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AN ANALYSIS OF DIFFERENT CRITERIA FOR RANKING FUZZY NUMBERS

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Abstract : Fuzzy set theory has been used to model systems that are hard to define precisely. As a methodology, fuzzy set theory incorporates imprecision and subjectivity into the model formulation and solution process. Fuzzy set theory represents an attractive tool to aid research in many fields when the dynamics of the production environment limit the specification of model objectives, constraints and the precise measurement of model parameters. This paper provides a survey of the application of different criteria for ranking fuzzy numbers using fuzzy set theory. A classification scheme for fuzzy applications in research is defined. Multi-Criteria Decision Making (MCDM) methods have evolved to accommodate various types of applications. Dozens of methods have been developed, with even small variations to existing methods causing the creation of new branches of research. This paper performs a literature review of common Multi-Criteria Decision Making methods, examines the advantages and disadvantages of the identified methods, and explains how their common applications relate to their relative strengths and weaknesses. The analysis of MCDM methods performed in this paper provides a clear guide for how MCDM methods should be used in particular situations.
Keywords: Fuzzy set theory, multi-criteria decision making; multi-criteria decision analysis, Decision Makers.

I. INTRODUCTION

Most of our traditional tools for formal modeling, reasoning and computing are crisp, deterministic, and precise in character. Crisp means dichotomous, that is, yes-or-no type rather than more-or-less type. In traditional dual logic, for instance, a statement can be true or false—and nothing in between. In set theory, an element can either belong to a set or not; in optimization a solution can be feasible or not. Precision assumes that the parameters of a model represent exactly the real system that has been modeled. This generally implies that the model is unequivocal, that is it contains no ambiguities. Certainty eventually indicates that we assume the structures and parameters of the model to be definitely known and that there are no doubts about their values or their occurrence.

Most of the early interest in fuzzy set theory pertained to representing uncertainty in human cognitive processes[5]. Fuzzy set is now applied to problems in engineering, business, medical and related health sciences, and the natural sciences. In an effort to gain a better understanding of the use of fuzzy set theory in research and to provide a basis for future research, a literature review of fuzzy set theory has been conducted. While similar survey efforts have been undertaken for other areas. Over the years there have been successful applications and implementations of fuzzy set theory in production management. Fuzzy set theory is being recognized as an important problem modeling and solution technique.

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As evidenced by the large number of citations found, fuzzy set theory is an established and growing research discipline. The use of fuzzy set theory as a methodology for modeling and analyzing decision systems is of particular interest to researchers in engineering and management due to fuzzy set theory's ability to quantitatively and qualitatively model problems which involve vagueness and imprecision.

II. MODELING UNCERTAINTY USING FUZZY SET THEORY

According to [3][4][6], a decision problem is said to be complex and difficult, if there exist:

- Multiple criteria—both qualitative and quantitative in nature;
- Multiple decision makers;
- Uncertainty and risk; and
- Incomplete information, imprecise data, and vagueness surrounding the decision making.

The ideal assessment methodology should therefore be capable of synthesizing multi-factors to reach an overall evaluation; it should have a way to process vague information and expert judgment; and it should be flexible enough to handle situations which are slightly different from past experience.

III. OVERVIEW OF FUZZY DECISION FRAME WORK

Fuzzy set theory does not replace probability theory but rather provides a solution to problems that lack the mathematical rigor required by probability theory. Membership function, linguistic variables, natural language computation, linguistic approximation, fuzzy integrals and fuzzy weighted sum are main concepts of fuzzy set theory applied to approximate characterization and decision making. A linguistic variable differs from numerical variable in that its values are not numbers but words or sentences in a natural or artificial language[2][7]. Linguistic variables such as "poor management," "good performance," and "moderate risk" describe the vague concept. A fuzzy decision-making framework generally consists of the following steps:

- Defining and specifying the types of fuzzy numbers and their membership functions to be used by Decision Makers;
- Establishing the scale of preference structure to be used by Decision Makers;
- Assigning the fuzzy values to attributes based on their performance on the decision criteria;
- Aggregating fuzzy numbers across the Decision Makers;
- Determination of global importance or overall value of each of the decision criteria;
- Defuzzification; and
- Ranking of alternatives.

IV. FUZZY MEMBERSHIP FUNCTION

Membership function of an element represents a degree to which the element belongs to a set. Let a_i be a fuzzy number such that $\forall a_i \in R$ (set of real numbers) and considered in the form of

$$a_i = \{x_1, x_2, x_3, x_4\}, \qquad \text{for } i = 1,2, \ldots, m$$

where $x_1 < x_2 < x_3 < x_4$= scale of preference structure to be used by Decision Makers and m = number of fuzzy number to be used in the analysis. Figs. 1 and 2 show the graphical representation of trapezoidal and triangular membership function $\mu(x)$, respectively.

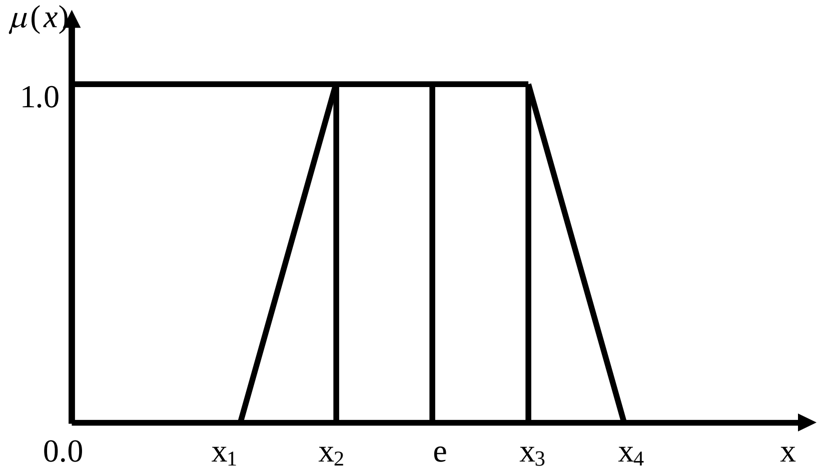


Figure 1: Graphical representation of trapezoidal membership function

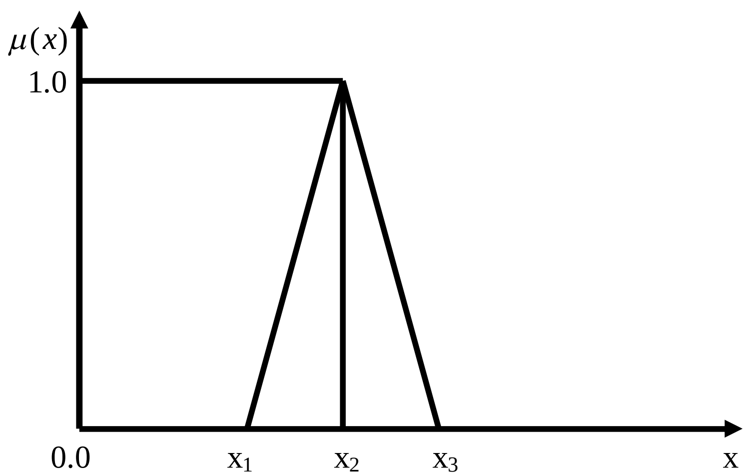


Figure 2 :Graphical representation of triangular Membership function

The normalized trapezoidal membership function of an alternative a_i can be expressed in the form of

$$\mu_{ai}(x) = \begin{cases} 0, & x < x_1 \\ (x - x_1)/(x_2 - x_1), & x_1 < x < x_2 \\ 1, & x_2 < x < x_3 \\ (x_4 - x)/(x_4 - x_3), & x_3 < x < x_4 \\ 0, & x > x_4 \end{cases} \quad (1)$$

V. RANKING FUZZY NUMBERS

If all the grades are real numbers, a total grade which is on a linear scale can be obtained. When the grades are represented by fuzzy sets, the overall grade which is also a fuzzy set can be obtained. One is then faced with another ranking problem in which the grade is unique but fuzzy with uncertainty. In the statistical decision analysis problems also, there are several decision making criteria to rank the alternatives. And in that all the criteria for ordering random variables are designed to define a function to convert the probability distribution to a single value (index) by which the decision could be made based on the largest index value. The same idea can be carried over to the ordering problem of fuzzy variables. There are many criteria to define the index function with different emphases and no single criterion is satisfactory for all situations. The choice of criteria is context dependent.

VI. DEFUZZIFICATION

Defuzzification is an operation that produces a non-fuzzy or crisp value that adequately represents the degree of satisfaction of the aggregated fuzzy number. A fuzzy number can be defined by a crisp quantity that represents the “defuzzified” or “expected” value of the fuzzy number. Fuzzy numbers are a generalization of the concept of the interval of confidence. As we are dealing with a number represented by an interval, ranking this number is not a straightforward process. Therefore, calculating the “expected value” of the fuzzy number will render the fuzzy number ranking and comparison much easier. Different methodologies have been developed to capture an expected value of a fuzzy number[1]. For example a trapezoidal fuzzy number be parameterized by x_1, x_2 ,x_3, x_4 as shown in Fig. 1, then its defuzzified value e is given by the following equation [1]:

$$e = (x_1 + x_2 + x_3 + x_4) / 4 \quad (2)$$

Similarly, for triangular fuzzy number as represented in Fig. 2

$$e = (x_1 + 2x_2 + x_3) / 4 \quad (3)$$

A. THE PSEUDO-EXPECTATION CRITERION

Under this criterion, we define an index function as follows:

$$D_1(A_i) = \int_0^1 x\mu_{Ai}(x)dx \tag{4}$$

Where $\mu_{Ai}(x)$ is the membership function for the fuzzy risk A_i.

As an example, suppose that two alternatives, A_1 and A_2, have fuzzy risk values as shown in figure 2. For triangular memberships, Equation 4 can be reduced to

$$D_1(A_i) = \frac{(b-a)(a+b+c)}{6} \tag{5}$$

hence for alternative A_1,

$$D_1(A_1) = \frac{(1.0-0.2)(0.2+0.4+1.0)}{6} = 0.213$$

and for alternative A_2,

$$D_1(A_2) = \frac{(0.8-0.1)(0.1+0.6+0.8)}{6} = 0.175$$

since $D_1(A_1) > D_1(A_2)$, thus A_1 is preferred to A_2 (interpreted as A_1 has higher risk than A_2).

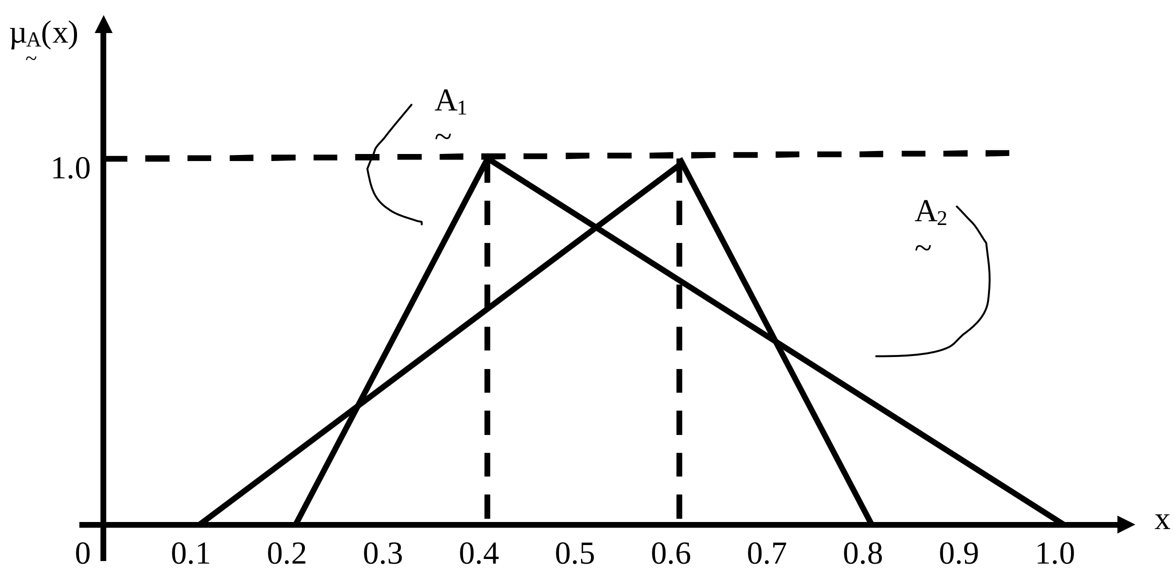


Figure 3: Fuzzy risk values for alternatives A_1 and A_2

Note that the membership functions are not required to be normalized. Hence $D_1(A_i)$ is not weighted average in the usual sense. The process may lead to a pathological ordering in some cases. For instance, suppose that the two fuzzy risks, A_1 and A_2, are as shown in figure 3.

Then, we have

$$D_1(A_1) = \frac{(0.8-0)(0+0.4+0.8)}{6} = 0.16$$

$$D_1(A_2) = \frac{(1-0.9)(0.9+0.95+1.0)}{6} = 0.048$$

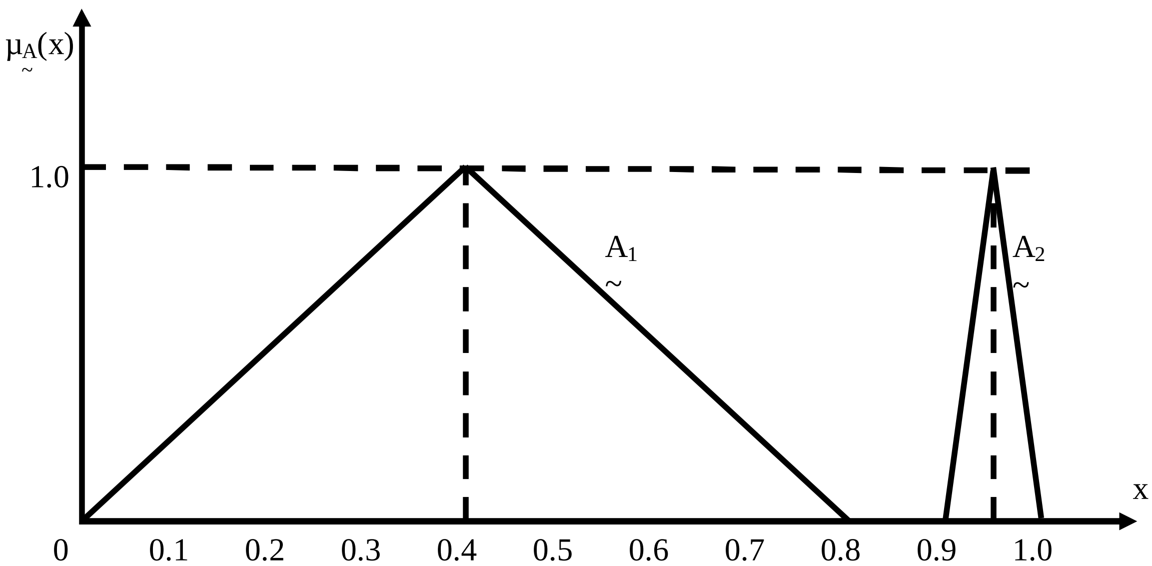


Figure 4 : A pathological case for criterion 1

Since $D_1(A_1) > D_1(A_2)$, A_1 is preferred to A_2. Needless to say, this decision does not agree with our intuition at all.

B. THE GRAVITY CENTER CRITERION

Define the following mapping function as index function to order the different alternatives Ai:

$$D_2(A_i) = \frac{\int_0^1 x\mu_{Ai}(x)dx}{\int_0^1 \mu_{Ai}(x)dx} \qquad (6)$$

Which is nothing but the abscissa of the center of gravity of Ai. When the membership function is triangular, Equation 6 can be reduced to

$$D_2(A_i) = \frac{(a+b+c)}{3} \quad (7)$$

For the example of Figure 3, we have

$$D_2(A_1) = \frac{(0.2+0.4+1.0)}{3} = 0.53$$

$$D_2(A_2) = \frac{(0.1+0.6+0.8)}{3} = 0.5$$

Since $D_2(A_1) > D_2(A_2)$, alternative A_1 is preferred to alternative A_2, a conclusion which is not different from the previous result. However, for Figure 4, we have

$D_2(A_1) = 0.4$; $D_2(A_2) = 0.95$

Under this criterion, since $D_2(A_2) > D_2(A_1)$, we prefer A_2 to A_1 which is an intuitively reasonable ordering. This criterion eliminates the effect of the area under the membership function and is better than criterion 1 in general.

C. THE AVERAGE MEAN CRITERION

Define the mean of an α-cut as follows (In figure 5)

$m_\alpha = ½ (a_\alpha + b_\alpha)$ (8)

then, the average mean value is defined by

$$D_8(A_i) = \int_0^1 m_\alpha \, d\alpha \quad (9)$$

Which is used as the index value. For triangular membership functions, Equation 9 reduces to

$$D_8(A_i) = ¼ (a + 2c + b) \quad (10)$$

And for trapezoidal membership functions, we have

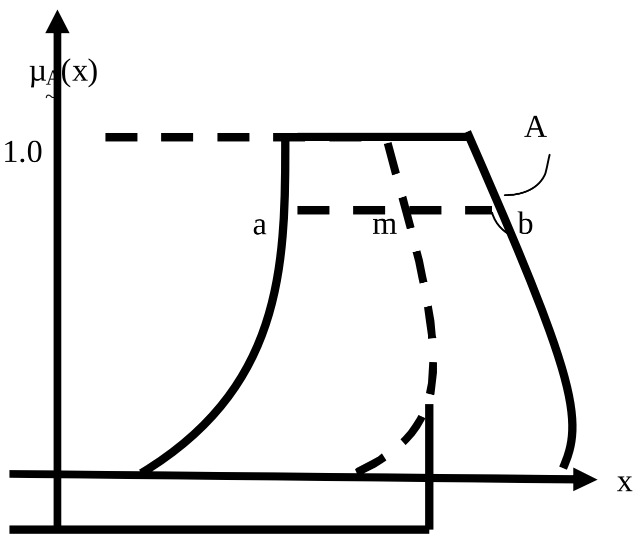
$$D_8(A_i) = ¼ (a + c + d + b) \quad (11)$$

for the example of figure 5, this criterion will lead to the conclusion that alternative A_2 is preferable since

$D_8(A_1) = ¼ (0.2 + 2*0.4 + 1.0) = 0.5$,

$D_8(A_2) = ¼ (0.1 + 2*0.6 + 0.8) = 0.525$

And, $D_8(A_2) > D_8(A_1)$



0 D_8 (A)

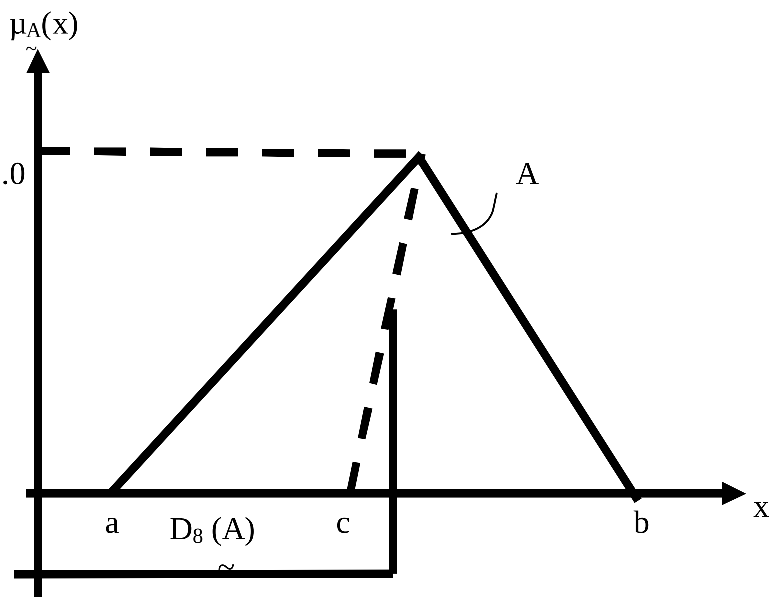


Figure 5: Average mean criterion

VII. CONCLUSION

Throughout the study, it has been observed that fuzzy set theory has been applied to most traditional areas of research on fuzzy set theory in different fields has grown in recent years. Fuzzy research in quality management , forecasting , and job scheduling have experienced tremendous growth in recent years. The fuzzy set theory which deals with a set of objects characterized by a membership function that assigns to each object a grade of membership ranging between zero and one is introduction and attempt is made to model uncertainty by using fuzzy numbers.

The methodology of extracting fuzzy numbers from experts is presented. The concepts that must be considered when modeling uncertainty using fuzzy arithmetic are introduced. Various approaches for representing uncertainty are discussed. A brief discussion of evaluation of multi-attribute based on multi-attributes decision theory is made.

The knowledge based are condensed into the fuzzy system description in which the, linguistic information can be processed using fuzzy set theory. Fuzzy algorithms for processing ill-defined information are developed when modeling uncertainty using fuzzy arithmetic are introduced. Methods for defuzzifying a fuzzy number is described, to define a fuzzy number by crisp quantities that represent the defuzzified or expert value of the fuzzy number.

The appropriateness and contribution of fuzzy set theory to problem solving in research may be seen by paralleling its use in operation research. Many researchers identified that fuzzy set theory can be used in operation research as a language to model problem which contain phenomena or relationships, as a tools to analyze such models in order to gain better insight into the problem and as an algorithmic tool to make solution procedures more stable or faster.

Numerous MCDM methods have been created and utilized over the last several decades. In recent years, because of ease of use due to advancing technologies, combining different methods has become common place in MCDM. The combination of multiple methods addresses deficiencies that may be seen in certain methods. These methods, along with the methods in their original forms, can be extremely successful in their applications, but only if their strengths and weaknesses are properly assessed. Certain problems could easily utilize a method that may not be best suited to solve it. This paper assessed the more common methods of MCDM in order to benefit practitioners to choose a method for solving a specific problem. Identification of common MCDM methods and identification of strengths and weaknesses is a major step in establishing the foundation of research. In this area, but it is only the first step.

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