

# A STUDY OF VARIOUS METHODS IN FINDING OPTIMAL SOLUTION TO ASSIGNMENT PROBLEM

Laveena D'Costa<sup>1</sup>, Ashwith Joel DSouza<sup>2</sup> & Smitha Kamath<sup>3</sup>

**Abstract** – We are conducting a comparative study of various methods of assignment problem. Through problem solving. Algorithms of One's Assignment Method for assignment problem, A Primal Method for the Assignment Problem and The Hungarian method are studied and implemented using multiple inputs which are recorded in different time. Assignment problem is a commonly encountered problem in mathematics and is also discuss in real world. In this paper, we attempt to study and find the best optimal ways of using these assignment methods proposed. We examine a numerical example by using these existing three methods. Also, we compare the optimal solutions among this new method and three existing method. The proposed method is a systematic procedure, easy to apply for solving assignment problem.

**Keywords** – Written Implementation, multiple inputs, comparison, Mathematical solving ways, algorithm.

## 1. INTRODUCTION

Out of the many ways to solve an assignment method we choose the following three One's assignment method, Primal method, and the Hungarian method. Assignment problem is an extraordinary sort of linear programming issue which manages the designation of the different assets to the different exercises on coordinated one to one basis. It does it such that the cost or time associated with the procedure is least and benefit or deal is most extreme. This paper presents a primal method for the assignment and transportation problems which is a method "dual to" the Hungarian Method. The Hungarian method is a combinatorial optimization algorithm that solves the assignment problem. we propose a theory that states the best optimal use of all the above three methods.

The rest of the paper is organized as follows. Proposed Theory and brief on all the three-assignment method we chose are in section II. Experimental results are presented in section III. Concluding remarks are given in section IV.

## 2. PROPOSED THEORY

Mathematically an assignment problem can be stated as follows:

$$\sum_{i=1}^n \sum_{j=1}^n c_{ij}x_{ij}$$

$$\left. \begin{aligned} \sum_{j=1}^n x_{ij} &= 1 & i = 1, \dots, n \\ \sum_{i=1}^n x_{ij} &= 1 & j = 1, \dots, n \\ x_{ij} &= 0 & \text{or } 1 & J = 1, \dots, n. \end{aligned} \right\} 2$$

where  $c_{ij}$  is the cost or effectiveness of assigning  $i^{\text{th}}$  job to  $j^{\text{th}}$  machine, and  $x_{ij}$  is to be some positive integer or zero, and the only possible integer is one, so the condition of  $x_{ij} = 0$  or 1, is automatically satisfied. Associated to each assignment problem there is a matrix called cost or effectiveness matrix  $[c_{ij}]$  where  $c_{ij}$  is the cost of assigning  $i^{\text{th}}$  job to  $j^{\text{th}}$  facility.

### 2.1 One's Assignment Method –

One's assignment method is one of the classic method of solving different complex transportation problems, logistic and assignment problems. It uses square matrix; thus, each task can be assigned to only one machine. In fact, any solution of this assignment problem will contain exactly non-zero positive individual allocations. It is in based on add or subtract a constant to each component of a row or column of the cost matrix's in a minimization demonstrate, and create some zeros in the given cost matrix and then try to find a complete assignment in terms of zeros. Its aim is to create ones in place of zeroes, and try to assign them in our problem.

---

<sup>1</sup> Assistant Professor, Department of IT, AIMIT, St. Aloysius College, Mangaluru– 575022, Karnataka, India

<sup>2</sup> Student, MCA 3<sup>rd</sup> Semester, Department of MCA, AIMIT, St. Aloysius College, Mangaluru– 575022, Karnataka, India

<sup>3</sup> Student, MCA 3<sup>rd</sup> Semester, Department of MCA, AIMIT, St. Aloysius College, Mangaluru– 575022, Karnataka, India

$$\begin{matrix}
 & \begin{matrix} 1 & 2 & 3 & \dots & n \end{matrix} \\
 \begin{matrix} 1 \\ 2 \\ \vdots \\ n \end{matrix} & \begin{pmatrix} c_{11} & c_{12} & c_{13} & \dots & c_{1n} \\ c_{21} & c_{22} & c_{23} & \dots & c_{2n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ c_{n1} & c_{n2} & c_{n3} & \dots & c_{nn} \end{pmatrix}
 \end{matrix}$$

### 2.2 Primal Method –

As is well known, the  $n \times n$  assignment problem is equivalent to the dual linear programs.

$$\begin{aligned}
 (1) \quad & \text{minimize } a(X) = \sum_{ij} a_{ij}x_{ij} \\
 & \text{constrained by } \sum_i x_{ij} = 1, \text{ all } i \\
 & \sum_j x_{ij} = 1, \text{ all } j \\
 & x_{ij} \geq 0 \text{ all } (i, j)
 \end{aligned}$$

or

$$\begin{aligned}
 (2) \quad & \text{maximize, } B(U, V) = \sum_i u_i + \sum_j v_j - \sum_{ij} a_{ij}x_{ij} \\
 & \text{constrained by } u_i + v_j - a_{ij}, \text{ all } (i, j).
 \end{aligned}$$

The criterion for optimality for an  $X$  satisfying the constraints of (1) and  $(U, V)$  satisfying the constraints of (2) is the orthogonality condition: (3)  $(a_{ij} - u_i - v_j)x_{ij} = 0$  all  $(i, j)$ .

The primal method described below obtains, in a finite number of steps, optimal solutions to

### 2.3 Hungarian Method –

Hungarian Method is for allotting employments by a one-for-one coordinating to recognize the most reduced cost arrangement. Each job must be assigned to only one machine. It is assumed that every machine can handle every job, and that the costs or values associated with each assignment combination are known and fixed. The number of rows and columns must be the same.

Steps of solving:

1. Subtract the most modest number in each row from each number in the line. This is known as a row reduction. Enter the outcomes in another table.
2. Subtract the most modest number in every segment of the new table from each number in the segment. This is known as a column reduction. Enter the outcomes in another table.
3. Test whether an ideal task can be made. You do this by deciding the base number of lines expected to cover (i.e., cross out) every one of the zeros. If the quantity of lines equals with the quantity of row, an ideal task is conceivable. All things considered, go to stage 6. Generally, go ahead to stage 4.
4. If the quantity of lines is not as much as the quantity of rows, alter the table along these lines:
  - a) Subtract the littlest unrevealed number from each uncovered number in the table.
  - b) Add the littlest unrevealed number to the numbers at crossing points of covering lines.
  - c) Numbers crossed out yet not at convergences of traverse unaltered to the following table.
5. Repeat steps 3 and 4 until an optimal table is obtained.
6. Make the assignments. Begin with rows or columns with only one zero. Match items that have zeros, using only one match for each row and each column. Cross out both the row and the column after the match.

## 3. EXPERIMENT AND RESULT

We have taken multiple number similar problems from different possibilities that can be encountered during assignment problems. The assignment problem is classified into balanced assignment problem and unbalanced assignment problem. If the number of rows is equal to the number of columns, then the problem is termed as a balanced assignment problem; otherwise, an unbalanced assignment problem. If the problem is unbalanced, like an unbalanced transportation problem, then necessary number of dummy row(s)/column(s) is added such that the cost matrix is a square matrix. The values for the entries in the dummy row(s)/column(s) are assumed to be zero. Under such a condition, while implementing the solution, the dummy row(s) or the dummy column(s) will not have assignment(s). Following are the examples that we used for our study:

Example: Assign the four tasks to four operators.

Operators:

1	2	3	4
20	28	19	13
15	30	31	28
40	21	20	17

21	28	26	12
----	----	----	----

Task	Operator	Cost
A	3	19
B	1	15
C	2	21
	4	12
Total Cost = Rs. 67.00		

From our experiments we have found the following: -

Pros:

One's Assignment Method –

- This method can be used for all kinds of assignment problems, whether maximize or minimize objective function.
- The new method is based on creating some ones in the assignment matrix, and find an assignment in terms of the ones.

Primal Method –

- This method is close to Hungarian, primal dual gives a clear cut not accurate but close enough (approximate) values.
- When in minimum or short problems error finding is possible or state has easy.

Hungarian Method -

- you find the second minimum cost among the remaining choices.
- Gives more precise answers. Most of the time its exactly that is what the finder would want.
- Easier solving methods and quick.

Cons:

One's Assignment Method –

- When taken in  $m \times n$  into consideration some rational values would yield not accurate but rather close or unmatched outputs.
- Consume more time than Hungarian and not optimal.

Primal Method –

- Complex but more discreet. Yields accurate values.
- Takes long time in execution due to different constrains. Lot more steps compare to other to methods.

Hungarian Method -

- Works only over square matrix.
- The number of steps to solves is directly proportional to the number of rows.

#### 4. CONCLUSION

Hungarian method tops up over the other two methods. But when there are odd number of unbalanced entries are formed in the assignment problems. They are only solver by ones Assignment method which is more compatible with unbalanced matrix over the other two methods. On certain occasion primal beats ones but not over Hungarian due to the complex steps of primal whereas on cost reduce Hungarian and one's assignment top over prima. Thus, Hungarian best on assignment methods, one's assignment is small scale logistics and transportation problems and primal on large, extra prices for complex more accurate ans.

#### 5. REFERENCES

- [1] Ones assignment method for solving assignment problems (PDF Download Available). Available from: [https://www.researchgate.net/publication/265498020\\_Ones\\_assignment\\_method\\_for\\_solving\\_assignment\\_problems](https://www.researchgate.net/publication/265498020_Ones_assignment_method_for_solving_assignment_problems) [accessed Nov 02 2017].
- [2] A primal method for the assignment and transportation problems. m. l. balinski' and r. e. gomory2.
- [3] G. B. Dantzig, L. R. Ford, and D. R. Fulkerson. A primal-dual algorithm for linear programs. In H. W. Kuhn and A. W. Tucker, editors, *Linear Inequalities and Related Systems*, pages 171–181. Princeton University Press, Princeton, NJ, 1956.
- [4] THE PRIMAL-DUAL METHOD FOR APPROXIMATION ALGORITHMS AND ITS APPLICATION TO NETWORK DESIGN PROBLEMS Michel X. Goemans David P. Williamson
- [5] Munkre's Assignment Algorithm Modified for Rectangular Matrices.
- [6] Problem statement: Reep, D. (1997). Technical writing : Principles, strategies, and readings (3rd ed.). Boston: Allyn and Bacon
- [7] E. L. Lawler, J. K. Lenstra, A. H. G. Rinnooy Kan, D. B. Shmoys (ed.), "The Travelling Salesman: A Guided Tour of Combinatorial Optimization", John Wiley 1985.