

Concepts of Quadratic Assignment Problem Relevant to Wireless Sensor Networks

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Abstract - The issues in Wireless Sensor Networks (WSNs) always bring the attention of research community. The fundamental issues of connectivity, energy consumption, routing, network lifetime is worth to study. Quadratic Assignment problem (QAP) plays an important role to study these fundamental issues. This paper highlights the concepts of QAP that are employed to address these fundamental issues. This paper also portraits the importance of WSN and its applications in various fields.

Keywords: Quadratic Assignment problem, NP-complete, formulation, Wireless Sensor Networks and energy efficiency.

I. INTRODUCTION

WSN is widely considered as one of the most important technologies of the 21st century. WSNs serve as the most attractive field for many researchers. But it consists of many micro devices called sensor nodes which are powered by batteries. They are widely employed in many applications such as environmental monitoring, earthquake detection etc where replacement of batteries is not practical. Therefore they should be managed carefully to minimize the consumption of energy. In any case, energy is a very significant resource and must be used sparingly. Therefore the main issues in wsn is how to prolong the network lifetime of wsn with a certain energy source and how to maintain coverage and connectivity. Optimizing the energy consumption in wsn has recently become an important concern. Hence energy management is a key issue in wsn. And this can be accomplished using Quadratic Assignment problem

Sensor node is a device that includes three basic components. a) sensing subsystem, b) processing subsystem c) wireless communication system. One of the fundamental goals for wsn is to collect information from the physical world. Comparing to existing infrastructure based networks, wsn can virtually work in any environment especially those where wired connections are now possible. WSNs are often deployed to sense, process and disseminate information of targeted physical environment.

Energy consumption is one of the biggest constraints of wireless sensor node and this limitation combined with a typical deployment of large number of nodes has added many challenges to the design and management of wsn.

One of the biggest problems of sensor networks is power consumption which is greatly affected by communication between nodes. In general the sensed data from multiple sensors are transmitted to the Base station. The base stations are one or more distinguished components of the WSN with much more computational energy and communication resources. They act as a gateway between sensor nodes and the end user as they typically forward data from the wsn onto a server.

Data generated from neighboring sensor is often redundant and highly correlated. Sensor nodes are energy constrained. It is inefficient for all the sensors to transmit the data directly to the Base station. Hence we need methods for combining data into high quality information and this can be accomplished by data aggregation.

The strength of WSNs lies in their flexibility and scalability. Wireless sensor nodes communicate directly with other nodes in their vicinity and reach far away nodes through multihop communication. This allows new sensor nodes to be easily added to a wsn to expand the area covered by the network while wsn's promise a wide range of applications, they also present a new collection of challenges.

Clustering provides an efficient method for utilizing the energy efficiently and balancing the energy consumption among the sensor nodes in the networks. Existing clustering algorithms select the cluster head with a high residual energy and rotate the cluster heads at regular intervals to distribute the energy consumption among the nodes. The responsibility of cluster head is to gather the data from the cluster members and then forwarding them to the base station.

II. THE QUADRATIC ASSIGNMENT PROBLEM

The Quadratic Assignment Problem (QAP) was originally introduced in 1957 by Tjalling C. Koopmans and Martin Beckman who were trying to model a facilities location problem. Since then, it has been among the most studied problems in all of combinatorial optimization. Many scientists including mathematicians, computer scientists, operations research analysts, and economists have used the QAP to model a variety of optimization problems. Sahni and Gonzalez (1976) showed that the QAP is *NP*-hard and that there is no ϵ -approximation polynomial algorithm for the QAP unless $P = NP$. This paper will present a general overview of the QAP.

As previously mentioned, the focus of this paper is the more complicated generalization of the Linear Assignment Problem, known as the Quadratic Assignment Problem. In addition to a cost matrix, as in the LAP, there is a so-called distance matrix involved.

In 1957, the Quadratic Assignment Problem (QAP) was introduced by Koopmans and Beckmann as a mathematical model for the location of a set of indivisible economic activities. As the QAP is a generalization of the Traveling Salesman Problem (TSP), it is also an NP complete problem (Sahni and Gonzales, 1976). In its basic interpretation, the problem seeks to assign n facilities to n locations with the cost being proportional to the flow between the facilities multiplied with their distances. The objective is to allocate each facility to a location such that the total cost is minimized. In general, QAP can be described as a one-to-one assignment problem of n facilities to n locations, which minimizes the sum of the total quadratic interaction cost, the flow between the facilities multiplied with their distances, and the total linear cost associated with allocating a facility to a certain location.

Over the years, the QAP has drawn the researcher's attention worldwide and extensive research has been done. From the theoretical point of view, it is because of the high computational complexity: QAP is NP-hard, and even finding an approximate solution is a hard problem. Moreover, many well-known classical combinatorial optimization problems such as the traveling salesman problem, the graph partitioning problem, the maximum clique problem can be reformulated as special cases of the QAP. From the practical point of view, it is because of the diversified applications of the QAP. The techniques which can be used to find the optimal solution are limited to branch and bound and cutting planes methods: with current hardware, problems of order greater than 20 cannot be solved in an acceptable time (Burkard et al., 1994). For this reason, in recent years many heuristic algorithms have been proposed which, though not ensuring that the solution found is the best one, give good results in an acceptable computation time (Maniezzo et al., 1994).

III . MATHEMATICAL FORMULATION OF QAP

Consider the set $N = \{1,2,3,\dots,n\}$ and three $n \times n$ matrices $F = (f_{ik})$, $D = (d_{jl})$ and $C = (c_{ij})$. The Quadratic Assignment problem with coefficient matrices F , D , and C shortly denoted by QAP can be stated as follows :

$$\begin{aligned} \text{Min}_{x \in X} & \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n \sum_{l=1}^n f_{ik} d_{jl} x_{ij} x_{kl} + \sum_{i=1}^n \sum_{j=1}^n c_{ij} x_{ij} \\ \text{such that} & \sum_{i=1}^n x_{ij} = 1, \quad i \in N, \quad \sum_{j=1}^n x_{ij} = 1, \quad j \in N, \quad x_{ij} \in \{0,1\}, \quad i, j \in N \end{aligned}$$

f_{ik} denotes the amount of flow between facilities i and k , d_{jl} denotes the distance between locations j and l and c_{ij} denotes the cost of locating facility i at location j .

$$x_{ij} = \begin{cases} 1, & \text{if facility } i \text{ is assigned to location } j \\ 0, & \text{otherwise} \end{cases}$$

IV. WIRELESS SENSOR NETWORKS (WSN)

A Wireless Sensor Network (WSN) is a network of thousands of small low-cost sensor nodes whose communications with a central station are conveyed by means of wireless signals. The first WSN was designed and used in 70s, in military held during the Vietnam war. WSN consist of nodes, from few to several one, which work together to capture data from an environment region and send this data to a base station. These sensor nodes use to track and monitor heat, temperature, vibratory movement, etc. They are small with limited computing resources and base on a routing algorithm, they can transmit data to the user.

WSN Architecture : The sensor network consists of the sensor fields, sensor nodes, sink and Task manager. In a typical WSN we see the following network components.

Sensor nodes (Field devices) : Each sensor network node has typically several parts, a radio transceiver with an internal antenna or connection to an external antenna, a micro controller, an electronic circuit for a) Interfacing with the sensors and an energy source usually a battery or an embedded form of energy harvesting. b) Gateway or Access points – A gateway enables communication between host application and field devices. c) Network manager – A network manager is responsible for configuration of the network, scheduling communication between devices, management of the routing tables and monitoring and reporting the health of the network. d) Security manager – The security manager is responsible for the generation, storage and management of keys.

Sensor nodes in WSN mainly use a broadcast communication paradigm where the sensor signals are used in further analysis of the sensed environment. WSN is preferred as the sensor system architecture with regard to its inherent redundancy but is susceptible to disadvantages caused by limited operation life-time.

V. APPLICATIONS OF QAP AND WSN

This model finds applications in parallel and distributed computing, combinatorial data analysis, facility location, computer manufacturing, scheduling, building layout design and process communications. Other optimization combinatorial problems such as travelling salesman problem, maximal clique isomorphism and graph partitioning

can be formulated as a QAP. Additional applications of QAP include (i) the allocation of plants to candidate locations; (ii) layout of plants; (iii) backboard wiring problem; (iv) design of control panels and typewriter keyboards; (v) balancing.

Wireless Sensor Networks (WSN s) are large-scale, dynamic and limited in Power. These WSN s can be used for various applications such as military, environmental, health, home and other commercial applications. With the high degree of deployment flexibility, applications of WSN are vast and can be broadly classified into the monitoring and tracking categories.

Monitoring applications include environmental monitoring such as forest fire detection, bio complexity mapping of the environment, flood detection, precision agriculture, health monitoring contains rele-monitoring of human physiological data, monitoring doctors and patients conditions and drug administration in hospitals.

Tracking applications include objects, animals, humans, vehicles and military enemy tracking. For example, in a battle field, a commander can be aware of the status of Friendly troops or the availability of equipment by sing the sensor networks. Another application is forest fire early detection system. Smoke or temperature sensors can be Deployed into a fire-susceptible forest area to detect a forest fire on its early stage. Lastly the technology of WSN also can be used in health applications. The physiological data are collected by wireless sensors are stored for a long period and used for medical exploration. Hence in the future this wide range of applications will make sensor networks an integral part of our lives.

VI. EXPERIMENTAL RESULTS

To know the effectiveness of each surveyed protocol it is compared with the various performance metrics like packet delivery ratio, end to end delay, shortest path, routing overhead and lifetime.

Packet delivery ratio : Packet delivery ratio is the ratio of the number of delivered data packet to the destination. The greater value of packet delivery ratio means the better performance of the protocol.

The Packet delivery ratio can be calculated as follows :

$$\text{Packet delivery ratio} = \frac{\text{Total number of packets received}}{\text{Total number of packets sent}}$$

End to End delay : End to End delay is the average time taken by a data packet to arrive in the

Destination. It also includes the delay caused by route discovery process and the queue in data packet transmission. It can be calculated as follows :

$$\text{End to End delay} = \frac{\text{Total Arrival time} - \text{Total sent time}}{\text{Total number of connections}}$$

Throughput : Throughput is the average data rate of successful data or message delivery over a specific communications link. Network throughput is measured in bits per second(bps). Throughput can be calculated as follows :

$$\text{Throughput} = \frac{\text{File size (sec)}}{\text{Transmission time (bps)}}$$

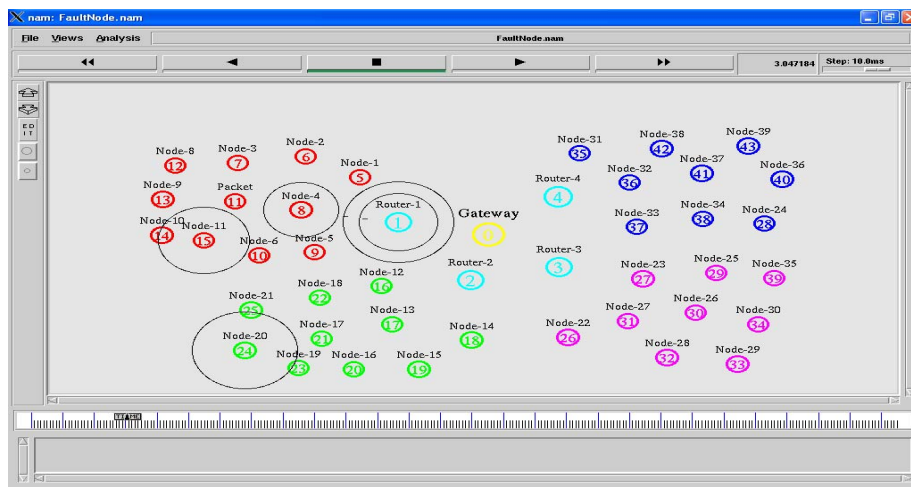
Life time : Life time is the time (number of rounds) of network disconnection due to the failure of one or more sensor nodes. Life time can be calculated as follows :

$$\text{Life time} = \frac{\text{File size (sec)}}{\text{Band width (sec)}}$$

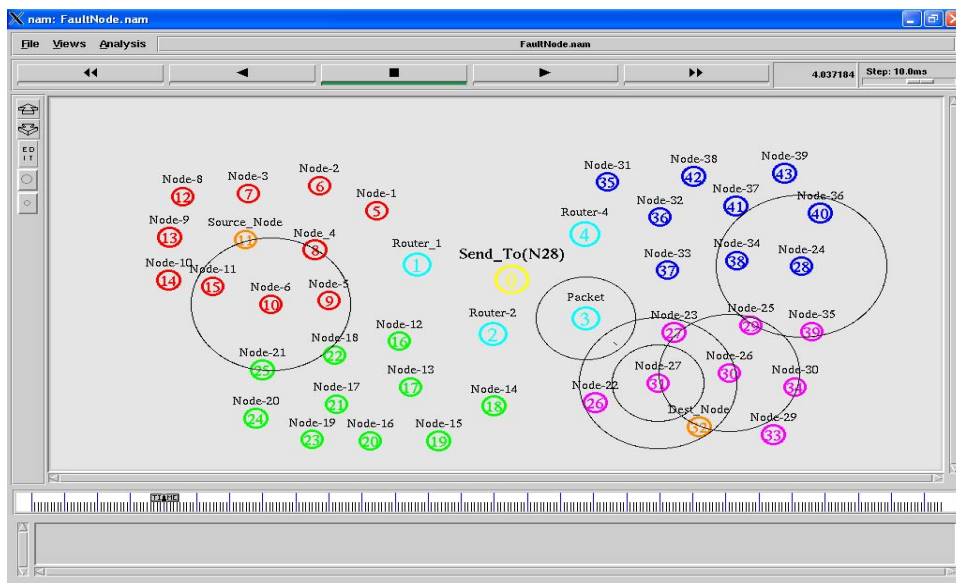
Routing overhead : Routing overhead is the nodes that often change their location within network. So some stale routes are generated in the routing table which leads to unnecessary routing overhead.

Transmission energy : Transmission energy refers to energy taken for transmitting the data from source to destination.

VII. NODES DEPLOYMENT



Here the monitoring gateway sends the data to the sub routers and each routers are connected to the group of sensor nodes. So, once router connection established from all sensors then the data transmission can be occurred.



Here the Source Node sends the packets to the Router 1 and the router connected to the Direct Gateway connects the neighbor router to the destination node via router 3 which is available nearby to reach the data.

VIII. CONCLUSION

In this paper we have presented the importance of QAP to address the fundamental issues of WSN. As a future scope of work we will further apply the NP-hard Quadratic Assignment problem to study the topology control issues of WSN. Data latency and accuracy are important in many applications such as environment monitoring, where the freshness of data is also an important factor. It is critical to develop energy efficient and fast data aggregation algorithm. Since nodes are deployed in the open environment there may be a chance for many attacks. So we need the techniques to provide data aggregation in a secured manner.

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