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INTUITIONISTIC FUZZY sgp- CLOSED SETS

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Abstract: In 1970, Levine introduced the concept of generalized closed sets in general topology. He observed that the family of all closed sets in a topological space X is a subfamily of the family of all generalized closed sets. He generalized some of well-known results of general topology replacing closed set by generalized closed sets, for instance, generalized closed subset of a compact space is compact and generalized closed subspace of a normal space is normal. Many authors utilized g-closed sets for the generalization of various topological concepts in general topology. The concept of intuitionistic fuzzy sets was introduced by Atanassov as a generalization of fuzzy sets . In 1997 Coker introduced the concept of intuitionistic fuzzy topological spaces. In 2008, Thakur and Chaturvedi introduced the notion of intuitionistic fuzzy generalized closed set in intuitionistic fuzzy topological space. After that different mathematicians worked and studied in different forms of intuitionistic fuzzy g-closed set and related topological properties. The aim of this paper is to introduce the new class of intuitionistic fuzzy closed sets called intuitionistic fuzzy sgp- closed sets in intuitionistic fuzzy topological space. The class of all intuitionistic fuzzy strongly sgp--closed sets lies between the class of all intuitionistic fuzzy closed sets and class of all intuitionistic fuzzy gsp-closed sets. We also introduce the concepts of intuitionistic fuzzy sgp-- open sets in intuitionistic fuzzy topological spaces.

Key words: *Intuitionistic fuzzy g-closed sets, Intuitionistic fuzzy sg-closed sets, Intuitionistic fuzzy sgp-closed sets and intuitionistic fuzzy sgp-open sets.*

I. INTRODUCTION

After the introduction of fuzzy sets by Zadeh [22] in 1965 and fuzzy topology by Chang [2] in 1967, several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy sets was introduced by Atanassov [1] as a generalization of fuzzy sets. In the last 25 years various concepts of fuzzy mathematics have been extended for intuitionistic fuzzy sets. In 1997 Coker [3] introduced the concept of intuitionistic fuzzy topological spaces. In 2008 Thakur and Chtuvedi introduced the concepts of intuitionistic fuzzy generalized closed sets [15] in intuitionistic fuzzy topology. After that many weak and strong forms of intuitionistic fuzzy g-closed sets such as intuitionistic fuzzy rg-closed sets[16],intuitionistic fuzzy sg-closed sets[17], intuitionistic fuzzy g*-closed sets[11], intuitionistic fuzzy αg-closed sets[14], intuitionistic fuzzy gα-closed sets[7], intuitionistic fuzzy w-closed sets[18], intuitionistic fuzzy rw-closed sets[19], intuitionistic fuzzy gpr-closed sets[20], intuitionistic fuzzy rgα-closed sets[21], intuitionistic fuzzy gsp-[12]closed sets, intuitionistic fuzzy gp[9] and intuitionistic fuzzy strongly g*-closed sets [5] have been appeared in the literature.
In the present paper we extend the concepts of sgp-closed sets due to Navalagi G. and Bhat M.[8] in intuitionistic fuzzy topological spaces . The class of intuitionistic fuzzy sgp-closed sets is properly placed between the class of intuitionistic fuzzy closed sets and intuitionistic fuzzy gsp-closed sets. We also introduced the concepts of intuitionistic fuzzy sgp-open sets, and obtain some of their characterization and properties.

II. PRELIMINARIES

Let X be a nonempty fixed set. An intuitionistic fuzzy set A[1] in X is an object having the form $A = \{<x, \mu_A(x), \gamma_A(x)> : x \in X\}$, where the functions $\mu_A : X \to [0,1]$ and $\Upsilon_A : X \to [0,1]$ denotes the degree of

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membership $\mu_A(x)$ and the degree of non membership $\gamma_A(x)$ of each element $x \in X$ to the set A respectively and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$. The intutionistic fuzzy sets $\tilde{\mathbf{0}} = \{< x, 0, 1 > : x \in X\}$ and $\tilde{\mathbf{1}} = \{<x, 1, 0> : x \in X\}$ are respectively called empty and whole intuitionistic fuzzy set on X. An intuitionistic fuzzy set $A = \{<x, \mu_A(x), \gamma_A(x)> : x \in X\}$ is called a subset of an intuitionistic fuzzy set $B = \{<x, \mu_B(x), \gamma_B(x)> : x \in X\}$ (for short $A \subseteq B$) if $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for each $x \in X$. The complement of an intuitionistic fuzzy set $A = \{<x, \mu_A(x), \gamma_A(x)> : x \in X\}$ is the intuitionistic fuzzy set $A^c = \{ <x, \gamma_A(x), \mu_A(x) >: x \in X\}$. The intersection (resp. union) of any arbitrary family of intuitionistic fuzzy sets $A_i = \{< x, \mu_{Ai}(x), \gamma_{Ai}(x) > : x \in X, (i \in \wedge) \}$ of X be the intuitionistic fuzzy set $\cap A_i = \{<x, \wedge \mu_{Ai}(x), \vee \gamma_{Ai}(x) >: x \in X\}$ (resp. $\cup A_i = \{ <x, \vee \mu_{Ai}(x), \wedge \gamma_{Ai}(x) >: x \in X \}$). Two intuitionistic fuzzy sets $A = \{<x, \mu_A(x), \gamma_A(x)> : x \in X\}$ and $B = \{<x, \mu_B(x), \gamma_B(x)> : x \in X\}$ are said be q-coincident (A_qB for short) if and only if $\exists$ an element $x \in X$ such that $\mu_A(x) > \gamma_B(x)$ or $\gamma_A(x) < \mu_B(x)$. A family $\Im$ of intuitionistic fuzzy sets on a non empty set X is called an intuitionistic fuzzy topology [3]on X if the intuitionistic fuzzy sets $\tilde{\mathbf{0}}, \tilde{\mathbf{1}} \in \Im$, and $\Im$ is closed under arbitrary union and finite intersection. The ordered pair $(X,\Im)$ is called an intuitionistic fuzzy topological space and each intuitionistic fuzzy set in $\Im$ is called an intuitionistic fuzzy open set. The compliment of an intuitionistic fuzzy open set in X is known as intuitionistic fuzzy closed set .The intersection of all intuitionistic fuzzy closed sets which contains A is called the closure of A. It denoted cl(A). The union of all intuitionistic fuzzy open subsets of A is called the interior of A. It is denoted int(A) [3].

Lemma 2.1 [3]**:** Let A and B be any two intuitionistic fuzzy sets of an intuitionistic fuzzy topological space $(X, \Im)$. Then:

(a) $(A_qB) \Leftrightarrow A \subseteq B^c$.
(b) A is an intuitionistic fuzzy closed set in X $\Leftrightarrow$ cl (A) = A
(c) A is an intuitionistic fuzzy open set in X $\Leftrightarrow$ int (A) = A.
(d) $cl(A^c) = (int(A))^c$.
(e) $int(A^c) = (cl(A))^c$.

Definition 2.1[3]: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space $(X,\Im)$ is called:

(a) An intuitionistic fuzzy semi open of X if there is an intuitionistic fuzzy set O such that $O \subseteq A \subseteq cl(O)$.
(b) An intuitionistic fuzzy semi closed if the compliment of A is an intuitionistic fuzzy semi open set.
(c) An intuitionistic fuzzy regular open of X if int(cl(A)) = A .
(d) An intuitionistic fuzzy regular closed of X if cl(int(A))= A.
(e) An intuitionistic fuzzy pre open if $A \subseteq int(cl(A))$.
(f) An intuitionistic fuzzy pre closed if $cl(int(A)) \subseteq A$
(g) An intuittionistic fuzzy α-open $A \subseteq int(cl(intA))$
(h) intuitionistic fuzzy α- closed if $cl(int(cl(A))) \subseteq A$

Definition 2.2[3] If A is an intuitionistic fuzzy set in intuitionistic fuzzy topological space$(X,\Im)$ then

(a) scl (A) = $\cap$ { F: $A \subseteq F$, F is intuitionistic fuzzy semi closed}
(b) pcl (A) = $\cap$ { F: $A \subseteq F$, F is intuitionistic fuzzy pre closed}
(c) αcl (A) = $\cap$ { F: $A \subseteq F$, F is intuitionistic fuzzy α closed}
(d) spcl(A) = $\cap$ { F: $A \subseteq F$, F is intuitionistic fuzzy semi pre-closed}

Definition 2.3: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space $(X,\Im)$ is called:

(a) Intuitionistic fuzzy g-closed if cl (A) $\subseteq$ O whenever $A \subseteq O$ and O is intuitionistic fuzzy open.[15]
(b) Intuitionistic fuzzy rg-closed if cl (A) $\subseteq$ O whenever $A \subseteq O$ and O is intuitionistic fuzzy regular open.[16]
(c) Intuitionistic fuzzy sg-closed if scl (A) $\subseteq$ O whenever $A \subseteq O$ and O is intuitionistic fuzzy semi open.[17]
(d) Intuitionistic fuzzy g*-closed if cl (A) $\subseteq$ O whenever $A \subseteq O$ and O is intuitionistic fuzzy g-open.[11]
(e) Intuitionistic fuzzy αg-closed if αcl (A) $\subseteq$ O whenever $A \subseteq O$ and O is intuitionistic fuzzy - open.[14]
(f) Intuitionistic fuzzy gα-closed if αcl (A) $\subseteq$ O whenever $A \subseteq O$ and O is intuitionistic fuzzy α- open.[7]
(g) Intuitionistic fuzzy w-closed if cl (A) $\subseteq$ O whenever $A \subseteq O$ and O is intuitionistic fuzzy semi open.[18]
(h) Intuitionistic fuzzy rw-closed if cl (A) $\subseteq$ O whenever $A \subseteq O$ and O is intuitionistic fuzzy regular semi open.[19]
(i) Intuitionistic fuzzy gpr-closed if pcl (A) $\subseteq$ O whenever $A \subseteq O$ and O is intuitionistic fuzzy regular open.[20]
(j) Intuitionistic fuzzy rgα-closed if αcl (A) $\subseteq$ O whenever $A \subseteq O$ and O is intuitionistic fuzzy regular α-open.[21]

(k) Intuitionistic fuzzy gsp-closed if spcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy - open.[12]
(l) Intuitionistic fuzzy gp-closed if pcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy open.[9]
(m) Intuitionistic fuzzy gs-closed if scl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy open.[13]
(n) Intuitionistic fuzzy strongly g*-closed set if if cl (int(A))$\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy g-open in X.[5].
(o) Intuitionistic fuzzy αgs-closed set if αcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy semi-open.[6]
(p) Intuitionistic fuzzy gspr-closed set if spcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy – regular open.[10]
The complements of the above mentioned closed set are their respective open sets.

III. INTUITIONISTIC FUZZY sgp -CLOSED SET

Definition 3.1: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X,$\Im$) is called an intuitionistic fuzzy g**-closed if pcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy semi-open in X.

First we prove that the class of intuitionistic fuzzy sgp- closed sets properly lies between the class of intuitionistic fuzzy closed sets and the class of intuitionistic fuzzy gsp-closed sets.

Theorem 3.1: Every intuitionistic fuzzy closed set is intuitionistic fuzzy sgp-closed.
Proof: Let A is intuitionistic fuzzy closed set. Let A $\subseteq$ U and U is intuitionistic fuzzy semi-open sets in X. Since A is intuitionistic fuzzy closed set we have A= cl(A). Hence cl(A) $\subseteq$ U . But pcl(A) $\subseteq$ cl(A) , therefore pcl(A) $\subseteq$ A .whenever A $\subseteq$ U and U is intuitionistic fuzzy semi open in X. Therefore A is intuitionistic fuzzy sgp-closed set.

Remark 3.1: The converse of above theorem need not be true as from the following example.
Example 3.1: Let X = {a, b, c} and intuitionistic fuzzy sets O and U are defined as follows
O = { < a,0.9,0.1.> ,<b,0,1>,<c,0,1>}
U = { < a,0,1,> ,<b,0.8,0.1>,<c,0.6,0.3>}
$\Im$ = {$0_\sim$, O, U $1_\sim$ } be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set A = {<a, 0.7, 0.2>,<b ,0.6, 0.3> <c, 0, 1>} is intuitionistic fuzzy sgp -closed but it is not intuitionistic fuzzy closed .
Theorem 3.2: Every intuitionistic fuzzy α-closed set is intuitionistic fuzzy sgp-closed.
Proof: Let A is intuitionistic fuzzy closed set. Let A $\subseteq$ U and U is intuitionistic fuzzy semi-open sets in X. Since A is intuitionistic fuzzy α-closed set we have A= αcl(A). Hence αcl(A) $\subseteq$ U . But pcl(A) $\subseteq$ αcl(A) , therefore pcl(A) $\subseteq$ A .whenever A $\subseteq$ U and U is intuitionistic fuzzy semi open in X. Therefore A is intuitionistic fuzzy sgp-closed set.
Remark 3.2: The converse of above theorem need not be true as from the following example.
Example 3.2: Let X = {a, b, c} and intuitionistic fuzzy sets O and U are defined as follows
O = { < a,0.6,0.2.> ,<b,0,1>,<c,0,1>}
U = { < a,0.5,0.4,> ,<b,0.8,0.1>,<c,0,1>}
$\Im$ = {$0_\sim$, O, U, $1_\sim$ } be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set A = {<a, 0.7, 0.2>,<b ,0, 1> <c, 0.7, 0.2>} is intuitionistic fuzzy sgp -closed but it is not intuitionistic fuzzy α-closed .
Theorem 3.3: Every intuitionistic fuzzy pre-closed set is intuitionistic fuzzy sgp-closed.
Proof: Let A is intuitionistic fuzzy closed set. Let A $\subseteq$ U and U is intuitionistic fuzzy semi-open sets in X. Since A is intuitionistic fuzzy pre-closed set we have A= pcl(A). Hence pcl(A) $\subseteq$ A .whenever A $\subseteq$ U and U is intuitionistic fuzzy semi open in X. Therefore A is intuitionistic fuzzy sgp-closed set.
Remark 3.3: The converse of above theorem need not be true as from the following example.
Example 3.3: Let X = {a, b} and intuitionistic fuzzy sets O and U are defined as follows
O = { < a,0..2,0.8.> ,<b,0.1,0.9>}
U = { < a,0.5,0.4,> ,<b,0.6,0.4>}

$\Im = \{\mathbf{0}_\sim, O, U, \mathbf{1}_\sim\}$ be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set A = {<a, 0.9, 0.1>,<b ,0.6, 0.3> } is intuitionistic fuzzy sgp -closed but it is not intuitionistic fuzzy pre-closed .

Theorem 3.4: Every intuitionistic fuzzy regular -closed set is intuitionistic fuzzy sgp-closed.

Proof: It follows from the fact that every intuitionistic fuzzy regular closed set is intuitionistic fuzzy closed set and Theorem 3.1.

Remark 3.4: The converse of above theorem need not be true as from the following example.

Example 3.4: Let X = {a, b, c} and intuitionistic fuzzy sets O and U are defined as follows

O = { < a,0..2,0.8.> ,<b,0.1,0.9>, ,<c,0.,1>, }

U = { < a,0.5,0.4,> ,<b,0, 1>,<c,0.4.,0.3>, }

$\Im = \{\mathbf{0}_\sim, O, U, \mathbf{1}_\sim\}$ be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set A = {<a, 0.9, 0.1>,<b ,0.6, 0.3> ,<c,0.,1>, } is intuitionistic fuzzy sgp -closed but it is not intuitionistic fuzzy regular-closed .

Theorem 3.5: Every intuitionistic fuzzy w-closed set is intuitionistic fuzzy sgp-closed.

Proof: Let A is intuitionistic fuzzy w-closed set. Let A $\subseteq$ U and U is intuitionistic fuzzy –semi open sets in X. By definition of intuitionistic fuzzy w-closed set, cl(A) $\subseteq$ U. Note that pcl(A) $\subseteq$ cl(A) is always true. Therefore pcl(A) $\subseteq$ U. Hence A is intuitionistic fuzzy sgp-closed set.

Remark 3.5: The converse of above theorem need not be true as from the following example.

Example 3.5: Let X = {a, b, c} and intuitionistic fuzzy sets O and U defined as follows

O = {< a , 0.9, 0.1.> ,< b, 0,1>,< c, 0, 1>}

U = {< a, 0.8, 0.1,> ,<b, 0.7, 0,2>,<c, 0, 1>}

$\Im = \{\mathbf{0}_\sim, O, U, \mathbf{1}_\sim\}$ be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set A = {<a ,0, 1>,<b, 0.8, 0.1> <c, 0, 1>} is intuitionistic fuzzy sgp -closed but it is not intuitionistic fuzzy w-closed .

Theorem 3.6: Every intuitionistic fuzzy αgs-closed set is intuitionistic fuzzy sgp-closed.

Proof: Let A is intuitionistic fuzzy αgs-closed set. Let A $\subseteq$ U and U is intuitionistic fuzzy –semi open sets in X. By definition of intuitionistic fuzzy αgs-closed set, αcl(A) $\subseteq$ U. Note that pcl(A) $\subseteq$ αcl(A) is always true. Therefore pcl(A) $\subseteq$ U. Hence A is intuitionistic fuzzy sgp-closed set.

Remark 3.6: The converse of above theorem need not be true as from the following example.

Example 3.6: Let X = {a, b, c, d } and intuitionistic fuzzy sets O,U, V,W defined as follows

O = { < a,0.9 ,0.i> ,<b,0, 1>,<c,0, 1>,<d, 0, 1>}

U = { < a ,0, 1,> ,<b, 0.8, 0.1>,<c, 0, 1>,<d ,0, 1>}

V = { < a, 0.9, 0.1> ,<b, 0.8 ,0.1,>,<c, 0, 1>,<d, 0, 1>}

W = { < a, 0.9, 0.1,> ,<b, 0.8 ,0.1>,<c ,0.7, 0.2>,<d, 0, 1>}

$\Im = \{\mathbf{0}_\sim, O, U, V, W, \mathbf{1}_\sim\}$ be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set A = {<a ,0, 1>,<b, 0, 1> <c, 0.7, 0.2>,<d ,0, 1>} is intuitionistic fuzzy sgp-closed set but it is not intuitionistic fuzzy αgs-closed.

Theorem 3.7: Every intuitionistic fuzzy sgp-closed set is intuitionistic fuzzy gp-closed.

Proof: Let A is intuitionistic fuzzy sgp-closed set. Let A $\subseteq$ U and U is intuitionistic fuzzy open sets in X. Since every intuitionistic fuzzy open set is intuitionistic fzzy semi-open sets, U is intuitionistic fuzzy semi open sets such that A $\subseteq$ U. Now by definition of intuitionistic fuzzy sgp-closed sets pcl(A) $\subseteq$ U. We have pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is intuitionistic fuzzy open in X. Therefore A is intuitionistic fuzzy gp-closed set.

Remark 3.7: The converse of above theorem need not be true as from the following example.

Example 3.7: Let X = {a, b, c} and intuitionistic fuzzy sets O is defined as follows

O = { < a,0.6,0.2.> ,<b,0,1>,<c,0,1>}

$\Im = \{\mathbf{0}_\sim, O, \mathbf{1}_\sim\}$ be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set A = {<a, 0.7, 0.2>,<b ,0.7, 0.3> <c, 0, 1>} is intuitionistic fuzzy gp -closed but it is not intuitionistic fuzzy sgp-closed .

Theorem 3.8: Every intuitionistic fuzzy sgp-closed set is intuitionistic fuzzy gpr-closed.

Proof: Let A is intuitionistic fuzzy sgp-closed set. Let A $\subseteq$ U and U is intuitionistic fuzzy regular open sets in X. Since every intuitionistic fuzzy regular open set is intuitionistic fzzy semi-open sets, U is intuitionistic fuzzy semi open sets such that A $\subseteq$ U. Now by definition of intuitionistic fuzzy sgp-closed sets pcl(A) $\subseteq$ U. We have pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is intuitionistic fuzzy regular open in X. Therefore A is intuitionistic fuzzy gpr-closed set.

Remark 3.8: The converse of above theorem need not be true as from the following example

Example 3.8: Let X = {a, b, c, d, e} and intuitionistic fuzzy sets O,U, V defined as follows

O = { < a, 0.9, 0.1> ,<b, 0.8, 0.1>,<c ,0, 1>,<d, 0, 1> <e ,0, 1>}
U = { < a ,0, 1, > ,<b, 0, 1>,<c, 0.8 ,0.1>,<d, 0.7, 0.2> <e, 0, 1>}
V = { < a ,0.9, 0.1> ,<b, 0.8, 0.1>,<c, 0.8 ,0.1>, <d, 0.7 ,0.2>,<e, 0, 1>}

Let $\Im$ = {**0**□, O, U, V, **1**□} be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set A ={ < a ,0.9, 0.1 > ,<b, 0, 1 >, <c, 0, 1>,<d, 0, 1> ,<e, 0, 1 >} is intuitionistic fuzzy gpr-closed but it is not intuitionistic fuzzy sgp-closed.

Theorem 3.9: Every intuitionistic fuzzy sgp-closed set is intuitionistic fuzzy gspr-closed.

Proof: Let A is intuitionistic fuzzy sgp-closed set. Let A $\subseteq$ U and U is intuitionistic fuzzy regular open sets in X. Since every intuitionistic fuzzy regular open set is intuitionistic fzzy semi-open sets, U is intuitionistic fuzzy semi open sets such that A $\subseteq$ U. Now by definition of intuitionistic fuzzy sgp-closed sets pcl(A) $\subseteq$ U. Note that spcl(A) $\subseteq$ pcl(A) is always true. We have spcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is intuitionistic fuzzy regular open in X. Therefore A is intuitionistic fuzzy gspr-closed set.

Remark 3.9: The converse of above theorem need not be true as from the following example

Example 3.9: Let X = {a, b, c, d} and intuitionistic fuzzy sets O,U defined as follows

O = { < a, 0.7, 0.1> ,<b, 0.6, 0.2 >,<c ,0, 1>,<d, 0, 1> }
U = { < a, 0, 1,> ,<b, 0, 1>,<c, 0.7, 0.2>,<d, 0.6, 0.1> }

Let $\Im$ = {**0**▪, O, U, **1**▪} be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set A ={<a, 0.7, 0.1 >,<b, 0, 1 >,<c, 0, 1>,< d, 0, 1>} is intuitionistic fuzzy gspr -closed but it is not intuitionistic fuzzy sgp-closed.

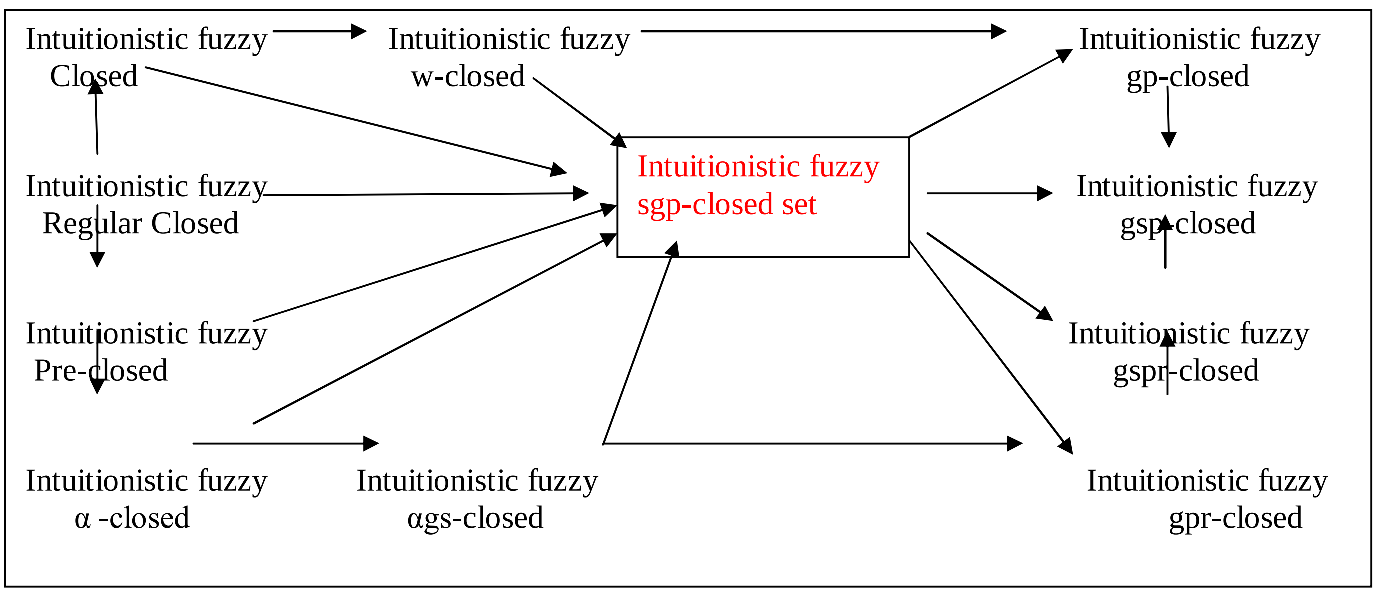
Theorem 3.10: Every intuitionistic fuzzy sgp-closed set is intuitionistic fuzzy gsp-closed.

Proof: Let A is intuitionistic fuzzy sgp-closed set. Let A $\subseteq$ U and U is intuitionistic fuzzy semi open sets in X. Now by definition of intuitionistic fuzzy sgp-closed sets pcl(A) $\subseteq$ U. Note that spcl(A) $\subseteq$ pcl(A) is always true. We have spcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is intuitionistic fuzzy semi open in X. Therefore A is intuitionistic fuzzy gsp-closed set.

Remark 3.10: The converse of above theorem need not be true as from the following example.

Example 3.10: Let X = {a, b} and $\Im$ = {**0**▪, U **1**▪} be an intuitionistic fuzzy topology on X, where U={<a, 0.5, 0.3>,<b, 0.2, 0.3>}.Then the intuitionistic fuzzy set A = {<a, 0.2 ,0.4>,<b, 0.6, 0.1>} is intuitionistic fuzzy gsp -closed but it is not intuitionistic fuzzy sgp- closed.

Remark 3.11: From the above discussion and known results we have the following diagram of implications:



Theorem 3.11: Let $(X,\Im)$ be an intuitionistic fuzzy topological space and A is an intuitionistic fuzzy set of X. Then A is intuitionistic fuzzy sgp-closed if and only if $\rceil(AqF) \Rightarrow \rceil(pcl(A)qF)$ for every intuitionistic fuzzy semi - closed set F of X.

Proof: **Necessity**: Let F be an intuitionistic fuzzy semi- closed set of X and $\rceil(AqF)$. Then by Lemma 2.1(a), $A \subseteq F^c$ and F^c is intuitionistic fuzzy semi- open in X. Therefore $pcl(A) \subseteq F^c$ by Def 3.1 because A is intuitionistic fuzzy sgp-closed. Hence by lemma 2.1(a), $\rceil(pcl(A)qF)$

Sufficiency: Let O be an intuitionistic fuzzy semi open set of X such that $A \subseteq O$ i.e. $A \subseteq ((O)^c)^c$ Then by Lemma 2.1(a), $\rceil(A_q O^c)$ and O^c is an intuitionistic fuzzy semi- closed set in X. Hence by hypothesis $\rceil(pcl\ (A)\ _q\ O^c)$. Therefore by Lemma 2.1(a), $pcl\ (A) \subseteq ((O)^c)^c$ i .e. $pcl\ (A) \subseteq O$ Hence A is intuitionistic fuzzy sgp-closed in X.

Remark 3.12: The intersection of two intuitionistic fuzzy sgp-closed sets in an intuitionistic fuzzy topological space $(X,\Im)$ may not be intuitionistic fuzzy sgp-closed. For,

Example 3.11: Let X = {a, b, c, d } and intuitionistic fuzzy sets O,U, V,W defined as follows

O = { < a ,0.9, 0.1 > ,<b, 0 ,1>,<c, 0, 1>,<d, 0 ,1>}

U = { < a, 0, 1,> ,<b ,0.8 ,0.1>,<c ,0, 1> ,<d ,0, 1>}

V = { < a ,0.9, 0.1> ,<b, 0.8, 0.1,>,<c, 0, 1>,<d, 0, 1>}

W = { < a, 0.9, 0.1,> ,<b,0.8, 0.1 >,<c, 0.7, 0.2>,< d, 0, 1>}

Let $\Im$ = {**0**, O, U, V, W **1**} be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set A = {<a, 0.9, 0.1 >,<b, 0.8, 0.1 > < c, 0 ,1>,< d, 0, 1>} and
B = {<a, 0.9, 0.1 >, < b, 0, 1>, < c, 0.7, 0.2 > ,< d, 0.9 ,0.1>}
are intuitionistic fuzzy sgp-closed in $(X, \Im)$ but $A \cap B$ is not intuitionistic fuzzy sgp-closed.

Theorem 3.12: Let A be an intuitionistic fuzzy sgp-closed set in an intuitionistic fuzzy topological space $(X, \Im)$ and $A \subseteq B \subseteq pcl\ (A)$ .Then B is intuitionistic fuzzy sgp-closed in X.

Proof: Let O be an intuitionistic fuzzy semi -open set in X such that $B \subseteq O$. Then $A \subseteq O$ and since A is intuitionistic fuzzy sgp-closed, $pcl(A) \subseteq O$. Now $B \subseteq pcl\ (A) \Rightarrow cl\ (B) \subseteq pcl\ (A) \subseteq O$.. Consequently B is intuitionistic fuzzy sgp-closed**.**

Definition 3.2: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space $(X, \Im)$ is called intuitionistic fuzzy sgp--open if and only if its complement A^c is intuitionistic fuzzy sgp-closed.

Remark 3.13: Every intuitionistic fuzzy -open set is intuitionistic fuzzy sgp-open but its converse may not be true.

Example 3.12: Let X = {a, b} and $\Im$ = {**0**, U, **1**} be an intuitionistic fuzzy topology on X, where U={<a ,0.7, 0.2 >,<b ,0.6 ,0.3>}.Then the intuitionistic fuzzy set A={<a, 0.2, 0.7 >,< b, 0.1, 0.8 >} is intuitionistic fuzzy sgp-open in $(X, \Im)$ but it is not intuitionistic fuzzy open in $(X, \Im)$.

Theorem 3.13: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space $(X, \Im)$ is intuitionistic fuzzy sgp-open if $F \subseteq pcl(A)$ whenever F is intuitionistic fuzzy semi-closed and $F \subseteq A$.

Proof: Follows from definition 3.1 and Lemma 2.1

Theorem 3.14: Let A be an intuitionistic fuzzy sgp-open set of an intuitionistic fuzzy topological space $(X ,\Im)$ and $pint\ (A) \subseteq B \subseteq A$. Then B is intuitionistic fuzzy sgp-open.

Proof: Suppose A is an intuitionistic fuzzy sgp-open in X and $pint(A) \subseteq B \subseteq A$. $\Rightarrow A^c \subseteq B^c \subseteq (pint(A))^c \Rightarrow A^c \subseteq B^c \subseteq pcl(A^c)$ by Lemma 2.1(d) and A^c is intuitionistic fuzzy sgp-closed it follows from theorem 3.12 that B^c is intuitionistic fuzzy sgp-closed .Hence B is intuitionistic fuzzy sgp-open.

IV. CONCLUSION

The theory of g-closed sets plays an important role in general topology. Since its inception many weak and strong forms of g-closed sets have been introduced in general topology as well as fuzzy topology and intuitionistic fuzzy topology. The present paper investigated a new form of intuitionistic fuzzy closed sets called intuitionistic fuzzy sgp-closed sets which contain the classes of intuitionistic fuzzy closed sets, intuitionistic fuzzy pre-closed sets, intuitionistic fuzzy α-closed sets, intuitionistic fuzzy w-closed sets, intuitionistic fuzzy αgs-closed sets, and contained in the classes of intuitionistic fuzzy gp-closed sets, intuitionistic fuzzy gpr-closed sets , intuitionistic fuzzy gspr-closed sets and class of all intuitionistic fuzzy gspr-closed sets. Several properties and application of intuitionistic fuzzy sgp--closed sets are studied. Many examples are given to justify the result.

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