Batch Arrival Retrial Queue with Fluctuating Modes of Service, Randomized Vacations and Orbital Search

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Abstract-Single server retrial queue with arrivals in batches of variable size and general service in fluctuating modes with different mean service rates is considered. Whenever the system becomes empty the server takes randomized vacations. During the idle period, the server searches for customer in the orbit with certain probability. Using supplementary variable technique, average number of customers in the orbit and average number of customers in the system are derived. Stochastic decomposition law is verified. Numerical results are presented.

Keywords-Retrial Queue, Fluctuating Modes of Service, Randomized Vacations, Orbital Search and Stochastic Decomposition Law

I. INTRODUCTION

Single server queueing system in literature assumes that the server provide one type of general service with same mean rate to all the customers. But in real life situations there could be variation in mean service rate due to many reasons. Baurah et al. (2014) studied a batch arrival single server queue with server providing general service in two fluctuating modes. Madan (2014) investigated a batch arrival queue with general service in three fluctuating modes, balking, randomized breakdowns and a stand-by server during breakdown periods.

In the retrial setup, each service is preceded and followed by the server's idle time because of the ignorance of the status of the server and orbital customers by each other. Server's idle time is reduced by the introduction of search of orbital customers immediately after a service completion. Deepak et al. (2013) obtained expected queue length of a batch arrival retrial queueing system with search of customers from the orbit. Rajadurai et al. (2015) investigated $M^{[X]}/G/1$ feedback retrial queue with two phase service, Bernoulli vacation, delayed repair and orbital search. Rajadurai et al. (2015a) studied $M^{[X]}/G/1$ unreliable retrial G-queue with orbital search and feedback under Bernoulli vacation schedule.

Allowing the server to take vacations makes the queueing model more realistic and flexible in studying the real world queueing situations. Applications arise especially in places like call centres with multitask employees, customized manufacturing, telecommunication and computer networks, production and quality control problems. In this paper batch arrival retrial queue with M fluctuating modes of service, randomized vacations and orbital search is analysed.

MODEL DESCRIPTION

Consider a single server retrial queueing system in which customers arrive in batches according to Poisson process with rate λ . Batch size X is a random variable with distribution function

$P(X = k) = C_k$ and first two moments m_1 and m_2 .

The server provides M heterogeneous types of service and a customer opts one of the types i with probability $p_i (1 \leq i \leq M)$. If the arriving batch finds the server free, then one of the customers in the batch receives service immediately and others join the orbit. Otherwise all the customers join the retrial queue. The retrial time is generally distributed with distribution A(x), density function a(x), Laplace Stieltjes transform $A^*(s)$ and conditional completion rate $\eta(x) = \frac{a(x)}{1-A(x)}$.

The service time of type i (i=1,2,…M) follows general distribution with distribution function $B_i(x)$, density function $b_i(x)$, Laplace Stieltje's transform $B_i^*(s)$, n^{th} factorial moments $\mu_{i,n}$ and conditional completion rate $\mu_i(x) = \frac{b_i(x)}{1-B_i(x)}$.

At a service completion epoch if the orbit becomes empty, the server leaves for a vacation of random length V. At the vacation completion epoch, if the orbit is still empty the server either remains idle in the system with probability q or leaves for another vacation with probability $\bar{q}(=1-q)$. This pattern continues until the number of vacations reaches a preassigned number J. If the system is empty at the end of the J^{th} vacation, the server is dormant idly in the system. Vacation time is generally distributed with distribution function V(x), Laplace Stieltjes transform $V^*(s)$, n^{th} factorial moment v_n and conditional completion rate $\gamma(x) = \frac{v(x)}{1-V(x)}$. During the idle period, if the orbit is non empty, server searches for customers in the orbit with probability θ or remains idle with probability $\bar{\theta}(=1-\theta)$.

The state of the system at time t can be described by the Markov process$\{X(t);t\geq 0\}=\{C(t),N(t), \xi_0(t), \xi_1(t), \xi_2(t); t \geq 0\}$ where C(t) denotes the server state 0, i or M+j accordingly as the server being idle, busy in type i service or on j^{th} vacation. X(t) corresponds to the number of the customers in the orbit. If C(t)=0 and X(t)>0 then $\xi_0(t)$ represents the elapsed retrial time. If C(t)=i$(1 \leq i \leq M)$, then $\xi_1(t)$ represents the elapsed service time. If C(t)= M+j (1≤j≤J) then $\xi_2(t)$ represents elapsed vacation time.

For the process {N(t); t≥0}, define the probability

$I_0(t) = P\{C(t) = 0, X(t) = 0\}$ and

the probability densities for t>0; x>0 as

$I_n(x,t) = P\{C(t) = 0, X(t) = n, x < \xi_0(t) < x + dx\}$, n ≥1.

$P_{i,n}(x,t) = P\{C(t) = i, X(t) = n, x < \xi_1(t) < x + dx\} n \geq 0, 1 \leq i \leq M.$

$V_{j,n}(x,t) = P\{C(t) = M + j, X(t) = n, x < \xi_2(t) < x + dx\} n \geq 0, j = 1,2,3, \dots, J.$

GOVERNING EQUATIONS

The system of steady state equations that governs the model under consideration is given below.

$$\lambda I_0 = \int_0^\infty V_{J,0}(x)\gamma(x)dx + q\sum_{j=1}^{J-1}\int_0^\infty V_{j,0}(x)\,\gamma(x)\,dx \tag{1}$$

$$\frac{d}{dx}I_n(x) = -(\lambda + \eta(x))I_n(x), \qquad n \geq 1 \tag{2}$$

$$\frac{d}{dx}P_{i,n}(x) = -(\lambda + \mu_i(x))P_{i,n}(x) + \lambda\sum_{k=1}^{n} C_k P_{i,n-k}(x), n \geq 0, 1 \leq i \leq M \tag{3}$$

$$\frac{d}{dx}V_{j,n}(x) = -(\lambda + \gamma(x))V_{j,n}(x) + \lambda\sum_{k=1}^{n} C_k V_{j,n-k}(x), n \geq 0, 1 \leq j \leq J \tag{4}$$

with boundary conditions

$$I_n(0) = \sum_{i=1}^{M}\int_0^\infty P_{i,n}(x)\mu_i(x)\,dx + \bar{\theta}\sum_{j=1}^{J}\int_0^\infty V_{j,n}(x)\gamma(x)dx, n \geq 1 \tag{5}$$

$$P_{i,0}(0) = p_i\left[\lambda C_1 I_0 + \int_0^\infty I_1(x)\eta(x)\,dx + \theta\sum_{j=1}^{J}\int_0^\infty V_{j,1}(x)\gamma(x)\,dx\right] \tag{6}$$

$$P_{i,n}(0) = p_i\left[\lambda C_{n+1}I_0 + \int_0^\infty I_{n+1}(x)\eta(x)\,dx + \lambda\sum_{k=1}^{n} C_k\int_0^\infty I_{n-k+1}(x)\,dx + \theta\sum_{j=1}^{J}\int_0^\infty V_{j,n+1}(x)\gamma(x)\,dx\right]$$

$$n \geq 1 \quad (7)$$

$$V_{1,n}(0) = \begin{cases}\sum_{i=1}^{M}\int_0^\infty P_{i,n}(x)\mu_i(x)dx, n = 0\\ 0 \quad , n \neq 0\end{cases} \quad (8)$$

$$V_{j,n}(0) = \begin{cases}\bar{q}\int_0^\infty V_{j-1,n}(x)\gamma(x)\,dx, n = 0, j = 2,3,\dots,J.\\ 0, n \neq 0\end{cases} \quad (9)$$

Define the probability generating functions

$$I(x,z) = \sum_{n=1}^{\infty} I_n(x)z^n \ , \ P_i(x,z) = \sum_{n=0}^{\infty} P_{i,n}(x)z^n ,$$

$$V_j(x,z) = \sum_{n=0}^{\infty} V_{j,n}(x)z^n \text{ and } C(z) = \sum_{k=1}^{\infty} c_k z^k$$

STEADY STATE PROBABILITY GENERATING FUNCTION

Multiplying equations (2) to (4) by z^n and summing over all possible values of n, we get the following differential equations.

$$\left[\frac{d}{dx} + (\lambda + \eta(x))\right] I(x,z) = 0 \quad (10)$$

$$\left[\frac{d}{dx} + \lambda(1 - C(z)) + \mu_i(x)\right] P_i(x,z) = 0, 1 \leq i \leq M \quad (11)$$

$$\left[\frac{d}{dx} + \lambda(1 - C(z)) + \gamma(x)\right] V_j(x,z) = 0, 1 \leq j \leq J \quad (12)$$

Solving the partial differential equations (10), (11) and (12), we get respectively

$$I(x,z) = I(0,z)\,e^{-\lambda x}[1 - A(x)] \quad (13)$$

$$P_i(x,z) = P_i(0,z)e^{-[\lambda(1-C(z))]x}[1 - B_i(x)] \quad (14)$$

$$V_j(x,z) = V_j(0,z)e^{-\lambda(1-C(z))x}[1 - V(x)] \quad (15)$$

Solving equation (4) at n=0, we get

$$V_{j,0}(x) = V_{j,0}(0)e^{-\lambda x}[1 - V(x)], j = 1,2,\dots,J \quad (16)$$

Multiplying equation (16) by $\gamma(x)$ and intrgrating with respect to x from 0 to ∞, we have

$$\int_0^\infty V_{j,0}(x)\,\gamma(x)dx = V_{j,0}(0)V^*(\lambda), j = 1,2,\dots,J \quad (17)$$

Using equation (1), (8) and (9), we get

$$\sum_{j=1}^{J} V_j(0,z) = \lambda I_0 T \quad (18)$$

where

$$T = \frac{[(\bar{q}V^*(\lambda))^J - 1]}{\left[\left((\bar{q}V^*(\lambda))^{J-1}\left((\bar{q}V^*(\lambda)) - 1\right) + q((\bar{q}V^*(\lambda))^{J-1} - 1)\right)\right]}$$

Equations (5), (6) and (7) yields

$$I(0,z) = \sum_{i=1}^{M} P_i(0,z)B_i^*(\lambda - \lambda C(z)) + \bar{\theta}\,\lambda I_0 T V^*(\lambda - \lambda C(z)) - \lambda I_0 T - \lambda I_0 \qquad (19)$$

and

$$P_i(0,z) = \frac{p_i}{z}[I(0,z)\left(C(z)(1 - A^*(\lambda)) + A^*(\lambda)\right) + \lambda I_0 C(z) + \theta\lambda I_0 T V^*(\lambda - \lambda C(z))] \qquad (20)$$

Solving equations (19) and (20) for $I(0,z)$ and $P_i(0,z)$, we get

$$I(0,z) = \frac{\lambda I_0 \begin{bmatrix} \sum p_i B_i^*(\lambda - \lambda C(z))\left(TV^*(\lambda - \lambda C(z))\theta + C(z)\right) + \\ zT\bar{\theta}V^*(\lambda - \lambda C(z)) - z(T+1) \end{bmatrix}}{\left[z - \left(A^*(\lambda) + C(z)(1 - A^*(\lambda))\right)\sum p_i B_i^*(\lambda - \lambda C(z))\right]} \qquad (21)$$

$$P_i(0,z) = p_i \lambda I_0 T_1(z) \qquad (22)$$

where

$$T_1(z) = \frac{\begin{bmatrix} A^*(\lambda)(C(z) - 1) + TV^*(\lambda - \lambda C(z))\theta + \\ T(V^*(\lambda - \lambda C(z))\bar{\theta} - 1)(A^*(\lambda) + C(z)(1 - A^*(\lambda))) \end{bmatrix}}{(1 - C(z))\left[z - \left(A^*(\lambda) + C(z)(1 - A^*(\lambda))\right)\sum p_i B_i^*(\lambda - \lambda C(z))\right]}$$

Substituting equations (21) and (22) in equations (13) and (14), we obtain

$$I(x,z) = \frac{\lambda I_0 \begin{bmatrix} \sum p_i B_i^*(\lambda - \lambda C(z))\left(TV^*(\lambda - \lambda C(z))\theta + C(z)\right) + \\ zT\bar{\theta}V^*(\lambda - \lambda C(z)) - z(T+1) \end{bmatrix}}{\left[z - \left(A^*(\lambda) + C(z)(1 - A^*(\lambda))\right)\sum p_i B_i^*(\lambda - \lambda C(z))\right]} e^{-\lambda x}[1 - A(x)] \qquad (23)$$

$$P_i(x,z) = p_i \lambda I_0 T_1(z) e^{-[\lambda(1 - C(z))]x}[1 - B_i(x)] \qquad (24)$$

PERFORMANCE MEASURES

Define the partial generating function $\psi(z) = \int_0^\infty \psi(z,x)dx$ for any generating function $\psi(z,x)$. Then we have

$$I(z) = \frac{I_0[1 - A^*(\lambda)] \begin{bmatrix} \sum p_i B_i^*(\lambda - \lambda C(z))\left(TV^*(\lambda - \lambda C(z))\theta + C(z)\right) + \\ zT\bar{\theta}V^*(\lambda - \lambda C(z)) - z(T+1) \end{bmatrix}}{\left[z - \left(A^*(\lambda) + C(z)(1 - A^*(\lambda))\right)\sum p_i B_i^*(\lambda - \lambda C(z))\right]} \qquad (25)$$

$$P(z) = I_0[T_1(z)]\sum_{i=1}^{M} p_i[1 - B_i^*(\lambda - \lambda C(z))] \qquad (26)$$

The server on vacation is given by

$$V(z) = \sum_{j=1}^{J} V_j(z) = \frac{I_0[1 - V^*(\lambda - \lambda C(z))]T}{(1 - C(z))} \qquad (27)$$

If I, P and V be the limiting values of $I(z)$, $P(z)$ and $V(z)$ as z tends to 1. Using the normalizing condition $I_0 + I + P + V = 1$, we get

$$I_0 = \frac{1 - m_1[1 - A^*(\lambda)] - \lambda m_1 \sum p_i \mu_{i,1}}{A^*(\lambda) - T\theta + T\theta A^*(\lambda) + T\lambda\gamma_1} \tag{28}$$

The following results are obtained from (25) to (27)

The steady state probability that the server is idle, when the system is not empty is

$$I = \frac{[1 - A^*(\lambda)][\lambda m_1 (T\theta + 1) \sum p_i \mu_{i,1} + T(\lambda\gamma_1 m_1 - \theta) + m_1 - 1]}{A^*(\lambda) - T\theta + T\theta A^*(\lambda) + T\lambda\gamma_1} \tag{29}$$

The steady state probability that the server is busy is

$$P = \frac{\lambda \sum p_i \mu_{i,1} [T\lambda\gamma_1 m_1 + m_1 A^*(\lambda) - T\theta m_1 (1 - A^*(\lambda))]}{A^*(\lambda) - T\theta + T\theta A^*(\lambda) + T\lambda\gamma_1} \tag{30}$$

The steady state probability that the server is on vacation is given by

$$V = \frac{[1 - m_1(1 - A^*(\lambda)) - \lambda m_1 \sum p_i \mu_{i,1}] T\lambda\gamma_1}{A^*(\lambda) - T\theta + T\theta A^*(\lambda) + T\lambda\gamma_1} \tag{31}$$

The probability generating function for the number of customers in the orbit is

$$\begin{aligned} P_q(z) &= I_0 + I(z) + P(z) + V(z) \\ &= I_0 (1 - z) T_1(z) \end{aligned} \tag{32}$$

Let N (z) and D (z) are the numerator and denominator of $P_q(z)$ .Then the mean number of customers in the orbit is

$$\begin{aligned} L_q &= \lim_{z \to 1} \frac{d}{dz} P_q(z) \\ &= \frac{D''(1) N'''(1) - N''(1) D'''(1)}{3(D''(1))^2} \end{aligned} \tag{33}$$

$$N''(1) = 2I_0 m_1 [T\theta(1 - A^*(\lambda)) - T\lambda\gamma_1 - A^*(\lambda)]$$

$$N'''(1) = 3I_0 [(T\theta)\left(m_2(1 - A^*(\lambda))\right) - A^*(\lambda) m_2 - T\lambda^2 m_1{}^2 \gamma_2 - T\lambda m_2 \gamma_1 - 2T\lambda\gamma_1 \bar{\theta} m_1{}^2 (1 - A^*(\lambda))]$$

$$D''(1) = 2m_1 [m_1 \sum p_i \lambda \mu_{i,1} + m_1 (1 - A^*(\lambda)) - 1]$$

$$\begin{aligned} D'''(1) = 3\{&[(m_1)(\sum p_i (\lambda^2 \mu_{i,2} m_1{}^2 + \lambda \mu_{i,1} m_2) + 2m_1{}^2 \lambda (1 - A^*(\lambda)) \sum p_i \mu_{i,1} \\ &m_2 (1 - A^*(\lambda)))] + [(m_2)(m_1(1 - A^*(\lambda)) + m_1 \lambda \sum p_i \mu_{i,1} - 1)]\} \end{aligned}$$

The probability generating function for the number of customers in the system is

$$\begin{aligned} P_s(z) &= I_0 + I(z) + zP(z) + V(z) \\ &= I_0 (1 - z) \sum p_i B_i{}^*(\lambda - \lambda C(z))\ T_1(z) \end{aligned} \tag{34}$$

The mean number of customers in the system can be obtained as

$$L_s = \lim_{z \to 1} \frac{d}{dz} P_s(z) = L_q + P$$

STOCHASTIC DECOMPOSITION

Theorem: The number of customer in the system (L_s) can be expressed as the sum of two independent random variables, one of which is the mean number of customers (L) in the batch arrival queue with fluctuating modes of service and orbital search and other is the mean number of customers in the orbit (L_I) given that the server is idle or on vacation.

Proof:The probability generating function $\pi(z)$ of the system size in the batch arrival queueing system with fluctuating modes of service and orbital search is given by

$$\pi(z) = \frac{(1 - \lambda m_1 \sum p_i \mu_{i,1})(1-z) \sum p_i B_i^*(\lambda - \lambda C(z))(C(z)-1)}{(z - \sum p_i B_i^*(\lambda - \lambda C(z)))(1 + T\lambda\gamma_1)(1 - C(z))} \tag{35}$$

The probability generating function $\psi(z)$ of number of customers in the orbit when the system is idle or on vacation is given by

$$\psi(z) = \frac{I_0 + I(z) + V(z)}{I_0 + I + V}$$

$$= [T_1(z)][T_2(z)](\sum p_i B_i^*(\lambda - \lambda C(z)) - z) \tag{36}$$

where

$$T_2(z) = \frac{[1 - m_1(1 - A^*(\lambda)) - \lambda m_1 \sum p_i \mu_{i,1}]}{(A^*(\lambda) - T\theta + T\theta A^*(\lambda) + T\lambda\gamma_1)(1 - \lambda m_1 \sum p_i \mu_{i,1})}$$

From equations (34), (35) and (36) we see that

$$P_s(z) = \pi(z).\ \psi(z) \tag{37}$$

Differentiating (37) with respect to z and taking limit as $z \to 1$, we get

$$L_s = L + L_I$$

SPECIAL CASES

Case (i): Let $\theta \to 0$(no orbital search) then our model will be reduced to batch arrival retrial queueing system with M fluctuating modes of service and randomized J vacation. In this case, the expressions of I_0, I, P, V, $P_q(z)$, $P_s(z)$ are

$$I_0 = \frac{1 - m_1[1 - A^*(\lambda)] - \lambda m_1 \sum p_i \mu_{i,1}}{A^*(\lambda) + T\lambda\gamma_1}$$

$$I = \frac{I_0[1 - A^*(\lambda)][\lambda m_1 \sum p_i \mu_{i,1} + (T\lambda\gamma_1 m_1) + m_1 - 1]}{1 - m_1[1 - A^*(\lambda)] - \lambda m_1 \sum p_i \mu_{i,1}}$$

$$P = \frac{I_0 \lambda \sum p_i \mu_{i,1}(T\lambda\gamma_1 m_1 + m_1 A^*(\lambda))}{1 - m_1[1 - A^*(\lambda)] - \lambda m_1 \sum p_i \mu_{i,1}}$$

$$V = I_0 T\lambda\gamma_1$$

$$P_q(z) = \frac{I_0(1-z)\begin{bmatrix} A^*(\lambda)(C(z)-1) + [A^*(\lambda) + C(z)(1 - A^*(\lambda))] \\ T(V^*(\lambda - \lambda C(z)) - 1) \end{bmatrix}}{(1 - C(z))\left[z - \left(A^*(\lambda) + C(z)(1 - A^*(\lambda))\right)\sum p_i B_i^*(\lambda - \lambda C(z))\right]}$$

Case (ii): Let $A^*(\lambda) \to 1$ (no retrial queue) then we get the results for a batch arrival classical queue with M fluctuating modes of service, randomized J vacation and orbital search.

$$I_0 = \frac{1 - \lambda m_1 \sum p_i \mu_{i,1}}{1 + T\lambda\gamma_1}$$

$$I = 0$$

$$P = \frac{I_0 \lambda \sum p_i \mu_{i,1}(T\lambda\gamma_1 m_1 + m_1)}{1 - \lambda m_1 \sum p_i \mu_{i,1}}$$

$$V = I_0 T\lambda\gamma_1$$

$$P_q(z) = \frac{I_0(1-z)[(C(z)-1) + T(V^*(\lambda - \lambda C(z)) - 1)]}{(1 - C(z))[z - \sum p_i B_i^*(\lambda - \lambda C(z))]}$$

Case (iii): Let C(z)=z, $\theta \to 0, A^*(\lambda) \to 1$(single arrival , no orbital search, no retrial queue) then we get a M/G/1 queueing system with fluctuating modes of service and randomized J vacation.

$$I_0 = \frac{1 - \lambda \sum p_i \mu_{i,1}}{1 + T\lambda\gamma_1}$$

$$I = 0$$

$$P = \frac{I_0 \lambda \sum p_i \mu_{i,1} (T\lambda\gamma_1 + 1)}{1 - \lambda \sum p_i \mu_{i,1}}$$

$$V = I_0 T\lambda\gamma_1$$

$$P_q(z) = \frac{I_0[(z-1) + T(V^*(\lambda - \lambda z) - 1)]}{[z - \sum p_i B_i^*(\lambda - \lambda z)]}$$

NUMERICAL RESULTS

Numerical results are presented to illustrate the effect of various parameters on the performance of the system by assuming the retrial time, service times and vacation time follow exponential distribution with respesctive rates $\eta, \mu_i\ (1 \le i \le M), \gamma$ set the arbitrary value $\lambda = 1.7;\ \eta = 1.12; p_1 = 0.62;\ p_2 = 0.38;\ m_1 = 0.09;\ m_2 = 1.22;\ \gamma = 0.11; J = 7; \theta = 0.95; q = 0.64; \mu_1 = 0.3;\ \mu_2 = 0.2; M = 2$

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The combined effect of (η, γ) and (λ, θ) on I_0, I, P, V, L_q and L_s are presented in Fig.1 to 12. From the figures it is observed that

i) Increase in γ increases I_0 and V, decreases I, L_s and L_q and has no effect on P.

ii) Increase in η increases V, L_s and L_q, decreases I and has no effect on I_0 and P.

iii) Increase in θ increases V, L_s and L_q and decreases I and has no effect on I_0 and P.

iv) Increase in λ increases I, P, L_s and L_q and decreases I_0 and V.

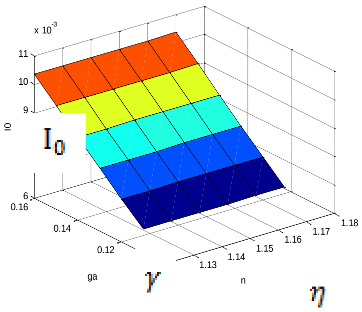

Fig.1 I_0 versus (η, γ)

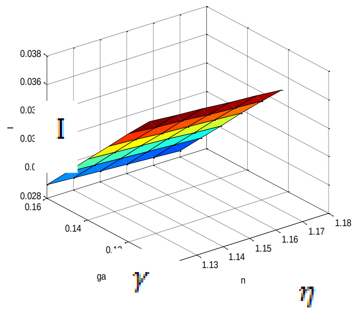

Fig.2 I versus (η, γ)

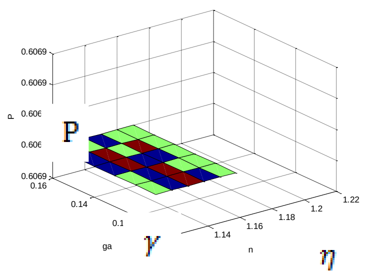

Fig.3 P versus (η, γ)

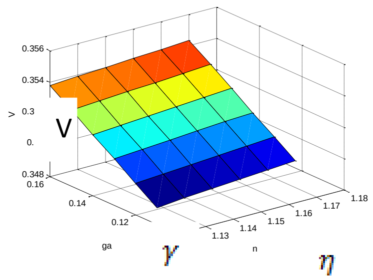

Fig.4 V versus (η, γ)

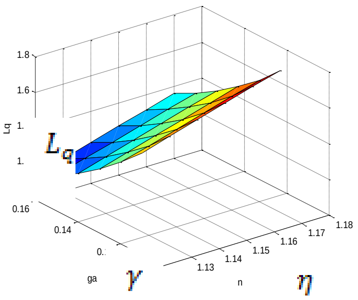

Fig.5 L_q versus (η, γ)

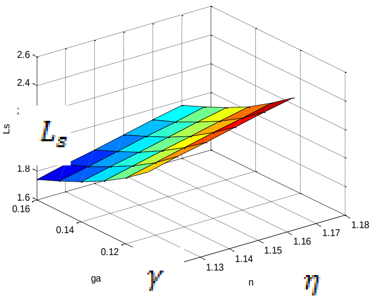

Fig.6 L_s versus (η, γ)

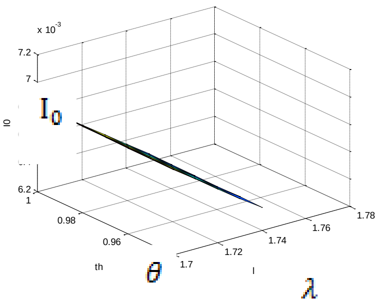

Fig.7 I_0 versus (λ, θ)

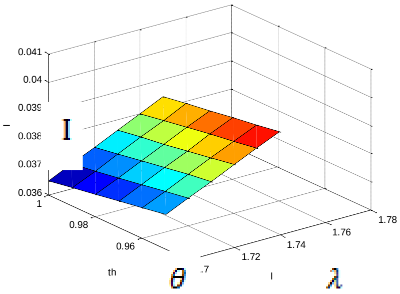

Fig.8 I versus (λ, θ)

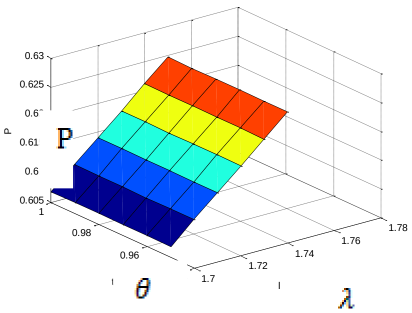

Fig.9 P versus (λ, θ)

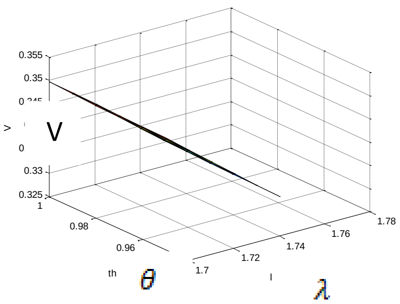

Fig.10 V versus (λ, θ)

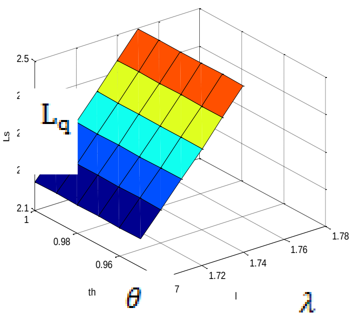

Fig.11 L_q versus (λ, θ)

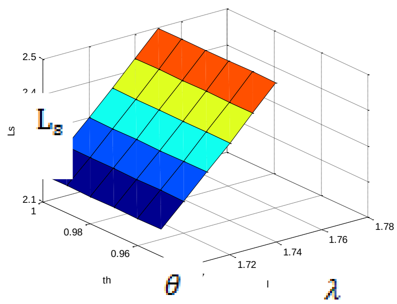

Fig.12 L_s versus (λ, θ)

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