Quantum Dot-A Promising Candidate for Realization of Quantum Computation: An Overview

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***Abstract*- A general introduction to low dimensional nano-structures is presented with a focus particularlyon the novel properties of quantum dots. The electron transport current and transmission probability is obtained for a system of double quantum dots coupled to each other. The results show that current through the system is quantized and transmission probability shows the discrete energy spectrum of system. Therefore, electrons occupying these discrete energy states get trapped on the dots. Further, based on the quantum bits also called qubits, represented by the spin states of electrons in quantum dots, a possibility of realization of quantum gates and hence the quantum computing is reviewed. An overview on recent achievements, current theoretical proposals, challenges and opportunities in the field of quantum computing with quantum dots is drawn. Study shows that amid enormous challenges, quantum dots seem to be promising candidates for materialization of quantum gates required for designing quantum computers.**

***Keywords*- Low-dimensional Nano-structures, quantum dots, quantum transport, quantum computing, qubit**

I. INTRODUCTION

During the last three decades, advances in micro-fabrication techniques have allowed researchers to achieve unique quantum confinement [1] and has thereby opened up the gates to a new realm of fundamental physical ideas.The nanoscale devices with dominant quantum mechanical effects, has attracted a lot of attention in the physics and engineering communities and has triggered a lot of theoretical and experimental research activities [2, 3, 4, 5].Driving force behind much of the research work on these systems is the expectation that the miniaturization will lead to new type of electronic devices which are much more advanced in their performance than the existing devices [6], specifically being much faster and dissipating less heat as was proposedby Nobel Laureate Richard Feynman in 1960 in his visionary and prophetic lecture entitled “There is Plenty of Room at the Bottom”, delivered at the meeting of American Physical Society. Precursor to all these developments has been MOSFET technology. Current state-of-the-art MOSFETs have dimensions that are well within the nano-scale regime. Length and widthof the channel are roughly 40 nm, and the channel can be as thin as 1-2 nm. If channel is represented as a potential well, then thelevel spacing in a MOSFET sized box is typically of the order of 10-5 eV. The connectionto the source and drain regions further broadens the levels into a continuum. Despitethe small size of modern MOSFETs, semi-classical transport calculations that require acontinuous band structure are still accurate.However, when devices shrink further, new features appear. A very small device, weakly coupled to leads, will have a discrete spectrumof energy states and the transport process depends crucially on the properties of each individual level [7]. Due tothe small size of device, Coulomb and spin interactions among electrons on the device are strongand can shift significantly the single particle spectrum [8].

In terms of theoretical research, new computational paradigms based on quantum effects, including quantum information science, have emerged [9]. Many quantum information proposals are converging to semiconductor nanostructures, in particular to quantum dots [10, 11], because of their potential scalability and the well established nanofabrication technology. Experimental requirements to build a quantum computer are difficult [12]. In fact, it remains to be seen whether a quantum computer will be our next-generation computer at all. But at the moment, nanostructures provide a new playground to study quantum physics with a great potential for scientific breakthroughs and applications [13].

The present work is an overview of the theoretical and experimental progress made in the quantum hetero-structuresspecifically an introduction to the quantum dots and its interesting properties. It is proposed to be a suitable system which has opened up the enormous possibilities with potential applications for future research and technological advances in almost all field of research [14]. The elements of quantum computation with quantum dots

[10], has also been discussed and thereby the work presented is of great interest and an important contribution to theinterdisciplinary research.
The remaining review work has been organized as follows: Section II describes the fabrication and introduction of different types of low dimensional nano-structures, Section-III introduces quantum dots and its properties. In section-IV, a brief introduction of quantum computing is presented whereas; quantum computing with quantum dots has been presented in section V. Finally in Section –VI conclusion and future prospects of present review has been presented.

II. LOW-DIMENSIONAL NANOSTRUCTURES

In bulk materials, the electrons are free to move in all the direction and have three degrees of freedom corresponding to the three directions of motion and hence electrons have no confinement effect. With the advanced modern semiconductor fabrication techniques and by virtue of unique electron confinement effect it has become possible to fabricate device with less than three degree of freedom of motion for electrons [15]. This confinement effect is usually carried out in a typical GaAs/AlGaAs hetero-structure, where at the hetero-junction some electrons get trapped in the depletion region as shown in Fig. 1.

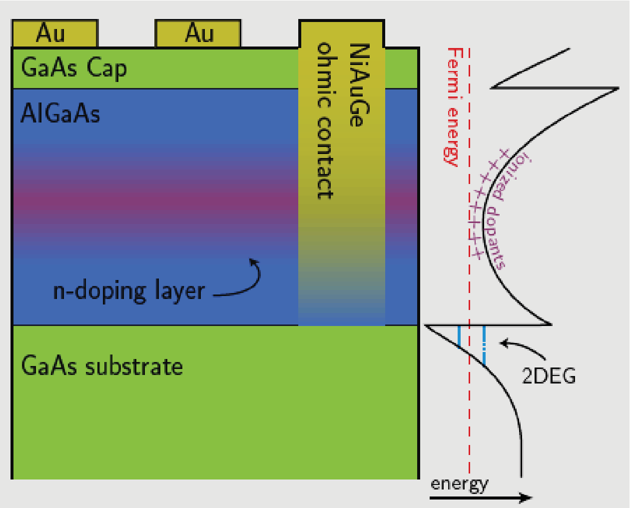

Fig. 1. Schematic layout of various layers showing formation of a 2DEG [R. Dingle et.al, Applied Physics Letters **7**, 665 (1978]

These trapped electrons have only two degrees of freedom of motion and one confinement. Hence it forms a two dimensional electron gas (2DEG), a basis for all the quantum devices. It is also called as Quantum Well.A metallic gate is deposited on top of the hetero-structure in order to control density of the 2DEG. If a positive (negative) bias voltage is applied to the gate, hetero-interface region underlying it will be populated (depleted) with the 2DEG, respectively. A new degree of control over 2DEG can be obtained by patterning the

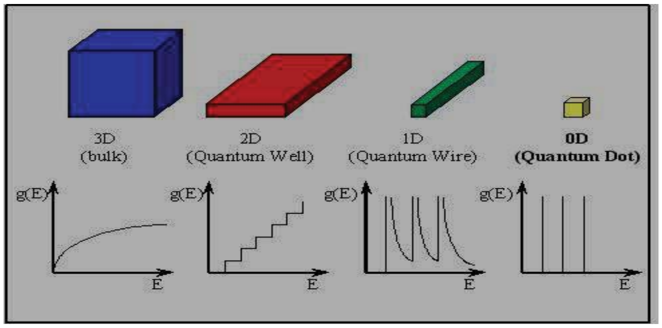

Fig. 2. Schematic representation of different low dimensional quantum nanostructures with corresponding density of states[Ph. D. thesis – Shyam Chand, H.P. University Shimla]

planar gate lithographically into a split gate, which only covers the hetero-structure partially. By changing the lithography pattern of the split gate, we can thereby reduce the 2DEG to 1D or 0D [16]. The confinement effect changes the density of states of system considerably and same is shown in Fig. 2.

III. QUANTUM DOTS

Quantum dots are nano-structures in which some electrons are trapped such that their motion is confined to zero dimensions. A QD is known by many names such as zero-dimensional electron gas, quantum box, artificial atom, Coulomb island etc. The confined electrons on QD have discrete energy spectrum and show shell filling effect analogous to a real atom. Hence, QDs are also named as artificial atoms [18]. But this analogy cannot be carried too far because of some differences between the real atom and a QD. First difference being the size itself, as quantum dot contains 103−106 atoms; the second is the confining potential which in real atom is a nearly spherically symmetric Coulomb potential of positively charged nucleus, whereas in the case of QD it is the potential set across gate terminals. The size and nature of confining potential in QD can be varied over wide ranges, which is not possible at all in case of real atoms. The properties of QDs, such as shape, size, number of confined electrons and energy levels, strongly depend on the fabrication conditions suchas the amount and composition of deposited materials, temperature, pressure, the rate of growth and can be further controlled by applying electric and magnetic fields externally [5].

On the other hand, QDs have proved to be flexible structures for various device applications. These are interesting tools by which quantum behavior can be probed on a scale 10 to 100 times larger than atomic scale. Consequently, physics of devices is somewhat closer to the classical physics than to atomic physics. But they are still sufficiently small enough to clearly exhibit quantum phenomena. The QDs have triggered a lot of theoretical and experimental research work in almost all fields including optics, superconductivity, transport, electronics, opto-electronics, quantum computers, biology etc.

A quantum dot or coupled quantum dots can be connected to external leads (which can be normal or superconducting) via tunnel barriers and setting a small bias leads to the tunneling of electrons from the left lead to QD system and from QD system to the right lead resulting into a current flow in the device (Fig. 3). The classical Ohms law and other conventional transport equation cannot be applied to study the transport through QDs as the electron transport through the system is dominated by quantum mechanical effect [19]. It requires special theoretical tools like Non-equilibrium Green function formalism [20] and has revealed some novel phenomena like resonant tunneling, Coulomb blockade, Fano effect, Kondo effect etc [21, 22, 23, 24, 25]. Further, calculation of transmission probability *T(ω)* curves show that the electrons on the QD have discrete levels and transport occurs through each level near Fermi energy. Interestingly, current and conductance have quantized maximum limits [26, 27, 28]. Calculations of these quantities are done using non-equilibrium Green function approach which results into an expression for current as:

$$I = \frac{e}{h}\sum_{\sigma}\int (f_L - f_R)\,T(\omega)\,d\omega \qquad (1)$$

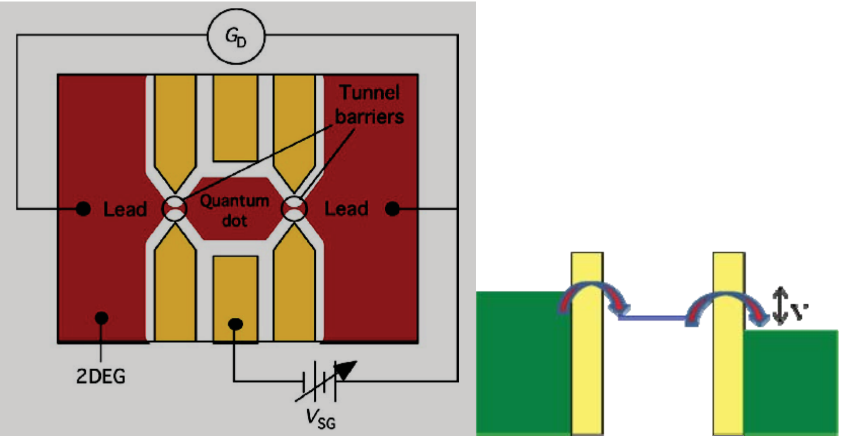

Fig. 3.Diagrams showing the coupling of QD to leads and electron transport through tunnel barriers.

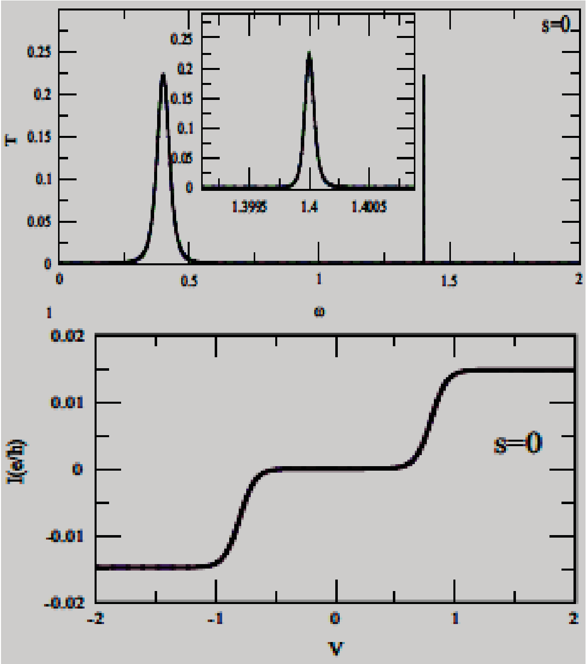

Fig. 4. Variation of transmission probability *T* with energy *ω* and *I-V* characteristics of a double QD system

Where, $T(\omega) = T_r(G^a\Gamma^R G^r\Gamma^L)$ is transmission probability and f_L and f_R being the Fermi distribution function of left and right lead respectively decided by net bias voltage. The quantities $G^{a(r)}$ are advanced (retarded) Greens function and $\Gamma^{L(R)}$ are the left (right) lead- dot coupling strengths. The transmission probability and I-V characteristics for a typical quantum dot system is shown in Fig. 4.

IV. QUANTUM COMPUTING

Quantum computation and quantum information is the study of tasks of information processing, accomplished using the laws of quantum mechanics [29]. Fundamental idea to quantum computation and quantum information involves the deep rooted knowledge of the fields like quantum mechanics with vector algebra, computer science, information theory and cryptography. In the early 1980's,Richard Feynman observed that certain quantum mechanical effects cannot be simulated efficiently on a classical computer and he speculated that computation could be done perhaps more efficiently if it made use of these quantum effects. But, building quantum computers and computational machines that use such quantum effects, proved tricky, and as no one was sure how to use the quantum effects to speed up computation. The field reached more digital ground when David Deutsch [30]proposed the universal quantum Turing machine in 1985, and later followed by the development of the first algorithms by Deutsch, Jozsa and by Simon. In the year 1994, Peter Shor [31] surprised the world by introducing a polynomial time quantum algorithm. Thus, the field of quantum computing came into existence at least in abstract theories. This discovery prompted a flurry of research activities, both among experimentalists trying to build components of quantum computers and theoreticians looking to find similar other quantum algorithms.

In spite of a highly challenging task, following are the different motivations which encourage researchers to study quantum computers:

1. The miniaturization of devices in ICs has already reached to a limit where quantum effects have become significant.
2. Making use of quantum effects allows usto speed-up certain computation tasks and processes enormously and enables some things that are otherwise impossible for classical computers.
3. Finally, the main goal of theoretical computer science is to *study the power and limitations of the strongest-possible computational devices that nature allows us*. Since our present understanding and visualization of nature is quantum mechanical, therefore theoretical computer science should explore the power of quantum computers, not that of classical ones.

Quantum computers and classical computers share many common features. In classical computers, information is encoded in binary state (0 or 1) and processed by various logic gates. In quantum computers, information is represented by state of the microscopic quantum systems called qubits and manipulated by various quantum gates. A qubit in quantum computers can be a two-level atom in the excited or ground state, a photon with horizontal or vertical polarization or a spin-½particles with up or down spins. The distinction between quantum and classical computer originate from the special characteristic of quantum mechanics. Due to a purely quantum mechanical phenomena called superposition, a quantum system can exist in different state at the same time. Thus, superposition enables quantum computers to process data in parallel and hence quantum computers solve certain problems faster than classical computers [32].

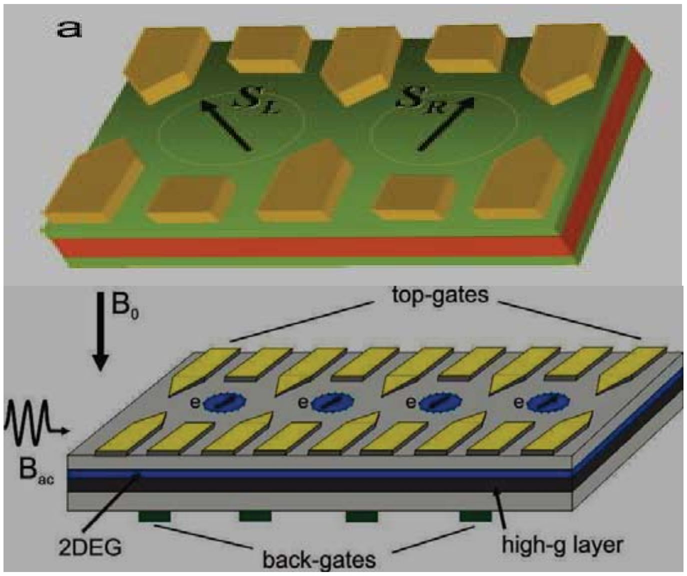

Fig. 5. Schematic layout of QDs with electron spins[33]

V. QUANTUM COMPUTING WITH QDs

Today's computers work by representing information as a series of 1’s and 0’s, or binary digits called "bits." This code is relayed with the help of transistors, which act as minute switches that can either be in an*on* or in*off* state, representing a 1 or a 0, respectively. Quantum computers would take advantage of an interesting phenomenon described by quantum theory called superposition, because of which objects, such as atoms or electrons, can be in two places at the same time, or they can exist in two states at the same time. That means computers based on quantum physics would have quantum bits also called qubits, that simultaneously exist in both the *on* and *off* states, making it possible for them to process information much more faster than classical computers. Since the string of quantum bits can calculate every possible *on-off* combination simultaneously, hence it would be able to increase the computer's power and memory dramatically [33].

Further, switches in a typical quantum computer would be made of QDs, which are formed in the internal circuits of computer. Each QD in quantum computer is like a switch that defines a single qubit, analogous to each transistor in a conventional computer. For quantum computations to work, information will have to be exchanged between pairs of qubits. As electrons on a QDs possess a *spin* which is either*up* or *down*; therefore, direction of *spin* can be utilised to define a bit state instead of the *on* or *off* positions of a conventional computer circuit. Each QD can have a 1or a 0, because the spin can be *up* or *down*.

Now a day, it has become possible to couple two QDs to form a double QD system andto control the precise number of electrons on each QD and then detect the spin state in a double QD. In contrast to conventional computer circuitry in which information is carried and processed by electric current, QD based quantum computers would rely on the manipulation of the different electron spin states.Recent research shows that quantum information can be encoded in the spin states of single-electron QD [34]. The tunnel barrier between neighbouring QDs, which is controllable through external gates, inducesa time-dependent electron-electron interaction and via Heisenberg exchange coupling,affects the spin states. Research shows that such a setup enables us to realize a universal and scalable quantum computing, externally controllable purely through electrical means[35].

A single electron on a QD has two spin states $|\uparrow\rangle$and$|\downarrow\rangle$. Mainly two different approaches for encoding the qubit have emerged. The first one uses spin eigen states $|\uparrow\rangle$ and $|\downarrow\rangle$ of single electron; while, the second one uses the singlet$|S\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$and triplet $|T_0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$ states of two electron spins, which forms$S$-T_0qubit

With the help of coupling of QD system to external and internal magnetic fields,two eigenstates$|\uparrow\rangle$and $|\downarrow\rangle$of a single electron in a QD can be split by an effective Zeemanenergy. The resulting energy difference gives rise to coherent single-qubit rotations $\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle) \leftrightarrow \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle)$. Consequently, two-qubit gates like$\sqrt{SWAP}$ and *SWAP* can be implemented by controllingthe overlap of the wave functions and hence the exchange energy for neighboring electrons. The elementary unit of an all-electrical *spin-qubit processor* has now been implemented successfully, which independently demonstrates controllable single-spin rotations in combination with inter-dot exchange of spin in a Double QD.

In the S-T_0 approach, eachqubit is formed by two electrons in two adjacent QDs, where the charge configuration and electron wave functions overlap depend on the shape of the confining potential, which in turn is determined by the applied gate voltages. Within the scope of this work,it is sufficient to distinguish the two charge configurations (1,1) and (0,2), whichdenotes that the electrons are found either in different dots or both are residing over the *right* QD, respectively. Coherent rotations$|S\rangle \leftrightarrow |T_0\rangle$are inducted by a magnetic field gradient applied between the two QDs. When the barrier is reduced till the wavefunctions strongly overlap with each other;then, the finite exchange energy resultsinto the coherent rotations$|\uparrow\downarrow\rangle \leftrightarrow |\downarrow\uparrow\rangle$ i.e.$\frac{1}{\sqrt{2}}(|T_0\rangle + |S\rangle) \leftrightarrow \frac{1}{\sqrt{2}}(|T_0\rangle - |S\rangle)$, thus allowing arbitrary rotations on the Bloch sphere. These single-qubit operations are carriedout within a few *ns* only and two-qubit gates may be implemented by capacitive coupling, where the chargeconfiguration in one double QD affects the exchange energy and thereby the precession frequency of other double QD.

However, large-scale quantum computers must be capable of reachinga system size of several thousands of qubits. This raisesaserious issue of architectural challenges to the implementation of exchange-based QD scheme of computingsince the large amount of wires and metallic gates for external control needs to be installed on a very smallscale. A promising strategy to meet this challenge has recently been proposed [36] wherein a long-distance spin-spin coupling canbe achieved via floating gates, which allows usto move the QDs far apart. The floating gates may be installed on top of the sampleor can even be located within the2DEG. The qubit-qubit coupling can be switched on and off by altering the relative positions of QDs (charges) with respect tothe gates which enablesus to have all-electrical controls. A key feature of this architecture is that it consists of a two-dimensionallattice of spin qubits with nearest neighbor inter-qubit interactions.

VI. CONCLUSIONS

The results based on the calculations done in the present work show that QDs contain few electrons confined to zero dimensions with discrete energy spectrum, depicted from the transmission probability and transport through QDs [28] occurs via resonant tunneling with step-like I-V characteristics. A variety of quantum dot systems is currently under investigation and the scalability of exchange-based schemes for quantum computing has now been proven in experiments also. The QDs are now precisely controlled and in this review work it has been observed that single-qubit rotations around different axes, two-qubit operations, various initialization and readout schemes have successfully been implemented during the past few years. Withthe proposed floating gates technique,long-distance spin-spin coupling can be exploited to overcome architectural challenges of a large-scale quantum computer. Recent development in quantum computing with QDs has been very positive and one can be curious about the progress of the next few years and anticipate that a *complete spin-qubit processor* would see the light of the day soon.

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