Domination Number of Complement Graph

N.Thamaraikannan
Assistant Professor, Department of Mathematics
SSM College of Engineering, Namakkal, Tamilnadu, India.

J.Jagadeeswaran
Assistant Professor, Department of Mathematics
SSM College of Engineering, Namakkal, Tamilnadu, India.

S.Suresh
Assistant Professor, Department of Mathematics
SSM College of Engineering, Namakkal, Tamilnadu, India.

Abstract- In this paper the domination number of complement graph of a graph G are characterized and the relation between the domination number of complement graph of a graph G and the minimum degree of G are established.

Keywords: Domination number, Minimum degree, complement graph.

I . INTRODUCTION

Let G = (V, E) be a simple graph of order p and size q. The distance $d_G(u, v)$ or d(u, v) between two vertices u and v in G is the length of a shortest path joining u and v in the graph G. For $u \in V(G)$, let $N(u) = \{v \in V(G) : uv \in E(G)\}$ and d(u) = |N(u)|. The eccentricity of a vertex u is given by $e(u) = \max\{d(u, v) : v \in V(G)\}$ The radius and diameter of the graph G are given by $rad(G) = \min\{e(u) : u \in V(G)\}$ and $diam(G) = \max\{e(u) : u \in V(G)\}$ respectively. The minimum degree of a graph G is given by $\delta(u) = \min\{d(u) : v \in V(G)\}$.The complement graph $\overline{G}$ is a graph with vertex set V (G) and two vertices are adjacent in $\overline{G}$ if and only if $d_G(u, v) > 1$. In [3] the authors introduced the concept of Domination number of distance k-complement graph and in this paper the domination number of the complement graph $\overline{G}$ of a graph G are characterized and the relation between the domination number of $\overline{G}$ and the minimum degree of G are established.

Definition 1.1. For an integer $k \geq 0$, the distance k-complement graph, denoted by $G^c{}_k$, of a graph G = (V, E) is a graph whose vertex set is V (G) and two vertices are adjacent in $G^c{}_k$ if and only if $d_G(u, v) > k$. That is, the edge set is $\{uv : d_G(u, v) > k\}$.

Definition 1.2. Let *v* be a vertex of the graph *G* the degree $d(v)$ of *v* is the number of edges of *G* incident with *v*.

II. DOMINATION IN COMPLEMENT GRAPH

Definition 2.1. Let G = (V, E) be a graph. A subset D of V is said to be a dominating set of G if every vertex in V – D is adjacent to some vertex in D. The minimum cardinality of a dominating set is called the domination number of G and is denoted by $\gamma(G)$.

Definition 2.2. Let G = (V, E) be a graph and $k \geq 1$ be an integer. A subset D of V is said to be a distance k-dominating set of G if for every vertex u in V – D, there is some vertex v in D such that $d_G(u, v) \leq k$.

The minimum cardinality of a distance k-dominating set is called the distance k-domination number of G and is denoted by $\gamma_k(G)$.

By using the following theorem, we characterize the graphs attaining the upper bound on the domination number of the complement graph.

Theorem 2.3. [3] If a graph G has no isolated vertices and diam(G) ≥ 2k + 1, then $\gamma(G^c{}_k) = 2$

Theorem 2.4. [3] A graph G has an isolated vertex if and only if $\gamma(G^c{}_k) = 1$.

Theorem 2.5. A graph G has an isolated vertex if and only if $\gamma(\overline{G}) = 1$

Proof: Let u be an isolated vertex in G, then it has no adjacent vertices in G, clearly u is adjacent to all other vertices in $\overline{G}$, Now {u} is an dominant set of $\overline{G}$ with one element, hence $\gamma(\overline{G}) = 1$.

Proposition 2.6 Let G be a graph with $\delta(G) = 1$, then $\gamma(\overline{G}) = 2$

Proof: Let G be a graph with $\delta(G) = 1$, Let u be a vertex of G with degree 1, ie, u is adjacent with only one vertex in G (say v). Clearly u is adjacent to all other vertices in $\overline{G}$ except v, Now {u,v} is a dominant set of $\overline{G}$ with two elements, hence $\gamma(\overline{G}) = 2$

Theorem 2.7 Let G be a graph with $\delta(G) \geq 2$ **, then** $2 \leq \gamma(\overline{G}) \leq \delta(G) + 1$

Proof: Let u be a vertex of G with $N(u) = \delta(G)$,

Case : 1

If there is an vertex v such that N(u)∩N(v)=Φ, then all the vertices of N(u) are adjacent to v in $\overline{G}$, similarly all the vertices of N(v) are adjacent to u in $\overline{G}$, Now {u,v} is a dominant set of $\overline{G}$ with two vertices. Hence the lower bound holds. $\gamma(\overline{G}) = 2$

Case: 2

If all the vertices of N(u) has degree n-1 in G, clearly all the vertices of N(u) should be isolated vertices in $\overline{G}$ and u should be adjacent with all the vertices in{ V($\overline{G}$)-N(u) } Now $\overline{G}$ has a dominant set {u,N(u)} with (1+ N(u)) vertices, Now $\overline{G}$ has a dominant number , Hence the upper bound holds.
Hence the Theorem.

REFERENCES

[1] F. Haray , *Graph Theory*, Addison-Wesley Publishing Company. 1969.

[2] T. W. Haynes, S. T. Hedetniemi, and P. J. Slater, *Fundamentals of Domination in Graphs*, Marcel Dekker, Inc.. 1998.

[3] M. Kalanithi, V. Swaminathan, A. Selvam, Domination number of distance k-complement graph, *International Journal of Mathematics and Soft Computing, 2 (2013)*..

[4] B. Rajendran, *Topics in Graph Theory : Antipodal Graphs*, Ph. D. Thesis, Madurai Kamaraj Uni-versity, 1985.

[5] E. Sampathkumar, *K-dimensional Graph and Semi Graph Theory*, DST Project Report No. DST/MS/131/2K, 2005.