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Large-Scale Analysis of Linear Massive MIMO Precoders in the Presence of Phase Noise

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Abstract**—**We study the impact of phase noise on the downlink performance of a multi-user multiple-input multiple-output system, where the base station (BS) employs a large number of transmit antennas M. We consider a setup where the BS employs Mosc freerunning oscillators, and M/Mosc antennas are connected to each oscillator. For this configuration, we analyze the impact of phase noise on the performance of the zero-forcing (ZF), regularized ZF, and matched filter (MF) precoders when M and the number of users K are asymptotically large, while the ratio M/K = _ is fixed.

I. INTRODUCTION

MASSIVE multiple-input multiple-output (MIMO) is a promising technology for future wireless networks [1], [2]. This technology deploys antenna arrays containing hundreds of antennas, which can be exploited to significantly enhance the network throughput and energy efficiency performances [3]. In particular, employing massive antenna arrays at the base station (BS) is expected to provide significant array gains and improved spatial precoding resolution for downlink transmission in multi-user (MU) MIMO systems [4].

II. SYSTEM MODEL

In this section, we introduce the considered single-cell massive MIMO system with i.i.d. flat-fading channels, oscillator phase noise at the BS and the UEs, TDD operation, channel estimation at the BS, and linear precoding.

III. LARGE-SCALE ANALYSIS OF THE RECEIVED SIGNALAND ACHIEVABLE RATES

In this section, we use tools from random matrix theory [14] to analyze the received signal model in (6). Specifically, we present a simplification of the desired signal term in (6) for the GO setup when $M,K \rightarrow \infty$, while $M/K = \beta$. Notably, we will show that in the CO and DO setups, the multiple-input single-output (MISO) system model in (6) can be re-written as an equivalent single-input single-output (SISO) phase noise channel including the effects of phase noise,

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AWGN, and interference [15]. Furthermore, we define the effective SINR, and discuss the achievable rates for the GO setup.

A. Received Signal Model

For the RZF precoder in (7), the received signal at the kth UE in (6) becomes

$$
\begin{aligned}
y^k_{\mathrm{d},\tau} &= \mathbf{h}_k^{\mathrm{T}} \boldsymbol{\Theta}_{\tau,k} \mathbf{G}_0 \mathbf{c}_\tau + w_{\mathrm{d},\tau,k} \\
&= \underbrace{\sqrt{p_k}\mathbf{h}_k^{\mathrm{T}} \boldsymbol{\Theta}_{\tau,k} \xi \left(\hat{\mathbf{H}}_0^{\mathrm{H}} \hat{\mathbf{H}}_0 + M\alpha \mathbf{I}_M\right)^{-1} \hat{\mathbf{h}}_{0,k}^{*} c_{\tau,k}}_{\triangleq \mathrm{I}_{\mathrm{sig}}} \\
&\quad + \underbrace{\mathbf{h}_k^{\mathrm{T}} \boldsymbol{\Theta}_{\tau,k} \xi \left(\hat{\mathbf{H}}_0^{\mathrm{H}} \hat{\mathbf{H}}_0 + M\alpha \mathbf{I}_M\right)^{-1} \hat{\mathbf{H}}_{0,-k}^{\mathrm{H}} \mathbf{P}_{-k}^{\frac{1}{2}} \mathbf{c}_{\tau,-k}}_{\triangleq \mathbf{I}_{\mathrm{int}}} \\
&\quad + w_{\mathrm{d},\tau,k}. \qquad (8)
\end{aligned}
$$

In (8), we have introduced the following definitions: $\hat{\mathbf{H}}_{0,-k} = [\hat{\mathbf{h}}_{0,1}, \ldots, \hat{\mathbf{h}}_{0,k-1}, \hat{\mathbf{h}}_{0,k+1}, \ldots, \hat{\mathbf{h}}_{0,K}]$, $\mathbf{P}_{-k} = \mathrm{diag}\{p_1, \ldots, p_{k-1}, p_{k+1}, \ldots, p_K\}$, and $\mathbf{c}_{\tau,-k} = [c_{\tau,1}, \ldots, c_{\tau,k-1}, c_{\tau,k+1}, \ldots, c_{\tau,K}]^{\mathrm{T}}$. Furthermore, $\mathrm{I}_{\mathrm{sig}}$ □ $\mathbb{C}$ and $\mathbf{I}^{\mathrm{T}}_{\mathrm{int}}$ □ $\mathbb{C}^{M-1\times 1}$ denote the scaling factors associated with the desired symbol and the interfering symbols at the kth UE, respectively. The factor $\mathrm{I}_{\mathrm{sig}}$ is simplified in the following proposition. *Proposition 1:* Consider an RZF precoded downlink transmission from a BS having M antennas to K single-antenna UEs employing TDD in the presence of oscillator phase noise. Let $\alpha > 0$, $M/K = \beta$, $\beta \geq 1$, and q_0 □ $[0, 1]$. Assume that $\frac{1}{M}\hat{\mathbf{H}}^{\mathrm{H}}\hat{\mathbf{H}}$ has uniformly bounded spectral norm for all M. Then, the desired signal factor $\mathrm{I}_{\mathrm{sig}}$, for $M,K \to \infty$, can be simplified to

where

$$T_{\mathrm{PN}} \triangleq \lim_{M\to\infty} \frac{1}{M} \mathrm{tr}\{\Delta\boldsymbol{\Phi}_\tau\}, \qquad (10)$$

$$\xi = \lim_{M,K\to\infty} \sqrt{\frac{M(1+m(-\alpha))^2}{m'(-\alpha)\sum_{k=1}^{K} p_k}} \qquad (11)$$

$$m(-\alpha) = \frac{\beta - 1 - \alpha\beta + \sqrt{\beta^2\alpha^2 + 2(\beta+1)\alpha\beta + (1-\beta)^2}}{2\alpha\beta} \qquad (12)$$

$$t = \frac{m(-\alpha)}{m(-\alpha)+1}. \qquad (13)$$

In (10), $\Delta\boldsymbol{\Phi}_\tau = \mathrm{diag}\left\{ e^{\jmath(\phi_\tau^{(1)} - \phi_0^{(1)})} \mathbf{1}^{\mathrm{T}}_{M/M_{\mathrm{osc}}\times 1}, \ldots, e^{\jmath(\phi_\tau^{(M_{\mathrm{osc}})} - \phi_0^{(M_{\mathrm{osc}})})} \mathbf{1}^{\mathrm{T}}_{M/M_{\mathrm{osc}}\times 1} \right\}$, $m(-\alpha)$ in (12) is the Stieltjes Transform of the Marchenko-Pastur Law [14, Eqs. (1.12, 2.43)], and $m'(-\alpha) = \frac{dm(z)}{dz}\big|_{z=-\alpha}$.

Proof: Please refer to Theorem 1 for a discussion on the proof, and to [15, Proposition 1] for a detailed proof. *Remark 1:* The terms t and ξ in (9) depend on α and β, and capture the channel hardening effect [1], [4] that results from the averaging of the random fading channels when RZF precoding is used, and $M,K \to \infty$. The term T_{PN} in (10) captures the effects of phase noise variations at the BS between the training and the data transmission phases, and is given by

$$T_{\mathrm{PN}} = \frac{1}{M_{\mathrm{osc}}} \sum_{l=1}^{M_{\mathrm{osc}}} e^{\jmath(\phi_\tau^{(l)} - \phi_0^{(l)})}. \quad (14)$$

Specifically, for the CO setup, where $\Delta\Phi_\tau = e^{\jmath(\phi_\tau - \phi_0)}\mathbf{I}_M$, and the DO setup, where $\Delta\Phi_\tau =$

$\mathrm{diag}\left\{e^{\jmath(\phi_\tau^{(1)} - \phi_0^{(1)})}, \ldots, e^{\jmath(\phi_\tau^{(M)} - \phi_0^{(M)})}\right\}$, (14) reduces to [7]

$$T_{\mathrm{PN}} = \begin{cases} e^{\jmath(\phi_\tau - \phi_0)} & \text{CO setup} \\ e^{-\frac{\tau\sigma_\phi^2}{2}} & \text{DO setup} \end{cases}. \quad (15)$$

$$y_{\mathrm{d},\tau}^{k} = \sqrt{p_k q_0} T_{\mathrm{PN}} \xi t e^{\jmath(\varphi_\tau^{(k)} - \varphi_0^{(k)})} c_{\tau,k} + \mathbf{I}_{\mathrm{int}} \mathbf{c}_{\tau,\text{-}k} + w_{\mathrm{d},\tau,k}. \quad (16)$$

For the CO and the DO setups, the MISO system model in (6) and (8) becomes an equivalent SISO phase noise channel in (16) [15]. However, when $2 \leq$ Mosc $< \infty$, (16) still corresponds to a MISO phase noise channel, since TPN in (14) depends on the random phase noise variations of the multiple oscillators at the BS.

B. Effective SINR and Achievable Rates

For the CO and the DO setups, we define the effective SINR based on the SISO phase noise channel in (16) as [15]

$$\mathsf{SINR}_k = \frac{|\mathrm{I}_{\mathrm{sig}}|^2}{\|\mathbf{I}_{\mathrm{int}}\|^2 + \sigma_{\mathrm{w}}^2}. \quad (17)$$

Based on the effective SINR in (17), an upper bound for the achievable rate of the kth UE for the CO and the DO setups for a given block of symbols (i.e., given τ) is [15], [16]

$$C(\mathsf{SINR}_k) \leq \log_2(1 + \mathsf{SINR}_k). \quad (18)$$

This upper bound, which corresponds to the AWGN channel capacity, is generally tight for low-to-medium SINR values. Another upper bound for the achievable rate for the CO and the DO setups, which is generally tight at high SINR, was derived by Lapidoth *et al.* [13], and is given by

$$C(\mathsf{SINR}_k) \leq \frac{1}{2}\log_2(4\pi^2 \mathsf{SINR}_k) - \frac{1}{2}\log_2\left(2\pi e \tau(\sigma_\varphi^2 + \delta_{\mathrm{pn}}\sigma_\phi^2)\right), \quad (19)$$

$$C(\mathsf{SINR}_k) = \mathbb{E}_\phi \log_2\left(1 + \frac{|\mathrm{I}_{\mathrm{sig}}|^2}{\|\mathbf{I}_{\mathrm{int}}\|^2 + \sigma_{\mathrm{w}}^2}\right) \quad (20)$$

$$\approx \log_2\left(1 + \frac{\mathbb{E}_\phi|\mathrm{I}_{\mathrm{sig}}|^2}{\mathbb{E}_\phi\|\mathbf{I}_{\mathrm{int}}\|^2 + \sigma_{\mathrm{w}}^2}\right), \quad (21)$$

IV. SINR ANALYSIS

In this section, we introduce Theorem 1, which provides the analytical form for the effective SINR achievable at a given UE for the RZF precoder.

Theorem 1:

Consider an RZF precoded downlink transmission from a BS having M antennas to K single-antenna UEs employing TDD in the presence of oscillator phase noise. Let α > 0, β ≥ 1, q0 □ [0, 1], and SINRk denote the effective SINR at the kth UE. Then,

$$\mathsf{SINR}_k - \mathsf{SINR}_{\mathrm{rzf}_k} \overset{M,K\to\infty}{\longrightarrow} 0 \qquad (23)$$

almost surely, and the effective SINR associated with the kth UE for the GO setup is given as

$$\mathsf{SINR}_{\mathrm{rzf}_k} = \frac{p_k t^2 q_0 \mathbb{E}_\phi |T_{\mathrm{PN}}|^2}{\frac{t_2}{M}\left(1 - t q_0 \mathbb{E}_\phi |T_{\mathrm{PN}}|^2 - \frac{t q_0 \mathbb{E}_\phi |T_{\mathrm{PN}}|^2}{(1+m(-\alpha))}\right) + \frac{\sigma_w^2}{\xi^2}} \quad (24)$$

with

$$\mathbb{E}_\phi |T_{\mathrm{PN}}|^2 \triangleq \mathbb{E}_\phi \left| \frac{1}{M} \mathrm{tr}\left\{\Delta\Phi_\tau\right\}\right|^2 = \frac{1 - e^{-\tau\sigma_\phi^2}}{M_{\mathrm{osc}}} + e^{-\tau\sigma_\phi^2} \quad (25)$$

$$t_2 = \sum_{\substack{k_1=1,\\ k_1\neq k}}^{K} p_{k_1} \frac{m'(-\alpha)}{(1+m(-\alpha))^2}, \qquad (26)$$

where $t, m(-\alpha), \xi$, and T_{PN} are as given in (10)-(13). Specifically, $\mathbb{E}_\phi |T_{\mathrm{PN}}|^2 = \exp\left(-\tau\sigma_\phi^2\right)$ for the DO setup, and $\mathbb{E}_\phi |T_{\mathrm{PN}}|^2 = 1$ for the CO setup.

V. SIMULATION RESULTS

In this section, the analytical results for the RZF precoder presented in Section IV are verified by comparing them against the results obtained from MC simulations. Even though the analytical results are derived for M,K → ∞, in the sequel, we observe that these results concur with those from simulations for finite values of M and K. We simulate the system model specified in (6) using the RZF precoder, and numerically evaluate the effective SINR in (22). Then, the achievable rate in the downlink for a given UE is computed using (21) for all values of Mosc, unless otherwise stated. Recall that this evaluation only depends on the SINR achieved at the UE, and does not account for the reduced variance

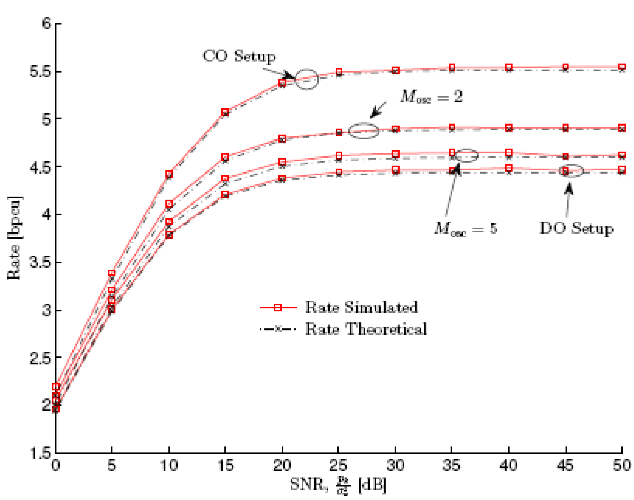

Fig. 2: $C(\mathsf{SINR}_k)$ for the optimized RZF precoder for $\beta = 5, M = 50, \sigma_\phi =$

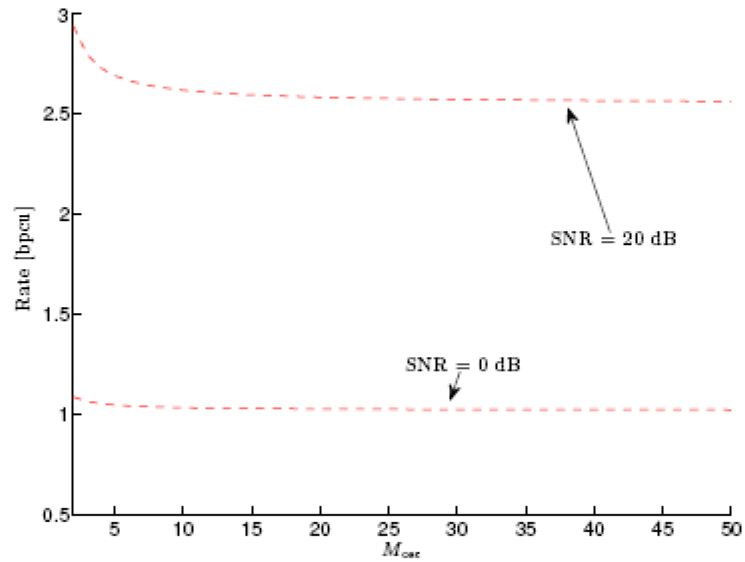

Fig. 3: $C(\mathsf{SINR}_k)$ for the RZF precoder in the GO setup, where $1 \le M_{\text{osc}} \le M, M/M_{\text{osc}} \in \mathbb{Z}^+$, for $M = 50, \beta = 2, q_0 = 0.9, \sigma_\phi = \sigma_\varphi = 0.06°$,

VI. CONCLUSIONS

In this work, we derived the effective SINR of the RZF precoder for the GO setup in the presence of phase noise. We showed that the effect of phase noise on the SINR can be expressed as an effective reduction in the CSI quality available at the BS. Importantly, the SINR degrades as the number of oscillators, Mosc, increases. This is because as Mosc increases, the desired signal power decreases, and the interference power increases. Furthermore, we showed that the variance of the random phase variations caused by the BS oscillators reduces with increasing Mosc. We demonstrated that the SINR obtained is tight and agrees with that obtained from simulations with remarkable accuracy for interesting, and practical values of M and K. Finally, we observed that for the RZF precoder, the CO setup has a higher achievable rate than the DO setup when β is small, while the DO setup outperforms the CO setup when the SNR at the UE is low and β is large.

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