlation:

$$N_u = \frac{\left(\frac{f}{s}\right)(Re - 1000)\,Pr}{1 + 12.7\sqrt{\frac{f}{s}}\left(Pr^{\frac{2}{3}} - 1\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

Where, Pr is Prandtl number given by the following relationship:

$$Pr = \frac{Cp\mu}{k} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

The friction factor f is evaluated using Moody [15] correlation given by

$$f = \frac{1.325}{\left[\log\left(\frac{e}{3.7D} + \frac{5.74}{Re^{0.9}}\right)\right]^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

For the saturated or superheated vapour flowing through the suction line, neglecting the elevation difference and external work in the capillary tube, term $\frac{G_s^2}{2}\frac{dv^2}{dz}$ is dropped from the energy equation. Inside the suction line, thermo physical properties of superheated vapour have not been assumed constant and are developed as the function of suction line temperature. For different suction line temperatures, thermo physical properties are plotted as a function of suction line temperature. The best fit line is fitted and the equation for each thermo physical property is derived and then is used in the computer program.

After the development of mathematical model, as described in the previous section, the physical domain shown in Fig. 2.1 is converted into computational domain by discretizing the heat exchange region into 'N' number of infinitesimal elements of equal lengths. The governing differential equations of momentum and energy have been converted into the difference equations using forward difference method of the Finite Differences. These difference equations have been solved simultaneously to obtain the pressure and temperature distribution along the capillary tube. Further, the capillary tube lengths in the adiabatic regions of the capillary tube are calculated. In the adiabatic regions of the capillary tube, refrigerant temperature is constant when the flow is single phase flow while it falls sharply in the two-phase flow.

Heat exchange process in the single phase flow region Fig 2.2 shows the schematic diagram of the capillary tube and suction line heat exchanger with the heat transfer starts in the single phase region. As can be seen from the figure that refrigerant enters the capillary tube in a sub cooled liquid state. There is a drop in pressure at the inlet of capillary tube due to sudden contraction. The pressure at point 2 is given by:

$$P_2 = P_1 - k\frac{G_c^2 v}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

Where k is the entrance loss coefficient and its value has been taken as 1.5

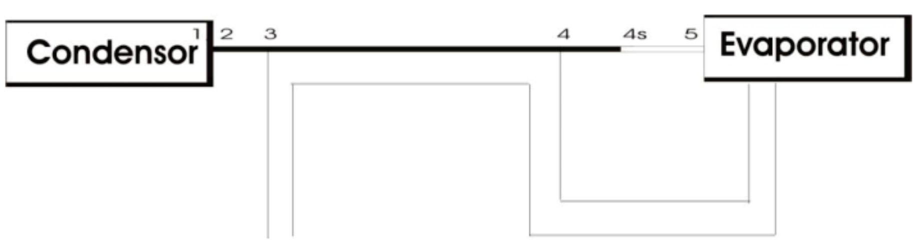


Fig. 2.2 Schematic diagram of capillary tube-suction line heat exchanger

The pressure drop in the initial length is evaluated using the momentum equation; therefore, pressure at point 3 can be written as

$$P_3 = P_2 - \frac{f_{sp}\, vG_c^2\, Lt_n}{2\,d_c} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

Since the flow in the capillary tube is adiabatic in the section 1-3, the refrigerant temperature will be constant in section 1-3.

$$T_3 = T_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

In section 3-4, the flow is diabatic, the temperature will not be constant and will vary along the length of capillary tube and in the suction line depending upon the rate of heat transfer. Since it is not known where the flash point location will lie in the heat exchange region, the single phase heat exchange region has been discretized into a number of infinitesimal elements of constant length $\Delta z = z_{i+1} - z_i$ Therefore, after each elemental length, the computer program checks the state of refrigerant. Later on all these infinitesimal elements are summed up to get the single phase heat exchange length. The pressure after each elemental length can be evaluated by converting the differential momentum equation into the difference equation using the forward difference formulation as:

$$-\frac{P_{i+1}-P_i}{Z_{i+1}-Z_i} = f\,\frac{G_c^2 v_i}{2d_c} + G_c^2\,\frac{v_{i+1}-v_i}{z_{i+1}-z_i} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (14)$$

The refrigerant temperature inside the capillary tube and the suction line can be calculated by applying the energy equation (shown in Table 2.1). As the energy equations are in differential form, they have been first converted difference equations as follows:

$$\frac{T_{ci+1}-T_{ci}}{Z_{i+1}-Z_i} = -\frac{h_c \pi d_c (T_{ci}-T_{wi})}{m_c c_{pc}} - \frac{G_c^2}{2}\,\frac{(v_{i+1}^2 - v_i^2)}{z_{i+1}-z_i} \quad \ldots\ldots\ldots\ (15)$$

$$\frac{T_{si+1}-T_{si}}{Z_{i+1}-Z_i} = -\frac{h_s \pi d_s (T_{wi}-T_{si})}{m_s c_{ps}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (16)$$

The wall temperature T_w has been calculated by the wall heat balance equation (6) as follows:

$$T_{wi} = -\frac{h_c d_c T_{ci} + h_s d_s T_{si}}{h_c d_c + h_s d_s} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (17)$$

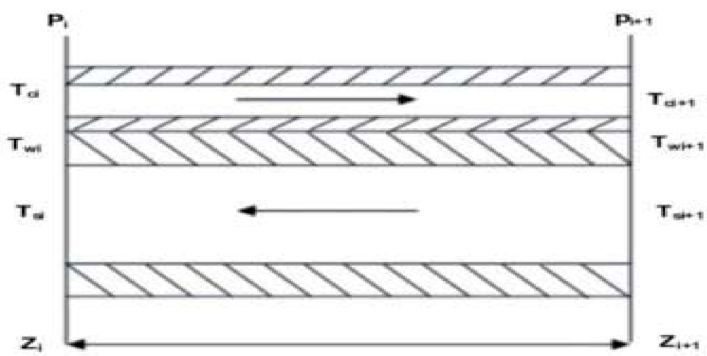


Fig. 2.3 Infinitesimal fluid element in the heat exchange region

The equations (15) to (17) have been solved simultaneously to obtain the distribution of temperature along the capillary tube-suction line heat exchanger. The temperature at the end of the heat exchange region i.e., T_4, will be known in this way. From 4 to 4s the single-phase flow is again adiabatic. Therefore -

$$T_{4s} = T_4 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (18)$$

In the section 4s-5, the flow of refrigerant is two-phase adiabatic flow. This section is again discretized into 'n' number of infinitesimal elements having constant pressure difference across them.

The pressure at any section 'i' is given by

$$P_i = P_{4s} - i\,dP \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (19)$$

Where, P_{4s} is the saturation pressure corresponding to the refrigerant temperature inside the capillary tube at the end heat exchange region, i.e., T_4. With the pressure P_i corresponding quality, x_i can be calculated and the entropy at the i-th section can be determined from

$$S_i = S_f + x_i S_{fg} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (20)$$

The incremental length, d_L, is calculated from section after section. For each section, pressure, temperature, vapour quality, viscosity, friction factor and entropy are calculated. For calculating viscosity McAdams[3] Correlation was used.

$$\frac{1}{\mu_{tp}} = \frac{x}{\mu_g} - \frac{1-x}{\mu_f} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (21)$$

It has been found that the entropy kept on increasing and after attaining certain value it started decreasing The calculations are done up to the point of maximum entropy. The pressure of the elemental section where entropy is maximum ($P_{i,smax}$), is then compared to the evaporator pressure (P_{evap})

If $P_{i\,smax} = P_e$ then $P_5 = P_e$
If $P_{i,\,smax} \neq P_e$ then $P_5 = P_{i,\,smax}$

Integrating momentum equation from table 2.1 for the section 4 – 5

$$L_{tp} = 2d\left(\frac{-1}{G^2}\int_{P_4}^{P_{smax}} \frac{\rho}{f_{tp}}\,dP + \int_{P_4}^{P_{smax}} \frac{dP}{\rho f_{tp}}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ (22)$$

The incremental length of each section is calculated using

$$\Delta L_i = \frac{2d}{f_{tpi}}\left(\frac{\rho_i \Delta P}{G^2} + \frac{\Delta P}{\rho_i}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

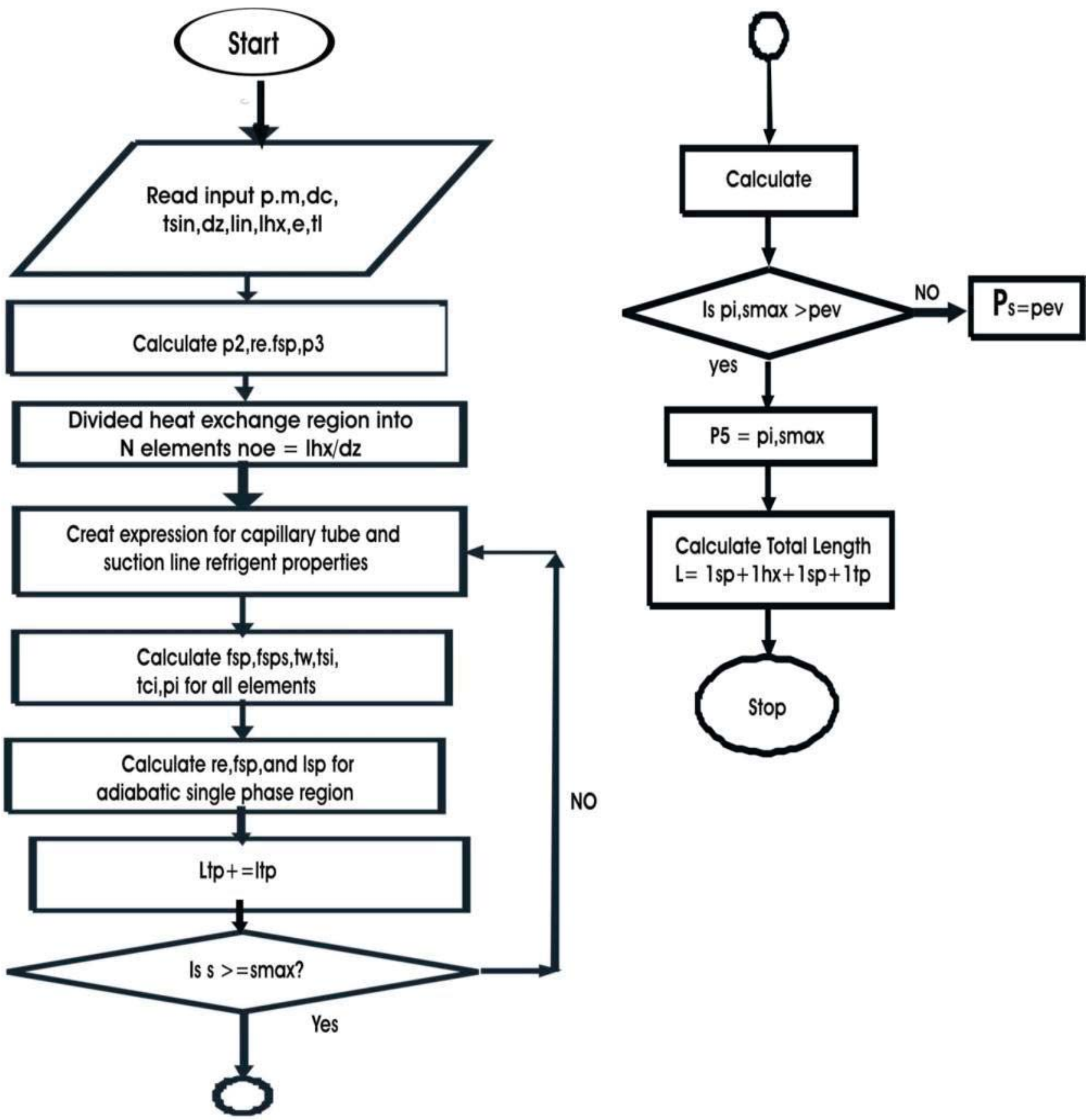
The total length of two-phase region is

$$L_{435} = L_{tp} = \sum_{t=1}^{n} \Delta L_i \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

The total length of capillary tubes is the sum of single and two-phase lengths, i.e.,

$$\mathrm{L} = \mathrm{L}_{13} + \mathrm{L}_{34} + \mathrm{L}_{443} + \mathrm{L}_{435} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

Following flow chart depicts the step by step procedure followed to obtain results with mathematical model.



III. CONCLUSION

In case of mathematical model of diabatic capillary tube, laws of conservation of mass, momentum and energy are used and lateral arrangement for capillary tube - suction line counter flow heat exchange region is used. The differential equations are converted into difference equations using forward difference formulation of Finite Difference Approximations. The solution is obtained by solving these difference equations simultaneously.

The phenomenon for recondensation and metastability should be incorporated for future research work on the development of mathematical model for diabatic capillary tubes.

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