MINIMUM TOTAL DOMINATING MAXIMUM DEGREE ENERGY FOR A GRAPH

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Abstract- The minimum dominating energy, Minimum total dominating (MTD) energy, minimum edge dominating energy and also minimum total dominating edge energy of a graph have been defined already related to graphs. Motivated by these articles and related work on energy of a graph, in the present research article we define minimum total dominating maximum degree energy for a graph. Here we compute above defined energy for some standard graphs. Such as Complete graph, star graph and crown graph.

Key Words: Minimum totals dominating maximum degree set, minimum total dominating maximum degree matrix, minimum total dominating maximum degree eigen values, minimum totals dominating maximum degree for a graphs.

I. INTRODUCTION

The idea of energy of a graph was introduced by I Gutaman [1,4,9], in the year 1978. Let \( G \) be a graph having \( n \)-vertices and \( m \)-edges. Let \( A = (a_{ij}) \) be matrix of adjacency for a graph. Let \( \lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n \) be eigen values of matrix of adjacency for a graph \( G \). The values are assumed to be in decreasing order that is \( \lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \ldots \geq \lambda_n \). Since \( A(G) \) is real and symmetric. Its eigen values are real numbers. The energy \( E(G) \) for a graph is defined to the sum of absolute values of its eigen values of graph \( G \). That implies \( E(G) = \sum_{i=1}^{n} |\lambda_i| \).

Many definitions based on energien energies are already existing such as Distance energy [6], Minimum covering energy [3], Incidence energy [7], Lalpacian energy [9], Minimum total dominating edge energy [2,5], Minimum total dominating complementary energy [8] Minimum total dominating maximum degree energy for a graph is defined in the present article and is computed for some standard graphs.

II. MINIMUM TOTAL DOMINATING MAXIMUM DEGREE ENERGY

The graphs under consideration for computation are finite, undirected, without loops and multiple edges. Let \( G = (V, E) \) be a graph with \( V \) as vertex set and \( E \) as edge set.

Let \( V = \{v_1, v_2, v_3, \ldots, v_p\} \). A set \( D \subseteq V \) be a total dominating set of \( G \) if each vertex in \( V \) is adjacent to some vertex in \( D \), and vertices in \( D \) are themselves adjacent. A total dominating set characterized by a set of minimum cardinality is considered as minimum total dominating set. Taking \( D, M \) as minimum total dominating
maximum degree set of $G$. Then Minimum total dominating maximum degree matrix for $G$ is an $\times \pi$ matrix defined as $A(\pi \pi)(G) = (a_{ij})$ where,

$$a_{ij} = \begin{cases} \max \{d(v_i), d(v_j)\} & \text{if } e_{ij} \in E \\ 1 & \text{if } i = j \text{ and } v_i \in \pi \pi \\ 0 & \text{otherwise} \end{cases}$$  \hspace{1cm} (1)

Characteristic polynomial for $A(\pi \pi)(G)$ is defined by

$$f_{\pi \pi}(\lambda, G) = (\lambda I - A(\pi \pi)(G))$$

Then eigen values of minimum total dominating maximum degree for a graph $G$ are the eigen values of $A(\pi \pi)(G)$. The matrix $A(\pi \pi)(G) = (a_{ij})$ is a real and symmetric. Then the eigen values of $A(\pi \pi)(G)$ are real numbers and are labeled in non-increasing order $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_\pi$. The minimum total dominating maximum degree energy of graph $G$ is defined as $E(\pi \pi)(G) = \sum_{i=1}^{\pi} \lambda_i$.

**Example 2.1:** Let $G$ be the graph with 6-vertices say $v_1, v_2, v_3, v_4, v_5, v_6$ as shown in the figure

![Fig. 1 Graph G](image)

The possible minimum total dominating sets for the graph $G$ are $T_1(M_\pi) = \{v_2, v_3\}$

$T_2(M_\pi) = \{v_2, v_5\}$ and $T_3(M_\pi) = \{v_3, v_5\}$

Then corresponding to minimum total dominating set, $T_1(M_\pi) = \{v_2, v_3\}$

The adjacency matrix is

$$T_1(M_\pi)(G) = \begin{bmatrix} 0 & 4 & 3 & 0 & 0 & 0 \\ 4 & 1 & 1 & 4 & 4 & 0 \\ 3 & 4 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 1 & 4 & 3 \\ 0 & 4 & 4 & 4 & 0 & 0 \\ 0 & 0 & 0 & 3 & 4 & 0 \end{bmatrix}$$  \hspace{1cm} (2)

The characteristic polynomial of $T_1(M_\pi)(G)$ is $\lambda^6 - 2\lambda^5 - 129\lambda^4 - 293\lambda^3 + 2024\lambda^2 + 5919\lambda + 3360 = 0$. The minimum total dominating maximum degree eigen values will be $\lambda_1 = -6.6692$, $\lambda_2 = -5.6391$, $\lambda_3 = -1.9863$, $\lambda_4 = 0.8077$, $\lambda_5 = 4.3752$, $\lambda_6 = 12.7272$.

Then, minimum total dominating maximum degree energy for the graph is given by

$$E_{T_1}(M_\pi)(G) = |-6.6692|(1) + |5.6391|(1) + |-1.9863|(1) + |0.8077|(1) + |4.3752|(1) + |12.7272|(1) \approx 32.2047$$
And corresponding to minimum total dominating set, \( T_2(M_x) = \{v_2, v_3\} \)

The adjacency matrix is

\[
T_2(M_x)(G) = \begin{bmatrix}
0 & 4 & 3 & 0 & 0 & 0 \\
4 & 1 & 4 & 4 & 0 & 0 \\
3 & 4 & 0 & 0 & 0 & 0 \\
0 & 4 & 0 & 0 & 4 & 3 \\
0 & 4 & 4 & 1 & 4 & 0 \\
0 & 0 & 0 & 3 & 4 & 0
\end{bmatrix}
\]

And characteristic polynomial for \( T_2(M_x)(G) \) is \( \lambda^6 - 2\lambda^5 - 129\lambda^4 - 316\lambda^3 + 1919\lambda^2 + 6078\lambda + 3969 \) and the characteristic equation is \( \lambda^6 - 2\lambda^5 - 129\lambda^4 - 316\lambda^3 + 1919\lambda^2 + 6078\lambda + 3969 = 0 \). The minimum total dominating maximum degree eigen values are \( \lambda_1 = -6.2915, \lambda_2 = -5.8820, \lambda_3 = -1.9479, \lambda_4 = -1.0000, \lambda_5 = 4.2915, \lambda_6 = 12.8295 \)

Therefore, minimum total dominating maximum degree energy for the graph is given by \( E_{T_2}(M_x)(G) = |-6.2915|^{(1)} + |-5.8820|^{(1)} + |-1.9479|^{(1)} + |-1.0000|^{(1)} + |4.2915|^{(1)} + |12.8295|^{(1)} \approx 22.2421 \)

Therefore minimum total dominating maximum degree energy for a graph depends on the minimum total dominating set of graph. And they are approximately same.

### III. MINIMUM TOTAL DOMINATING MAXIMUM DEGREE ENERGY OF SOME STANDARD GRAPHS

**Theorem 3.1:** If \( K_p \) is a complete graph with \( p \geq 2 \) vertices, then

\[
E_T(M_x)(K_p) = (p - 1)(p - 2) + \sqrt{(p^4 - 2p^3 - p^2 + 6p - 3)}.
\]

Proof: Let \( K_p \) be a complete graph with vertex set \( V = \{v_1, v_2, \ldots, v_p\} \). The minimum total dominating maximum degree set of \( K_p \) is \( TM_x = \{v_2\} \). (For \( K_p \) Minimum total dominating maximum degree set and minimum total dominating set are same) Then,

\[
TM_x(K_p) = \begin{bmatrix}
1 & p-1 & p-1 & p-1 & p-1 \\
p-1 & 0 & p-1 & p-1 & p-1 \\
p-1 & p-1 & 0 & p-1 & p-1 \\
p-1 & p-1 & p-1 & 0 & p-1 \\
p-1 & p-1 & p-1 & p-1 & 0
\end{bmatrix}_{p \times p}
\]

The characteristic polynomial of \( TM_x(K_p) \) is

\[
\begin{bmatrix}
\lambda & -1 & p-1 & p-1 & p-1 \\
p-1 & \lambda & p-1 & p-1 & p-1 \\
p-1 & p-1 & \lambda & p-1 & p-1 \\
p-1 & p-1 & p-1 & \lambda & p-1 \\
p-1 & p-1 & p-1 & p-1 & \lambda
\end{bmatrix}_{p \times p}
\]

Characteristic equation is
Minimum total dominating maximum degree eigen values are

\[ \lambda = -(p - 1) \left( (p - 2) \times \text{times} \right) \]

\[ \lambda = \frac{(p^2 - 3p + 3) \pm \sqrt{(p^4 - 2p^3 - p^2 + 6p - 3)}}{2} \]

Then, minimum total dominating maximum degree energy is given by

\[
\begin{align*}
E_T(M_x)(K_p) &= \left| -(p - 1) \right| (p - 2) + \frac{(p^2 - 3p + 3) \pm \sqrt{(p^4 - 2p^3 - p^2 + 6p - 3)}}{2} \\
&= (p - 1)(p - 2) + \sqrt{(p^4 - 2p^3 - p^2 + 6p - 3)} .
\end{align*}
\]

**Theorem 3.2**: If \( K_{1,p-1} \) is a star with \( p \geq 2 \) vertices, then

\[ E_T(M_x)(K_{1,p-1}) \approx \sqrt{4p^3 - 12p^2 + 12p - 3} \]

**Proof**: Let \( K_{1,p-1} \) be a star graph with vertex set \( V = \{v_1, v_2, \ldots, v_p\} \). The minimum total dominating maximum degree set of \( K_{1,p-1} \) is \( TM_x = \{v_1\} \). For \( K_{1,p-1} \) Minimum total dominating maximum degree set and minimum total dominating set is same. Then,

\[
A_T(M_x)(K_{1,p-1}) = \begin{bmatrix}
1 & p-1 & p-1 & \ldots & p-1 & p-1 \\
p-1 & 0 & 0 & \ldots & 0 & 0 \\
p-1 & 0 & 0 & \ldots & 0 & 0 \\
p-1 & 0 & 0 & \ldots & 0 & 0 \\
p-1 & 0 & 0 & \ldots & 0 & 0 \\
p-1 & 0 & 0 & \ldots & 0 & 0 \\
\end{bmatrix}_{p \times p}
\]

The characteristic polynomial of \( A_T(M_x)(K_{1,p-1}) \) is

\[
[\lambda - 1 \ p-1 \ p-1 \ \ldots \ p-1 \ p-1] \\
[\ p-1 \ \lambda \ 0 \ \ldots \ 0 \ 0] \\
[\ p-1 \ 0 \ \lambda \ \ldots \ 0 \ 0] \\
[\ p-1 \ 0 \ 0 \ \ldots \ \lambda \ 0] \\
[\ p-1 \ 0 \ 0 \ \ldots \ 0 \ \lambda]
\]

Characteristic equation is

\[ A_{p-2}(\lambda^2 - \lambda - (p - 1))^2 = 0 \]

Minimum total dominating maximum degree energy eigenvalues are

\[ \{\lambda_1 = 0 \ (p - 2 \times \text{times}), \ \lambda_2 = \frac{1 + \sqrt{4p^3 - 12p^2 + 12p - 3}}{2}, \ \lambda_1, \ \lambda_2 \}
\]

Therefore, the minimum total dominating maximum energy is \( E_T(M_x)(K_{1,p-1}) \) is
3.3. Definition: Crown Graph

A crown graph $S_p$ on $2p$ vertices is an undirected graph with two sets of vertices $u_i$ and $v_j$ for $i, j = 1, 2, 3, \ldots, p$ with an edge from $u_i$ to $v_j$ whenever $i \neq j$. And the crown graph is shown as below.

![Fig. 2 Crown graph](image)

\[ E_T(M_x)(K_{1,p-1}) = |0| \cdot (p - 2) \times (1 + \frac{\sqrt{4p^3 - 12p^2 + 12p - 3}}{2}) + (1 - \frac{\sqrt{4p^3 - 12p^2 + 12p - 3}}{2}) \]

**Theorem 3.4:** If $S_p$ is crown graph with $p \geq 2$, then $E_T(M_x)(S_p)$ is not defined.

**Proof:** Minimum total dominating maximum degree set for the crown graph itself does not exist. Therefore, minimum total dominating maximum degree energy of a crown graph cannot be defined.

Graphically, minimum total dominating maximum degree energies of complete graph $\{E_T(M_x)(K_p)\}$, Star graph $\{E_T(M_x)(K_{1,p-1})\}$ are shown.

![Fig. 3 Comparision of minimum total dominating maximum degree energy for complete and star graph](image)

Point of intersection of curves = $(2, 2.24)$
That is energy of both graphs coincides at $p = 2$.
IV. CONCLUSION

In this article, the new concept of energy namely minimum total dominating maximum degree energy for the graph is defined and has been obtained for the complete graph, Star graph and It is also shown that the same energy cannot be obtained for crown graph. Minimum total dominating maximum degree energy of several other family of graphs is an open problem.

REFERENCES


