OPTIMAL COORDINATION OF DIRECTIONAL OVERCURRENT RELAY USING MOTH FLAME OPTIMIZATION ALGORITHM

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Abstract- Over current relays are commonly used in distribution system whose function is not only to provide primary protection but also secure and accurate back up protection also. The overcurrent relays must be optimally coordinate among them to reduce the total operating time so that fault clearing time will be increased. In this paper, a Moth Flame optimization technique is proposed in order to minimize the total operating time of the overcurrent relays. The proposed method is tested on the data obtained from two widely adopted distribution system. Comparison assessment with other techniques shows the efficiency, accuracy, and speed of the proposed algorithm.

Keywords –Overcurrent relay coordination, Optimization, Moth-Flame algorithm, Transverse orientation

I. INTRODUCTION

In distribution system, sudden change in current due to shunt fault is an obvious phenomenon. In such case, overcurrent relays are frequently used in order to provide the primary and back up protection. This operation commonly called as relay coordination. Directional overcurrent relays are more common in distribution system, feeder, and sub-transmission systems. These relays should be selective, sensitive, and reliable and must be fast enough with inception of any fault. This needs proper coordination of the overcurrent relays among themselves. In this context, many literatures have been reported for proper coordination [1]-[27].

The formulation of the coordination problem of the directional overcurrent relay is presented in the Ref. [1, 2]. Ref. [3], presents a wide review on coordination of directional overcurrent relay. This coordination problem can be solved by employing optimization algorithms. In recent years, many optimization algorithms have been implemented for relay coordination problem. In addition to this different Evolutionary Algorithms (EA) have been proposed in recent years in order to solve the directional overcurrent relay coordination problem [4]. Methods based on Differential Evolution Algorithm (DEA), Modified Differential Evolution Algorithm (MDEA) , and Self-Adaptive Differential Evolutionary (SADE) algorithm for solving optimal relay coordination problem are successfully implemented and presented in [5]-[7]. In this context, swarm based techniques like Particle Swarm Optimization (PSO), Modified Particle Swarm Optimizer are illustrated in [8, 9], techniques based on Evolutionary Particle Swarm Optimization (EPSO) [10], Box-Muller Harmony Search (BMHS) [11], Zero-one Integer Programming (ZOIP) Approach [12], Covariance Matrix Adaptation Evolution Strategy (CMA-ES) [13], Seeker Algorithm (SA) [14],Teaching Learning-Based Optimization (TLBO), Chaotic Differential Evolution Algorithm (CDEA) [15], Artificial Bee Colony algorithm (ABC) [16], Firefly Optimization Algorithm (FOA) [17, 18], Modified Swarm Firefly Algorithm (MSFA) [19], and Biogeography Based Optimization (BBO) [20] are successfully implemented for solving optimal coordination of directional overcurrent relay problem. Further, many hybrid based methods such as Evolutionary Algorithm based on Tabu Search (EA-TS) [21], Evolutionary Algorithm based on Linear Programming (DE-LP) [22], Nelder-Mead and Particle Swarm Optimization (NM-PSO)

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[23], Linear Programming and Genetic Algorithms (LP-GA) [24], Particle Swarm Optimization and Linear Programming (PSO-LP) [25], and Genetic Algorithm and Particle Swarm Optimization algorithm (GA-PSO) is presented [26] in order to solve the optimal directional relay coordination problem. Recently, Ref. [27] uses chaotic behavior of the Firefly’s for directional overcurrent coordination. However, complexity, slow convergence characteristic, and local best trapping are some major disadvantages associated with these algorithms. Hence, a new algorithm based on the behavior of the Moth-Flame is proposed and implemented for this problem. The performance of the proposed method is evaluated for two most widely used distribution systems. Results and comparison assessment of the proposed method as compared to the other algorithms shows the efficiency, accuracy, and speed of the algorithm.

III. PROBLEM FORMULATION

Operating time of the IDMT relay is conversely proportional with current. Hence, overcurrent relay will operate fast after sensing the high current. Generally, there are two variables of such relays i.e. TDS and PS. The relay operating time is strongly linked to TDS, PS and the fault current (IF). In a multi-loop interconnected distribution power system the directional overcurrent relays should coordinate among themselves and hence formulated as an optimization problem. However, the objective function of relay coordination problem should satisfy some sets of constraints and can be formulated as:

$$\min f = \sum_{j=1}^{n} w_j T_{j,k}$$  \hspace{1cm} (1)

where $w_i$ and $T_i$ are representing the weight and operation of the directional overcurrent relays. For all the relays the value $w_i = 1$. The operating time of the IDMT type overcurrent relay is given by the equation:

$$T_{OP} = TMS_i \left\{ \frac{\alpha}{\left( \frac{I_{f}}{I_{pj}} \right)^k - 1} \right\}$$  \hspace{1cm} (2)

where $\alpha$ and $k$ are representing some constant parameters which values are supposed to be $\alpha = 0.14$ and $k = 0.02$ for a normal inverse type relay. The variables TMS, represents the time multiplier setting of the relay and $I_{pj}$ represents the pickup value of the current for the $i^{th}$ relay, while $I_{f}$ defines the amount of fault current flowing through relay $R_i$:

$$PSM = \left( \frac{I_{f}}{I_{pj}} \right)$$  \hspace{1cm} (3)

where $I_{pj}$ is the primary pickup current and PSM stands for the Plug Setting Multiplier:

$$T_{OP} = TMS_i \left\{ \frac{\alpha}{\left( PSM \right)^{\alpha} - 1} \right\}$$  \hspace{1cm} (4)

From the eq. (4) it can be stated that the above problem is highly non-linear in nature. If we consider the PS to be constant then it can be observed that the operating time of the relay is directly proportional to the TMS. Hence, the operating time of the relay can be represented as the linear combination of the TMS and given as:

$$T_{OP} = a_p \times TMS_i$$  \hspace{1cm} (5)

where

$$a_p = \left\{ \frac{\alpha}{\left( PSM \right)^{\alpha} - 1} \right\}$$  \hspace{1cm} (6)

Hence the objective function for relay coordination problem can be formulated as:

$$\min f = \sum_{i=1}^{n} a_p \left( TMS_i \right)$$  \hspace{1cm} (7)
The constraints for this problem are described as follows,

The objective of this problem is to minimize the total operating time of the directional overcurrent relays. However, to fulfill this objective some additional constraints must have to satisfy. Generally, relay coordination problems are having two types of constraints. Firstly, the TMS setting of the each and every relay should lies within its maximum and minimum limits. The boundary equation for TMS setting is given by:

$$T_{MS_{i_{min}}} \leq T_{MS_i} \leq T_{MS_{i_{max}}} \quad (8)$$

The second type of constraint is to adjust the minimum gap in operating time between the primary and backup relays. As a fault within the protective zone is not only sensed by the primary relay but also by the backup relay, there should be a proper time gap between them in order to prevent the relay maloperation. The is interval is commonly called as coordination time interval (CTI). This CTI is the combination of the circuit breaker operating time and maximum overshoot time of the relay. This CTI constraint for relay coordination study is defined as:

The coordination constraint is defined as follows:

$$T_j \geq T_i + CTI \quad (9)$$

where $T_i$ and $T_j$ are the operating times of the primary and backup relays, respectively. The value of the CTI for this work may vary within 0.2 to 0.5 s, considering different worst factors.

II. PROPOSED ALGORITHM

In proposed MFO algorithm, moths are the variables which can fly anywhere in the search space having some dimension. Since, MFO is a population orient algorithm, the set of moths are represented by the matrix as follows:

$$M = \begin{bmatrix}
m_{1,1} & m_{1,2} & m_{1,3} & \cdots & m_{1,d} \\
m_{2,1} & m_{2,2} & m_{2,3} & \cdots & m_{2,d} \\
\vdots & \vdots & \vdots & & \vdots \\
m_{n,1} & m_{n,2} & m_{n,3} & \cdots & m_{n,d}
\end{bmatrix}_{n \times d} \quad (10)$$

Where, $n$ is the moth’s number and $d$ is the dimension of the problem or called population size. The corresponding fitness values of all the moths are stored in an array, represented by

$$FM = \begin{bmatrix}
f_{m_{1,1}} \\
f_{m_{2,1}} \\
\vdots \\
f_{m_{n,1}}
\end{bmatrix}_{n \times 1} \quad (11)$$

It is to be noted that the fitness value is nothing but the objective function solved by each Moth. The positive vector is passed through the each Moth in order to obtain the fitness value.

Another critical element of the proposed technique is flame. The position vector of the flame is represented as follows

$$F = \begin{bmatrix}
F_{1,1} & F_{1,2} & F_{1,3} & \cdots & F_{1,d} \\
F_{2,1} & F_{2,2} & F_{2,3} & \cdots & F_{2,d} \\
\vdots & \vdots & \vdots & & \vdots \\
F_{n,1} & F_{n,2} & F_{n,3} & \cdots & F_{n,d}
\end{bmatrix}_{n \times d} \quad (12)$$

The dimensions of the matrices ‘$M$’ and ‘$F$’ are same. Similar to that of moth it has been assumed that there is also a corresponding fitness array for each flame and represented by,

$$FM = \begin{bmatrix}
f_{F_{1,1}} \\
f_{F_{2,1}} \\
\vdots \\
f_{F_{n,1}}
\end{bmatrix}_{n \times 1} \quad (13)$$

Both moth and flames are representing the solutions in this work. The subtraction factor between these two will be updated in each iteration. The basic difference in between these two is that moths represent the search space whereas flames indicate the best position of the moths so far. Therefore, each moth search around the best position (flames) and updated its position in order to find the best solution. With this behavior, a moth never loses its capability to find a best solution.
The MFO algorithm contains three elements in order to achieve the global best solution and can be formulated as:

$$MFO = (A, B, C)$$  (14)

A is the function which generates the corresponding random generation of the moth and its fitness values. The mathematical model is given by:

$$A : \varphi \Rightarrow \{M, fM\}$$  (15)

The function B moves the moths around the search space. This function provides the updated value of M matrix.

$$B : M \Rightarrow M$$  (16)

The C function is associated with the termination criterion i.e. it indicates whether termination condition has been satisfied or not.

$$C : M \Rightarrow \{true, false\}$$  (17)

The pseudo codes associated with these functions are described in below [28]:

$$M = A();$$
while C(M) is equal to false
$$M = B(M);$$
end

The function A generates random population in search space and find out the best solution from the search space. The pseudo code for random population generation within their respective limits is given as:

for i = 1: n
for j = 1: d
$$M(i, j) = (ub(i)-lb(i))*rand() + lb(i);$$
end
end

OM = Fitness Function (M);

The lower and upper limits of the variables are denoted by the term lb and ub respectively.

The expressions for lb and ub for n variables are represented by

$$lb = [lb_1, lb_2, lb_3, ... lb_{n-1}, lb_n]$$  (18)

$$ub = [ub_1, ub_2, ub_3, ... ub_{n-1}, ub_n]$$  (19)

After successful initialization of the population, the function B is iteratively run until the best solution is achieved. As mentioned the algorithm is inspired from transverse orientation behavior, the position is updated according to the equation

$$M_i = S(M_i, F_j)$$  (20)

where M_i indicate the i^{th} moth, F_j indicates the j^{th} flame, and S is the spiral function.

In this work, a logarithmic spiral is used for update mechanism. However, the spiral should satisfy the following conditions

- The initial point of the spiral should be started from a moth.
- The final point of the spiral should be position of a moth.
- Spiral should within its boundary.

With the above knowledge, the logarithmic spiral for MFO algorithm is defined as

$$S(M_i, F_j) = D_i e^{b \pi} \cos(2\pi t) + F_j$$  (21)

Where, D_i indicates the Euclidian distance between the i^{th} moth and j^{th} flame. The constant b signifies the shape of the spiral and should be between [-1, 1]. The mathematical representation of D_i is given as

$$D_i = |F_j - M_i|$$  (22)

If N represents the initial number of flame then the number of flame that should be updated adaptively and can be done using following formulae.

$$\text{flame no} = \text{round} \left( N - 1 * N - \frac{1}{C} \right)$$  (23)

The detail explanation with its mathematical formulations of MFO algorithm is provided in [26].

III. RESULTS AND DISCUSSION

In order to test the performance of the proposed MFO algorithm, two different distribution systems are taken into the consideration. In following subsections results for two different test systems are discussed briefly.
Test Case-1: The performance of the proposed MFO algorithm is validated for the test system shown in Figure 5.1. This test system is considered in many manuscripts. The data of the tested system is borrowed from [27].

To verify the efficiency of the proposed algorithm the test system shown in Figure 1 is taken into the consideration. The studied parallel feeder consists of 5 relays among which relays 1, 4, and 5 are directional in nature whereas relays 2 and 3 are non-directional in nature. In the case the plug setting is assumed to be constant i.e. 1. The current transformer placed at every relay location are having same ratio of 300:1. Three fault location has been taken into the consideration. F1 and F2 are at middle of the transmission line-1 and line-2 and F3 is at line between bus-B and bus-C. Relay 4 will back up relay 2 for fault at F1 and relay 1 will back up relay 3 for fault at F2. The total fault current is assumed to be 4000 A for this case. In proportion to their path impedences the total fault current is divided in 1000 A and 3000 A. The STI is assumed to be 0.2 for both the cases.

Let the variable TMSs of all relays are denoted as $y_1$-$y_5$. The value of constant $a_p$ and relay currents at each relay location are illustrated in Table 5.1. The studied power system is a standard distribution system and has been already used many studies [27].

$$3.106y_3 \geq 0.1 + 4.341y_4 - 2.004y_5 \geq 0.1 + 6.265y_4 \geq 0.1 + 2.004y_5$$

After choosing the corresponding values of the $a_p$ from the Table-1, the final objective function is represented as

$$\text{Table-1}
\begin{array}{|c|c|c|c|c|c|}
\hline
\text{Fault point} & \text{Response time (ms)} \\
\hline
\text{F1} & I_{\text{relay}} & 9.059 & 3.019 & -- & 3.019 & -- \\
& a_p & 3.106 & 6.265 & -- & 6.265 & -- \\
\hline
\text{F2} & I_{\text{relay}} & 3.019 & -- & 9.059 & 3.019 & -- \\
& a_p & 6.265 & -- & 3.106 & 6.265 & -- \\
\hline
\text{F3} & I_{\text{relay}} & 4.875 & -- & 4.875 & 4.875 & 29.25 \\
& a_p & 4.34 & -- & 4.34 & 4.34 & 2.004 \\
\hline
\end{array}$$

$$\min f = 3.106y_1 + 6.265y_2 + 3.106y_3 + 6.265y_4 + 2.004y_5$$

Subject to the constraints

- Coordination constraints

$$6.265y_4 - 3.106y_2 \geq 0.2$$

$$6.265y_1 - 3.106y_3 \geq 0.2$$

$$4.341y_1 - 2.004y_5 \geq 0.2$$

$$4.341y_4 - 2.004y_5 \geq 0.2$$

- Relay operating time constraints

$$3.106y_1 \geq 0.1$$

$$6.265y_2 \geq 0.1$$

$$3.106y_3 \geq 0.1$$
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The dimension of this problem is four. Now, the objective function is solved by the proposed MFO algorithm. The population size and number of iteration for this scenario are set to 40 and 50 respectively. The results obtained using proposed MFO algorithm is described below.

![Objective space](image)

The convergence characteristics obtained using various well known algorithm along with proposed MFO method is depicted in Figure. 2. From the convergence characteristic, it can be conferred that using proposed method the total operating time can be minimized. Also, proposed method is very fast in convergence as compared to other well known algorithms like PSO, APSO, GSA, and DE. The optimized values of the corresponding TMF obtained using proposed method are shown in the Table-1.

![Comparison results for various techniques for case 1 obtained using MFO](image)

As, observed from the Table-2, the TMSs of all the relays can be minimized which led to minimization of the total operating time using proposed MFO algorithm. Hence, it can be concluded from above results that proposed method is accurate and fast as compared to other well-known optimization algorithms.

Test Case-2: Further, to evaluate the performance of the proposed optimization algorithm as compared to other metaheuristic algorithms, a single end fed distribution system with six numbers of overcurrent relay case is taken into the consideration. The structure of the distribution system is shown in Figure. 3.
The line data for the system is borrowed from [27]. Transmission line charging current due to shunt admittances are neglected in this study. In this case, faults are four different locations are taken into the consideration. Table 3 shows the primary and back up protection relays for the studied distribution system. The corresponding CT ratio along with the PS are tabulated in Table 4. The currents seen by the relays and the ap constants for this scenario are shown in Table 5. The fault locations i.e. A, B, C, D are assumed at middle of the individual feeders. Only one relay is backing up for every fault. Relays are at position 1 and 5 are having directionality properties. In this scenario, there are six TMS constraints and five numbers of CTI constraints. Minimum operating time of each relay is assumed to 0.1 s. The TMS lies in between 0.025 and 1.2 s. For this case, CTI is taken as 0.3 s. The relay TMS are denoted through the variables $y_1$–$y_6$.

Table 3: Primary–Backup relays for case 2.

<table>
<thead>
<tr>
<th>Fault point</th>
<th>Primary relay</th>
<th>Back up relay</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>C</td>
<td>5</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>D</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>--</td>
</tr>
</tbody>
</table>

Table 4: CT ratio and PS of relays for case 2.

<table>
<thead>
<tr>
<th>Fault point</th>
<th>CT ratio (A/A)</th>
<th>Plug setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1000/1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>300/1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1000/1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>600/1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>600/1</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>600/1</td>
<td>1</td>
</tr>
</tbody>
</table>

For this case the objective function can be formulated as:

$$\min f = 102.4y_1 + 6.06y_2 + 98.75y_3 + 24.4y_4 + 35.31y_5 + 11.53y_6$$

Subject to the constraint
- Minimum operating time constraints
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- Coordination constraints

\[ 3.646 y_1 \geq 0.1 \]
\[ 6.055 y_2 \geq 0.1 \]
\[ 8.844 y_3 \geq 0.1 \]
\[ 8.844 y_4 \geq 0.1 \]
\[ 4.044 y_5 \geq 0.1 \]
\[ 11.539 y_6 \geq 0.1 \]

The above mentioned objective function is now solved using proposed MFO optimization algorithm and the results obtained using proposed method are compared with the existing optimization algorithms. The results using proposed method are discussed below. In this case, the dimension of the search space contains 40 number of population with six variables. The maximum number of iteration is set to 60. It can be depicted from the convergence characteristic that proposed MFO algorithm is faster convergence rate as compared to other existing algorithms. That signifies that proposed algorithm requires less time (less number of iterations) to achieve global best solution. This signifies that computational time of the proposed method is very less as compared to the other well known algorithms. Therefore, proposed method is better among the best as compared to the other existing algorithms in terms of the computational speed and accuracy.

The optimal TMS setting and total operating time computed by each algorithms are shown in Table-6 from which it can be observed that the optimal TMS setting for overcurrent relay coordination problem can be obtained using proposed method as compared to PSO, APSO, GSA, DE, and proposed method.

From the above analysis, it can be concluded that proposed method is efficient in terms of speed, accuracy, and computational burden proposed MFO algorithm is best among the other recent metaheuristic algorithms. Hence, proposed can be efficiently used for optimal coordination of the overcurrent relays in distribution system.
Moth-Flame based optimal time coordination of the directional overcurrent relay in distribution system is proposed in this paper. The optimal relay coordination problem is formulated as a highly constrained optimization problem. Earlier techniques are slow in convergence, highly complex, and trap by local minima. Therefore, an efficient algorithm based on the behavior of the moth and flame is proposed to accomplish this task. The proposed method is applied on two test systems: one is a simple parallel system and another one is a multiloop system. The results are obtained using MATLAB 2013. Results and comparison with other latest technique depicts the simplicity, accuracy, and speed of the proposed MFO algorithm.

## REFERENCES


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