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VERTEX POLYNOMIAL FOR THE DEGREE SPLITTING GRAPH OF SOME STANDARD GRAPHS

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Abstract-The vertex polynomial for the graph $G = (V, E)$ is defined as $V(G, x) = \sum_{k=0}^{\Delta(G)} v_k x^k$, where $\Delta(G) = \max\{d(v)/v\epsilon V\}$ and v_k is the number of vertices of degree k. In this paper we seek to find the vertex polynomial for thedegree splitting graph of Comb, Crown, Triangular snake, Double Triangular snake, Quadrilateral snake, Double Quadrilateral snake, $P_n \odot \bar{K}_m, (n \geq 2), C_n \odot \bar{K}_m, (n \geq 3)$.

Keywords- Comb, Crown, Triangular snake, Double Triangular snake, Quadrilateral snake, Double Quadrilateral snake.

1. Introduction

In a graph $G = (V, E)$, we mean a finite undirected simple graph. The vertex set is denoted by V and the edge set by E. For $v\epsilon V$, $d(v)$ is the number of edges incident with v, the maximum degree of G is defined as $\Delta(G) = \max\{d(v)/v\epsilon V\}$.For terms not defined here, we refer to Frank Harary [1]. The graph $G = (V, E)$with $V = S_1 \cup S_2 \cup \ldots \cup S_i \cup T$, where each $S_i$is a set of vertices having at least two vertices and having the same degree and$T = V \backslash \cup S_i$. The degree splitting graph of G denoted by $DS(G)$and is obtained from G by adding the vertices $w_1, w_2, \ldots, w_t$and joining $w_i$to each vertex ofS_i, $1 \leq i \leq t$. The graph $G = (V, E)$ is simply denoted by G.

Definition: 1.1

The graph obtained by joining a single pendent edge to each vertex of a path is called Comb.

Definition: 1.2

Any cycle with pendant edge attached to each vertex is called Crown($C_n \odot K_1$).

Definition: 1.3

A Triangular Snake T_n is obtained from a path $u_1 u_2 \ldots u_n$ by joining u_i and $u_{i+1}$to a new vertex v_i for $1 \leq i \leq n-1$. That is every edge of a path is replaced by a triangleC_3.

Definition: 1.4

A Quadrilateral Snake Q_n is obtained from a path $u_1 u_2 \ldots u_n$ by joining u_i and $u_{i+1}$to two new vertices v_i and w_i respectively and then joining v_i and w_i. That is every edge of a path is replaced by a cycleC_4.

Definition: 1.5

The Double Triangular Snake $D(T_n)$consists of two Triangular snakes that have common path.

Definition: 1.6

The Double Quadrilateral Snake $D(Q_n)$consists of two Quadrilateral snakes that have common path.

2. Main Results:

Theorem: 2.1

Let G be a Comb with order $2n, (n \geqslant 3)$. Then the Vertex Polynomial of $DS(G)$ is $V(DS(G), x) = x^{n-2} + x^n + (n-2)x^4 + 2x^3 + (n+1)x^2$.

Proof:

Let G be a Comb with order $2n, (n \geqslant 3)$. In G, $n-2$ vertices have degree 3; n vertices have degree 1 and 2 vertices have degree 2. We can construct the graph $DS(G)$ by introducing three newvertices, say w_1, w_2, w_3; make$w_1$adjacent to 3-degree vertices, $w_2$adjacent to 1-degree vertices, $w_3$adjacent to 2-degree vertices. Therefore, we have $n-2$ vertices have degree 4, $n+1$ vertices have degree 2, 2 vertices have degree 3, 1 vertex has degree $n-2$ and 1 vertex has degree$n$. This gives$V(DS(G), x) = x^{n-2} + x^n + (n-2)x^4 + 2x^3 + (n+1)x^2$.

Theorem: 2.2

Let G be a Crown with order $2n, (n \geqslant 3)$. Then the Vertex Polynomial of $DS(G)$ is $V(DS(G), x) = 2x^n + nx^4 + nx^2$.

Proof:

Let G be a Crown with order $2n, (n \geqslant 3)$. In G, n vertices have degree 3 and$n$ vertices have degree 1. We can construct the graph $DS(G)$ by introducing two new vertices, say w_1, w_2; make $w_1$adjacent to 3-degree vertices and$w_2$adjacent to 1-degree vertices. Therefore, we have n vertices have degree 4,n vertices have degree 2 and 2 vertices havedegree$n$. This gives $V(DS(G), x) = 2x^n + nx^4 + nx^2$.

Theorem: 2.3

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Let G be a Triangular Snake with order $2n-1, (n \geqslant 3)$. Then the Vertex Polynomial of $DS(G)$ is
$V(DS(G),x) = x^{n-2} + x^{n+1} + (n-2)x^5 + (n+1)x^3$.

Proof:

Let G be a Triangular Snake with order $2n-1, (n \geqslant 3)$.In $G, n-2$ vertices have degree 4 and $n+1$ vertices have degree 2. We can construct the graph $DS(G)$ by introducing two new vertices, say w_1, w_2; make $w_1$adjacent to 4-degree vertices and $w_2$adjacent to 2-degree vertices. Therefore, we have $n-2$ vertices have degree 5, $n+1$vertices have degree 3, 1 vertex has degree$n-2$ and 1 vertex have degree$n+1$. This gives
$V(DS(G),x) = x^{n-2} + x^{n+1} + (n-2)x^5 + (n+1)x^3$.

Theorem: 2.4

Let G be a Double Triangular snake with order $3n-2, (n \geqslant 3)$. Then the Vertex Polynomial of $DS(G)$ is
$V(DS(G),x) = x^{n-2} + x^{2n-2} + (n-2)x^7 + 2x^4 + (2n-2)x^3 + x^2$.

Proof:

Let G be a Double Triangular snake with order $3n-2, (n \geqslant 3)$. In $G, n-2$ vertices have degree 6; $2n-2$ vertices have degree 2 and 2 vertices have degree 3. We can construct the graph $DS(G)$ by introducing three new vertices, say w_1, w_2, w_3; make $w_1$adjacent to 6-degree vertices, $w_2$adjacent to 2-degree vertices, $w_3$adjacent to 3-degree vertices. Therefore, we have $n-2$ vertices have degree 7, $2n-2$ vertices have degree 3, 2 vertices have degree 4, 1 vertex has degree$n-2$, 1 vertex has degree $2n-2$ and 1 vertex has degree 2. This gives
$V(DS(G),x) = x^{n-2} + x^{2n-2} + (n-2)x^7 + 2x^4 + (2n-2)x^3 + x^2$.

Theorem: 2.5

Let G be a Quadrilateral Snake with order $3n-2, (n \geqslant 3)$. Then the Vertex Polynomial of $DS(G)$ is
$V(DS(G),x) = x^{n-2} + x^{2n} + (n-2)x^5 + 2nx^3$.

Proof:

Let G be a Quadrilateral Snake with order $3n-2, (n \geqslant 3)$. In $G, n-2$ vertices have degree 4 and $2n$ vertices have degree 2.We can construct the graph $DS(G)$ by introducing two new vertices, say w_1, w_2; make $w_1$adjacent to 4-degree vertices and $w_2$adjacent to 2-degree vertices. Therefore, we have $n-2$ vertices have degree 5, $2n$ vertices have degree 3, 1 vertex has degree$n-2$ and 1 vertex have degree $2n$. This gives $V(DS(G),x) = x^{n-2} + x^{2n} + (n-2)x^5 + 2nx^3$.

Theorem: 2.6

Let G be a Double Quadrilateral snake with order $5n-4, (n \geqslant 3)$. Then the Vertex Polynomial of $DS(G)$is
$V(DS(G),x) = x^{n-2} + x^{4n-4} + (n-2)x^7 + 2x^4$
$+(4n-4)x^3 + x^2$.

Proof:

Let G be a Double Quadrilateral snake with order $5n-4, (n \geqslant 3)$. In $G, n-2$ vertices have degree 6; $4n-4$ vertices have degree 2 and 2 vertices have degree 3. We can construct the graph $DS(G)$ by introducing three new vertices, say w_1, w_2, w_3; make $w_1$adjacent to 6-degree vertices, $w_2$adjacent to 2-degree vertices, $w_3$adjacent to 3-degree vertices. Therefore, we have $n-2$ vertices have degree 7, $4n-4$ vertices have degree 3, 2 vertices have degree 4, 1 vertex has degree$n-2$, 1 vertex has degree $4n-4$ and 1 vertex has degree 2. This gives
$V(DS(G),x) = x^{n-2} + x^{4n-4} + (n-2)x^7 + 2x^4 + (4n-4)x^3 + x^2$.

Theorem: 2.7

Let G be$P_n \odot \overline{K}_m, (n \geqslant 2)$. Then the Vertex Polynomial of $DS(G)$is
$V(DS(G),x) = x^{n-2} + x^{mn} + (n-2)x^{m+3} + 2x^{m+2} + (mn+1)x^2$.

Proof:

Let G be$P_n \odot \overline{K}_m, (n \geqslant 2)$. $n-2$ Vertices have degree $m+2$,2 vertices have degree $m+1$ and nm vertices have degree 1.We can construct the graph $DS(G)$ by introducing three new vertices, say w_1, w_2, w_3; make $w_1$adjacent to (m+2)-degree vertices, $w_2$adjacent to (m+1)-degree vertices, $w_3$adjacent to 1-degree vertices. Therefore, we have $n-2$ vertices have degree m+3, 2 vertices have degree m+2, $(mn+1)$ vertices have degree 2, 1 vertex has degree$n-2$ and 1 vertex has degreemn. This gives $V(DS(G),x) = x^{n-2} + x^{mn} + (n-2)x^{m+3} + 2x^{m+2} + (mn+1)x^2$.

Theorem: 2.8

Let G be$C_n \odot \overline{K}_m, (n \geqslant 3)$.Then the Vertex Polynomial of $DS(G)$ is $V(DS(G),x) = x^n + x^{mn+1} + nx^{m+3} + mnx^2$.

Proof:

Let G be$C_n \odot \overline{K}_m, (n \geqslant 3)$. In G, , n vertices have degree $m+2$ and nm vertices have degree 1. We can construct the graph $DS(G)$ by introducing two new vertices, say w_1, w_2; make $w_1$adjacent to (m+2)-degree vertices and $w_2$adjacent to 1-degree vertices. Therefore, we have n vertices have degree (m+3), mn vertices have degree 2,1 vertex has degree$n$ and 1 vertex has degree$mn+1$. This gives $V(DS(G),x) = x^n + x^{mn+1} + nx^{m+3} + mnx^2$.

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