International Journal of Latest Trends in Engineering and Technology
Vol.(7)Issue(3), pp. 354-363
DOI: http://dx.doi.org/10.21172/1.73.547
e-ISSN:2278-621X

ON INTUTIONISTIC FUZZY SUPRA PRE-OPEN SET AND INTUTIONISTIC FUZZY SUPRA-P RE-CONTINUITY ON TOPOLOGICAL SPACES

C. Indirani[1]

Abstract- In this paper, we introduce and investigate a new class of sets and functions between topological spaces called intuitionostic fuzzy supra pre-open set intuitionistic fuzzy supra pre-continious functions and intuitionistic fuzzy supra pre- open maps respectively.

Keywords –Intuitionistic fuzzy supra topological spaces, intuitionistic fuzzy supra pre-open sets, intuitionistic fuzzy supra pre-continuous mappings and intuitionistic fuzzy supra pre-open set.

1. Introduction and preliminaries

The concept of intuitionistic fuzzy set is defined by Atanassov as a general- ization of the concept of fuzzy set given by Zadeh [13]. Using the notation of intuitionistic fuzzy sets, Coker [3] introduced the notation of intuitionistic fuzzy topological spaces. In 1983 Mashhour et al [8] inrtoduced the supra topological spaces and studied s-continuous maps and s - continuous functions. In 1987, Abd El-Monsef et al. [1] introduced the supra topological spaces and studied fuzzy supra-continuous functions and obtained some properties and characteriza- tions. In 1996, Won Keun Min [12] introduced fuzzy s-continuous, fuzzy s-open and fuzzy s-closed maps and established a number of characterizations. In 2008, Devi et al [4] introduced and studied a class of sets and a maps between topo- logical spaces called supra α-open and supra α-continuous functions and studied some of the basic properties for this class of functions. In 1999, Necla Turanl [11] introduced the concept of intuitionistic fuzzy supra topological space. In this pa- per, we study the basic properties of intuitionistic fuzzy supra pre-open sets and introduce the notation of intuitionistic fuzzy supra pre-continuous maps and investigate several properties of intuitionistic fuzzy supra pre-continuous maps.

Throughout this paper, by (X, τ) or simply by X we will denote the intuitionistic fuzzy supra topological space (briefly, IFsTS). For a subset A of a space (X, τ), cl(A), int(A) and $\bar{A}$ denote the closure of A , the interior of A and the complement of A respectively. Each intuitionistic fuzzy supra set (briefly , IFsS) which belongs to (X, τ) is

[1] Bannari Amman Institute Of Technology, Sathyamangalam 638 401.

called intuitionistic fuzzy supra open set (briefly, IF- sOS) in X. The complement of an IFsOS A in X is called an intuitionistic fuzzy supra fuzzy supra closed set (IFsCS) in X. We introduce some basic notations and results that are used in the sequel.

Definition 1.1. [2] Let X be a non empty fixed set and I be the closed interval [0, 1]. An intuitionistic fuzzy set (IFS) A is an object of the following form

$$A = \{<x, \mu_A(x), \nu_A(x)> : x \in X\}.$$

where the mappings $\mu_A : X \to I$ and $\nu_A : X \to I$ denote the degree of membership (namely $\nu_A(x)$) and the degree of nonmembership (namely $\mu_A(x)$) for each element $x \in X$ to the set A respectively and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$. Obviously, every fuzzy set A on a nonempty set X is an IFS of the following form

$$A = \{<x, \mu_A(x), 1 - \mu_A(x)> : x \in X\}.$$

Definition 1.2. **[2]** Let A and B are IFSs of the form $A = \{<x, \mu_A(x), \nu_A(x)> : x \in X\}$ and $B = \{<x, \mu_B(x), \nu_B(x)> : x \in X\}$. Then

(i) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$;

(ii) $\bar{A} = \{<x, \mu_A(x), \nu_A(x)> : x \in X\}$

(iii) $A \cap B = \{<x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)> : x \in X\}$; (iv) $A \cup B = \{<x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)> : x \in X\}$; (v) $A = B$ iff $A \subseteq B$ and $B \subseteq A$;

(vi) $[\,]A = \{<x, \mu_A(x), 1 - \mu_A(x)> : x \in X\}$; (vii) $<>A = \{<x, 1 - \nu_A(x), \nu_A(x)> : x \in X\}$;

(viii) $1_\sim = \{<x, 1, 0>, x \in X\}$ and $0_\sim = \{<x, 0, 1>, x \in X\}$.

We will use the notation $A = <x, \mu_A(x), \nu_A(x)>$ instead of $A = \{<x, \mu_A(x), \nu_A(x)>, x \in X\}$.

Definition 1.3. **[10]** A family τ of IFsS's on X called an instuitionistic fuzzy supra topology (IFsT for short) on X if $0_\sim \in \tau$, $1_\sim \in \tau$ and τ is closed under arbitrary suprema. Then we call the pair (X, τ) an instuitionistic fuzzy supra topological space (IFsTS for short). Each member of τ is called an instuitionis- tic fuzzy supra open set and the complement of instuitionistic fuzzy supra open set is called instuitionistic fuzzy supra closed set. The instuitionistic fuzzy supra closure of an IFsS A is denoted by s-cl(A). Here s-cl(A) is the intersection of all instuitionistic fuzzy supra closed sets containing A. The instuitionistic fuzzy supra interior of A will be denoted by s-int(A). Here, s-int(A) is the union of all instuitionistic fuzzy supra open sets contained in A.

Definition 1.4. Let (X, τ) be an instuitionistic fuzzy supra topological space. An IFS $A \in IF(X)$ is called

(i) instuitionistic fuzzy supra α-open [7] iff $A \subseteq$ s-int(s-cl(s-int(A))),

(ii) instuitionistic fuzzy supra β-open [6] iff $A \subseteq$ s-cl(s-int(s-cl(A))),

(iii) instuitionistic fuzzy supra b-open [6] iff $A \subseteq$ s-int(s-cl(A)) $\cup$ s-cl(s-int(A)), Let f be a mapping from an ordinary set X into an ordinary set Y, if B =

$\{<y, \mu_A(y), \nu_B(y)> : y \in Y\}$ is an IFsT in Y, then the inverse image of B under f

is an IFsT defined by $f^{-1}(B) = \{<x, f^{-1}(\mu_B)(x), f^{-1}(\nu_B)(x)> : x \in X\}$.

The image of IFsT $A = \{<y, \mu_A(y), \nu_A(y)> : y \in Y\}$ under f is an IFsT defined by $f(A)$ =

$\{<y, f(\mu_A(y)), f(\nu_A(y))> : y \in Y\}$.

2. Intuitionistic fuzzy supra pre-open set.

In this section, we introduce a new class of open sets called intuitionistic fuzzy supra pre-open sets and study some of their basic properties.

Definition 2.1. Let (X, τ) be an intuitionistic fuzzy supratopological space. An intuitionistic fuzzy set A is called an intuitionistic fuzzy supra pre-open set (briefly IFsPOS) if A $\subseteq$ s-int(s-cl(A)). The complement of an intuitionistic fuzzy supra pre-open set is called an intuitionistic fuzzy supra pre-closed set (briefly IFsPCS).

Theorem 2.2. Every intuitionistic fuzzy supra-open set is an intuitionistic fuzzy supra pre-open.

Proof. Let A be an intuitionistic fuzzy supra-open set in (X, τ). Then A $\subseteq$ s-int(A), we get A $\subseteq$ s-int(s-cl(A)) then s-int(A) $\subseteq$ s-int(s-cl(A)) . Hence A is supra pre-open in (X, τ).

The converse of the above theorem need not be true as shown by the following example.

Example 2.3. Let $X = \{a, b\}$, $A = \{x, <0.5, 0.2>, <0.3, 0.4>\}$,

$B = \{x, <0.3, 0.4>, <0.6, 0.5>\}$ and $C = \{x, <0.3, 0.4>, <0.2, 0.5>\}$,

$\tau = \{0_\sim, 1_\sim, A, B, A \cup B\}$. Then C is called intuitionistic fuzzy supra pre-open set but it is not an intuitionistic fuzzy supra-open set.

Theorem 2.4. Every intuitionistic fuzzy supra α-open set is an intuitionistic fuzzy supra pre-open

Proof. Let A be an intuitionistic fuzzy supra α-open set in (X, τ). Then A $\subseteq$ s-int(s-cl(s-int(A)), it is obvious that s-int(s-cl(s-int(A)) $\subseteq$ s-int(s-cl(A)) and A $\subseteq$ s-int(s-cl(A)). Hence A is an intuitionistic fuzzy supra pre-open in (X, τ).

The converse of the above theorem need not be true as shown by the following example.

Example 2.5. Let $X = \{a, b\}$, $A = \{x, <0.3, 0.5>, <0.4, 0.5>\}$,

$B = \{x, <0.4, 0.3>, <0.5, 0.4>\}$ and $C = \{x, <0.4, 0.5>, <0.5, 0.4>\}$,

$\tau = \{0_\sim, 1_\sim, A, B, A \cup B\}$. Then C is called an intuitionistic fuzzy supra pre-open set but it is not an intuitionistic fuzzy supra α-open set.

Theorem 2.6. Every intuitionistic fuzzy supra pre-open set is an intuitionistic fuzzy supra β-open

Proof. Let A be an intuitionistic fuzzy supra pre-open set in (X, τ). It is obvious that s-int(s-cl(A)) $\subseteq$ s-cl(s-int(s-cl(A))). Then A $\subseteq$ s-int(s-cl(A)). Hence A $\subseteq$ s-cl(s-int(s-cl(A))).

The converse of the above theorem need not be true as shown by the following example.

Example 2.7. Let $X = \{a, b\}$, $A = \{x, <0.2, 0.3>, <0.5, 0.3>\}$,

$B = \{x, <0.1, 0.2>, <0.6, 0.5>\}$ and $C = \{x, <0.2, 0.3>, <0.2, 0.3>\}$,

$\tau = \{0_\sim, 1_\sim, A, B, A \cup B\}$. Then C is called an intuitionistic fuzzy supra pre-open set but it is not an intuitionistic fuzzy supra pre-open set.

Theorem 2.8. Every intuitionistic fuzzy supra pre-open set is an intuitionistic fuzzy supra b-open

Proof. Let A be an intuitionistic fuzzy supra pre-open set in (X, τ). It is obvious that A $\subseteq$ s-int(s-cl(A)) $\subseteq$ s-int(s-cl(A)) $\cup$ s-cl(s-int(A))), Then A $\subseteq$ s- int(s-cl(A)). Hence A $\subseteq$ s-int(s-cl(A)) $\cup$ s-cl(s-int(A))).

The converse of the above theorem need not be true as shown by the following example.

Example 2.9. Let $X = \{a, b\}$, $A = \{x, <0.5, 0.2>, <0.3, 0.4>\}$, $B = \{x, <0.3, 0.4>, <0.6, 0.5>\}$ and $C = \{x, <0.3, 0.4>, <0.4, 0.4>\}$, $\tau = \{0_\sim, 1_\sim, A, B, A \cup B\}$. Then C is called an intuitionistic fuzzy supra pre-open set but it is not an intuitionistic fuzzy supra pre-open set.

IFsOS $\rightarrow$ IFsαOS

& $\downarrow$

IFsbOS $\leftarrow$ IFsPOS $\rightarrow$ IFsβOS

Theorem 2.10.

(i) Arbitrary union of intuitionistic fuzzy supra pre-open sets is always an in- tuitionistic fuzzy supra pre-open.

(ii) Finite intersection of intuitionistic fuzzy supra pre-open sets may fail to be an intuitionistic fuzzy supra pre-open.

(iii) X is an intuitionistic fuzzy supra pre-open set.

Proof.

(i) Let A and B to be intuitionistic fuzzy supra pre-open sets. Then A $\subseteq$ s-int(s-cl(A)) and B $\subseteq$ s-int(s-cl(B)). Then A $\cup$ B $\subseteq$ s-int(s-cl(A)). Therefore, A $\cup$ B $\subseteq$ s-int(s-cl(A $\cup$ B)) $\subseteq$ s-int($\cup$ (s-cl(A $\cup$ B))) $\subseteq$s−int(s- cl($\cup$ (A $\cup$ B))) is an intuitionistic fuzzy supra pre-open sets.

(ii) Let $X = \{a, b\}$, $A = \{x, <0.3, 0.4>, <0.2, 0.5>\}$, $B = \{x, <0.3, 0.4>, <0.4, 0.4>\}$ and $\tau = \{0_\sim, 1_\sim, A, B, A \cup B\}$. Hence A and B are intuitionistic fuzzy supra pre-open but $A \cap B$ is not intuitionistic fuzzy supra pre-open set.

Theorem 2.11.

(i) Arbitrary intersection of intuitionistic fuzzy supra pre-closed sets is always an intuitionistic fuzzy supra pre-closed.

(ii) Finite union of intuitionistic fuzzy supra pre-closed sets may fail to be an intuitionistic fuzzy supra pre-closed.

Proof.
(i) This proof immediately from Theorem 2.10

(ii) Let $X = \{a, b\}$, $A = \{x, <0.2, 0.3>, <0.2, 0.4>\}$, $B = \{x, <0.5, 0.4>, <0.4, 0.5>\}$ and $\tau = \{0_\sim, 1_\sim, A, B, A \cup B\}$. Hence A and B are intuitionistic fuzzy supra pre-closed but $A \cup B$ is not an intuitionistic fuzzy supra pre-closed set.

Definition 2.12**.** The intuitionistic fuzzy supra pre-closure of a set A, denoted by s-pre-cl(A), is the intersection of an intuitionistic fuzzy supra pre-closed sets including A. The intuitionistic fuzzy supra pre-interior of a set A, denoted by s-pre-int(A), is the union of intuitionistic fuzzy supra pre-open sets included in A.

Remark 2.13. It is clear that s-pre-int(A) is an intuitionistic fuzzy supra pre-open set and s-pre-cl(A) is an intuitionistic fuzzy supra pre-closed set.

Theorem 2.14.

(i) $A \subseteq$ s-pre-cl(A); and A = s-pre-cl(A) iff A is an intuitionistic fuzzy supra pre-closed set;

(ii) s-pre-int(A) $\subseteq$ A; and s-pre-int(A) = A iff A is an intuitionistic fuzzy suprapre-open set;

(iii) $X -$s-pre-int(A) = s-pre-cl($X - A$);
(iv) $X -$s-pre-cl(A) = s-pre-int($X - A$).

Proof. Obvious.

Theorem 2.15.
(i) s-pre-int(A) $\cup$ s-pre-int(B) $\subseteq$ s-pre-int($A \cup B$); (i) s-pre-cl($A \cap B$) $\subseteq$ s-pre-cl(A) $\cap$ s-pre-cl(B).
Proof. Obvious.
The inclusions in (i) and (ii) in Theorem 2.15 can not replaced by equalities by let $X = \{a, b\}$, $A = \{x, <0.3, 0.4>, <0.2, 0.5>\}$, $B = \{x, <0.3, 0.4>, <0.4, 0.4>\}$ and $\tau = \{0_\sim, 1_\sim, A, B, A \cup B\}$, where s-pre-int(A) = $\{x, <0.2, 0.5>, <0.3, 0.4>\}$, s-pre- int(B) = $\{x, <0.5, 0.4>, <0.4, 0.5>\}$ and s-pre-int($A \cup B$) = $\{x, <0.5, 0.5>, <0.3, 0.4>\}$. Then s-pre-cl(A) $\cap$ s-pre-cl(B) = $\{x, <0.3, 0.4>, <0.2, 0.5>\}$ and s-pre-cl(A)=s-pre- cl(B) = $1_\sim$.
Proposition 2.16.

(i) The intersection of an intuitionistic fuzzy supra open set and an intuitionistic fuzzy supra pre-open set is an intuitionistic fuzzy supra pre-open set

(ii) The intersection of an intuitionistic fuzzy supra α-open set and an intu- itionistic fuzzy supra pre-open set is an intuitionistic fuzzy supra pre-open set

3. Intuitionistic suzzy supra pre-continuous mappings.

In this section, we introduce a new type of continuous mapings called a intu- itionistic fuzzy supra pre-continuous mappings and obtain some of their properties and characterizations.

Definition 3.1. Let (X, τ) and (Y, σ) be the two intuitionistic fuzzy topolog- ical sets and μ be an associated supra topology with τ. A map $f : (X, \tau) \to (Y, \sigma)$ is called an intuitionistic fuzzy supra pre-continuous mapping if the inverse image of each open set in Y is an intuitionistic fuzzy supra pre-open set in X.

Theorem 3.2. Every intuitionistic fuzzy supra continuous map is an intu- itionistic fuzzy supra pre-continuous map .

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ is called intuitionistic fuzzy continuous map and A is an IFsOS in Y. Then $f^{-1}(A)$ is an IFsOS in X . Since μ is associated with τ, then τ □ μ. Therefore, $f^{-1}(A)$ is an IFsOS in X which is an IFsPOS set in X . Hence f is an intuitionistic fuzzy supra pre-continuous map.

The converse of the above theorem is not true as shown in the following exam ple.

Example 3.3. Let $X = \{a, b\}$, $Y = \{u, v\}$ and $A = \{<0.5, 0.2>, <0.3, 0.4>\}$, $B = \{<0.3, 0.4>, <0.6, 0.5>\}$, $C = \{<0.5, 0.4>, <0.3, 0.4>\}$, $D = \{<0.3, 0.4>, <0.6, 0.5>\}$. Then $\tau = \{0_\sim, 1_\sim, A, B, A \cup B\}$ be an intuitionistic fuzzy supra topology on X. Then the intuitionistic fuzzy supra topology σ on Y is defined as follows: $\sigma = \{0_\sim, 1_\sim, C, D, C \cup D\}$. Define a mapping $f(X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The inverse image of the IFsOS in Y is not an IFsOS in X but it is an IFsPOS. Then f is an intuitionistic fuzzy supra pre-continuous map but not be an intuitionistic fuzzy supra continuous map.

Theorem 3.4. Every intuitionistic fuzzy supra α-continuous map is an intu- itionistic fuzzy supra pre-continuous map .

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ is called intuitionistic fuzzy supra α-continuous map and A is an IFsOS in Y. Then $f^{-1}(A)$ is an IFsαOS in X . Since μ is as- sociated with τ, then τ □ μ. Therefore, $f^{-1}(A)$ is an IFsαOS in X which is an IFsPOS in X. Hence f is an intuitionistic fuzzy supra pre-continuous map.

The converse of the above theorm is not true as shown in the following exam- ple.

Example 3.5. Let $X = \{a, b\}$, $Y = \{u, v\}$ and $A = \{<0.5, 0.2>, <0.3, 0.4>\}$, $B = \{<0.3, 0.4>, <0.6, 0.5>\}$, $C = \{<0.5, 0.4>, <0.3, 0.4>\}$, $D = \{<0.3, 0.4>, <0.6, 0.5>\}$. Then $\tau = \{0_\sim, 1_\sim, A, B, A \cup B\}$ be an intuitionistic fuzzy supra topology on X. Then the intuitionistic fuzzy supra topology σ on Y is defined as follows: $\sigma = \{0_\sim, 1_\sim, C, D, C \cup D\}$. Define a mapping $f(X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The inverse image of IFsOS in Y is not an IFsαOS in X but it is an IFsPOS. Then f is an intuitionistic fuzzy supra pre-continuous map but not be an intuitionistic fuzzy supra α-continuous map.

Theorem 3.6. Every intuitionistic fuzzy supra pre-continuous map is an in- tuitionistic fuzzy supra b-continuous map .

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ is called intuitionistic fuzzy supra pre- continuous map and A is an IFsOS in Y. Then $f^{-1}(A)$ is an IFsPOS in X . Since μ is associated with τ, then τ □ μ. Therefore, $f^{-1}(A)$ is an IFsPOS in X which is an IFsbOS in X. Hence f is an intuitionistic fuzzy supra b-continuous map.

The converse of the above theorm is not true as shown in the following exam- ple.

Example 3.7. Let $X = \{a, b\}$, $Y = \{u, v\}$ and $A = \{<0.5, 0.2>, <0.3, 0.4>\}$, $B = \{<0.3, 0.4>, <0.6, 0.5>\}$, $C = \{<0.5, 0.4>, <0.3, 0.4>\}$, $D = \{<0.3, 0.4>, <0.6, 0.5>\}$. Then $\tau = \{0_\sim, 1_\sim, A, B, A \cup B\}$ be an intuitionistic fuzzy supra topology on X. Then the intuitionistic fuzzy supra topology σ on Y is defined as follows: $\sigma = \{0_\sim, 1_\sim, C, D, C \cup D\}$. Then $\tau = \{0_\sim, 1_\sim, A, B, A \cup B\}$ be an intuitionistic fuzzy supra topology on X. Then the intuitionistic fuzzy supra topology σ on Y is defined as follows: $\sigma = \{0_\sim, 1_\sim, C, D, C \cup D\}$. Define a mapping $f(X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The inverse image of IFsOS in Y is not an IFsPOS in X but it is an IFsbOS. Then f is an intuitionistic fuzzy supra b-continuous map but not be an intuitionistic fuzzy supra pre-continuous map.

Theorem 3.8. Every intuitionistic fuzzy supra pre-continuous map is an in- tuitionistic fuzzy supra β-continuous map .

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ is called intuitionistic fuzzy supra pre-continuous map and A is an IFsOS in Y. Then $f^{-1}(A)$ is an IFsPOS in X . Since μ is associated with τ, then $\tau \subseteq \mu$. Therefore, $f^{-1}(A)$ is an IFsPOS in X which is an IFsβOS in X. Hence f is an intuitionistic fuzzy supra β-continuous map.

The converse of the above theorem is not true as shown in the following exam ple.

Example 3.9. Let $X = \{a, b\}$, $Y = \{u, v\}$ and $A = \{<0.5, 0.2>, <0.3, 0.4>\}$, $B = \{<0.3, 0.4>, <0.6, 0.5>\}$, $C = \{<0.5, 0.2>, <0.3, 0.4>\}$, $D = \{<0.3, 0.4>, <0.6, 0.5>\}$. Then $\tau = \{0_\sim, 1_\sim, A, B, A \cup B\}$ be an intuitionistic fuzzy supra topology on X. Then the intuitionistic fuzzy supra topology σ on Y is defined as follows: $\sigma = \{0_\sim, 1_\sim, C, D, C \cup D\}$. Define a mapping $f(X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The inverse image of IFsOS in Y is not an IFsPOS in X but it is an IFsβOS. Then f is an intuitionistic fuzzy supra b-continuous map but not be an intuitionistic fuzzy supra pre-continuous map.

IFs continuous $\to$ IFsα continuous

& $\downarrow$

IFsb continuous $\leftarrow$ IFsP continuous $\to$ IFsβ continuous

Theorem 3.10. Let (X, τ) and (Y, σ) be the two topological spaces and μ be an associated intuitionistic fuzzy supra topology with τ. Let f be a map from X into Y. Then the following are equivalent

(i) f is an intuitionistic fuzzy supra pre-continuous map.

(ii) The inverse image of an intuitionistic fuzzy supra closed sets in Y is an intuitionistic fuzzy supra pre-closed set in X;

(iii) s-pre-cl$(f^{-1}(A)) \subseteq f^{-1}(cl(A))$ for every set A inY;

(iv) f(s-pre-cl(A)) $\subseteq$ cl(f(A)) for every set A in X;

(v) $f^{-1}(int(B)) \subseteq$ s-pre-int$(f^{-1}(B))$ for every set B in Y.

Proof. (i) $\Rightarrow$ (ii): Let A be a closed set in Y, then $Y - A$ is open set in Y. Then $f^{-1}(Y - A) = X - f^{-1}(A)$ is s-pre-open set in X. It follows that $f^{-1}(A)$ is a supra pre-closed subset of X.

(ii) $\Rightarrow$ (iii): Let A be any subset of Y. Since cl(A) is closed in Y, then it follows that f^{-1}(cl(A)) is an intuitionistic fuzzy supra pre-closed set in X. Therefore s-pre-cl(f^{-1}(A)) □ (f^{-1}(cl(A))) $=$ (f^{-1}(cl(A)).

(iii) $\Rightarrow$ (iv): Let A be any subset of X. By (iii) we have f^{-1}(cl(f(A))) □ s-pre-cl(f^{-1}(f(A))) □ s-pre-cl(A) and hence f(s-pre-cl(A)) □ cl(f(A)).

(iv) $\Rightarrow$ (v): Let B be any subset of Y. By (4) we have f^{-1}(s-pre-cl($X - f^{-1}$(B))) □ cl(f($X - f^{-1}$(B))) and f($X -$s-pre-int(f^{-1}(B))) □ cl($Y - B$) $= Y -$int(B). There- fore we have $X -$s-pre-int(f^{-1}(B)) □ f^{-1}($Y -$int(B)) and hence f^{-1}(int(B)) □ s- pre-int(f^{-1}(B)).

(v) $\Rightarrow$ (i): Let B be a open set in Y and f^{-1}(int(B)) □ s-pre-int(f^{-1}(B)), hence f^{-1}(B) □ s-pre-int(f^{-1}(B)). Then f^{-1}(B) $=$ s-pre-int(f^{-1}(B)). But, s-pre- int(f^{-1}(B)) □ f^{-1}(B). Hence f^{-1}(B) $=$ s-pre-int(f^{-1}(B)). Therefore f^{-1}(B) is an intuitionistic fuzzy supra pre-open set in Y.

Theorem 3.11. If a map $f : (X, \tau) \rightarrow (Y, \sigma)$ is a s-pre-continuous and

$g : (Y, \sigma) \rightarrow (Z, \eta)$ is continuous ,then $(g \circ f)$ is s-pre-continuous.

Proof. Obvious.

Theorem 3.12. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an intuitionistic fuzzy s-pre- continuous map if one of the following holds:

(i) f^{-1}(s-pre-int(B)) □ int(f^{-1}(B)) for every set B in Y, (ii) cl(f^{-1}(A)) □ f^{-1}(s-pre-cl(B)) for every set B in Y,

(iii) f(cl(A)) □ s-pre-cl(f(B)) for every A in X.

Proof. Let B be any open set of Y, if the condition (i) is satisfied, then f^{-1}(s- pre-int(B)) □ int(f^{-1}(B)). We get, f^{-1}(B) □ int(f^{-1}(B)). Therefore f^{-1}(B) is an intuitionistic fuzzy open set. Every intuitionistic fuzzy open set is intuitionistic fuzzy supra pre-open set. Hence f is an intuitionistic fuzzy s-pre-continuous . If condition (ii) is satisfied, then we can easily prove that f is an intuitionistic fuzzy supra pre-continuous. Let condition (iii) is satisfied and B be any open set in Y. Then f^{-1}(B) is a set in X and then we can easily prove that f is an intuitionistic fuzzy s-pre- continuous function. If condition (iii) is satisfied, and B is any open set of Y. Then f^{-1}(B) is a set in X and f(cl(f^{-1}(B))) □ s-pre-cl(f(f^{-1}(B))). This implies f(cl(f^{-1}(B))) □ s-pre-cl(B). This is nothing but condition (ii). Hence f is an intuitionistic fuzzy s-pre-continuous.

4. Intuitionistic Fuzzy supra pre-open maps and supra pre-closed maps.

Definition 4.1. A map $f : X \longrightarrow Y$ is called intuitionistic fuzzy supra pre-open (res.intuitionistic fuzzy supra pre-closed) if the image of each open (resp.closed) set in X, is intuitionistic fuzzy supra pre-open(resp.intuitionistic fuzzy supra pre-closed) in Y.

Theorem 4.2. A map $f : X \longrightarrow Y$ is called an intuitionistic fuzzy supra pre-open if and only if $f(int(A)) \subseteq$ s-pre-int(A) for every set A in X.

Proof. Suppose that f is an intuitionistic fuzzy supra pre-open map. Since $int(A) \subseteq f(A)$. By hypothesis $f(int(A))$ is an intuitionistic fuzzy supra pre-open set and s-pre-int(f(A)) is the largest intuitionistic fuzzy supra pre-open set con- tained in f(A), then $f(int(A)) \subseteq$ s-pre-int(f(A))

Conversely, Let A be a open set in X. Then $f(int(A)) \subseteq$ s-pre-int(f(A)). Since $int(A) = A$, then $f(A) \subseteq$ s-pre-int(f(A)). Therefore f(A) is an intuitionistic fuzzy supra pre-open set in Y and f is an intuitionistic fuzzy supra pre-open.

Theorem 4.3. A map $f : X \longrightarrow Y$ is called an intuitionistic fuzzy suprapre-closed if and only if $f(cl(A)) \subseteq$ s-pre-cl(A) for every set A in X.

Proof. Suppose that f is an intuitionistic fuzzy supra pre-closed map. Since for each set A in X, cl(A) is closed set in X, then f(cl(A)) is an intuitionis- tic fuzzy supra pre-closed set in Y. Also, since $f(A) \subseteq f(cl(A))$, then s-pre- cl(f(A)) $\subseteq f(cl(A))$.

Conversely, Let A be a closed set in X. Since s-pre-cl(f(A)) is the smallest intuitionistic fuzzy supra pre-closed set containing f(A), then $f(A) \subseteq$ s-pre- cl(f(A)) $\subseteq f(cl(A)) = f(A)$. Thus $f(A) =$ s-pre-cl(f(A)). Hence f(A) is an intuitionistic fuzzy supra pre-closed set in Y. Therefore f is a intuitionistic fuzzy supra pre-closed map.

Theorem 4.4. Let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be two maps.

(i) If $g \circ f$ is an intuitionistic fuzzy supra pre-open and f is continuous surjective, then g is an intuitionistic fuzzy supra pre-open.

(ii) If $g \circ f$ is open and g is an intuitionistic fuzzy supra precontinuous injective, then f is an intuitionistic fuzzy supra pre-open.

(iii) f is an intuitionistic fuzzy supra pre-closed map;

Proof. (i)$\Rightarrow$ (ii). Suppose B is a closed set in X. Then $X - B$ is an open set in an open set in X. By (1), $f(X - B)$ is an intuitionistic fuzzy supra pre-open set in X. Since f is bijective, then $f(X - B) = Y - f(B)$. Hence f(B) is an intuitionistic fuzzy supra pre-closed set in Y. Therefore f is an intuitionistic fuzzy supra pre-closed map.

(ii)$\Rightarrow$ (iii). Let f is an intuitionistic fuzzy supra pre-closed map and B be closed set X. Since f is bijective, then $(f^{-1})^{-1}(B)=f(B)$ is an intuitionistic fuzzy supra pre-closed set in Y. By Theorem 3.7 f is an intuitionistic fuzzy supra pre-continuous map.

(iii)$\Rightarrow$ (i). Let A be an open set in X. Since f^{-1} is an intuitionistic fuzzy supra pre-continuous map, then $(f^{-1})^{-1}(A) = f(A)$ is an intuitionistic fuzzy supra pre- open set in Y. Hence f is an intuitionistic fuzzy supra pre-open.

REFERENCES

[1] M.E. Abd El-Monsef and A.E.Ramadan, "On fuzzy supra topological spaces",Indian J.Pure and Appl, vol. 4, 18(1987), 322-329.

[2] K.T. Atanassov, intuitionstic fuzzy sets, "Fuzzy sets and systems", 209 1986,87-96.

[3] D. Coker, " An introduction to intuitionstic fuzzy topological spaces", Fuzzy sets and systems, 88(1997), 81-89.

[4] R. Devi, S. Sampathkumar and M. Caldas, "On supra α- open sets and sα -continuous functions General Mathematics", vol. 16, 2 (2008), 77-84

[5] N. Levine, "Semi-open sets and semi-continuity in topological spaces", Amer.Math. Monthly, 70(1963), 36 - 41.

[6] M. Parimala, C.Indirani, "Intuitionistic fuzzy β-supra open sets and Intu- itionistic fuzzy β-supra continuous mapping", Notes on intuitionistic fuzzy sets, 20(2014), 6-12.

[7] M. Parimala, Jafari Syaeid, Intuitionistic fuzzy α-supra continuous maps,"Annals of fuzzy mathematics and informatics", vol.9, 5(2015), 745-751.

[8] A.S. Mashhour, A. A. Allam, F. S. Mahmoud and F. H. Khedr, "On supra topological spaces", Indian J. Pure and Appl. vol. 4, 14(1983), 502-510.

[9] O.R. Sayed, "Supra pre-open sets and Supra pre-continuity on topological spaces", scientific studies and research series mathematics and informatics, 20(2010), 79 -88.

[10] N. Turnal, "On intuitionistic Fuzzy Supra topological Spaces", International Conference on Modeling and Simulation, Spain, vol.2, (1999) 69-77.

[11] N. Turnal, "An over view of intuitionstic fuzzy Supra topological spaces". Hacettepe Journal of Mathematics and Statistics, vol.32 (2003), 17-26.

[12] Won Keun Min, "On fuzzy s- continuous functions", Kangweon - Kyungki, vol.1, 4(1996), 77-82.

[13] Zadeh, L. A. "Fuzzy sets", Information and Control, 8(1965), 338-353.