An Optimized Full State Feedback Controller for Ball and Beam System

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Abstract- The Ball and Beam system (BBS) is a nonlinear and unstable system which resembles with many real-time complicated systems, such as aircraft take off, aircraft turning in the space and robot walking with load in one arm. The simple structure of BBS offers easy modeling and testing different intelligent control techniques. Providing an appropriate beam angle to give the stability to the ball on the beam in a specific position with certain specifications, is a challenging task for the control system researchers. The performance of the system is greatly dependent on the accuracy of the control action. In this paper an optimized full state feedback controller has been designed with the help of Linear Quadratic Regulator (LQR) theory and Genetic Algorithm (GA) as an optimization method. LQR with GA will find the optimum feedback gain vectors which are associated with the all states of the ball and beam system. The stochastic operators (selection, crossover and mutation) will force the candidate solution in the n-dimensional search space to an optimal solution in the transition from one generation to the next generation. Global minima of the fitness function are confirmed by the elite count operator. Genetic Algorithm is coded for three types of fitness function, such as Integral Time and Absolute Error (ITAE), Integral Square Error (ISE) and Integral Absolute Error (IAE). The optimum performance, in terms of transient and steady state response is obtained by the ITAE fitness function. The simulation work is carried out in simulink environment of MATLAB (7.8.0) software. The simulation results are also validated in the real-time implementation of the ball and beam system, designed by Googol Technology.

Keywords – Ball and Beam system, Full state feedback control, Linear Quadratic Regulator, Genetic Algorithm, Selection, Crossover, Mutation, Integral Time and Absolute Error, Integral Square Error; Integral Absolute Error.

I. INTRODUCTION

The BBS is one of the most enduringly popular and important laboratory models for testing different control techniques, as its open loop operation shows instability. The mechanical plant [1] in Fig. 1 consists of a base, a beam, a ball, a lever arm, a gear box, a support block, a motor and an embedded electrical power supply. The ball can roll freely along the whole length of the beam. The beam is connected to the fixed support block at one end and to the movable lever arm at other end. The motion of the lever arm is controlled by the DC brush motor through the gear. The motor has built-in rotary optical incremental encoder that provides feedback information about current actual position of the motor shaft. There is a linear potentiometer sensor that senses current linear actual position of the ball on the beam. This measured position along with other states is fed back with appropriate gain, to the comparator to generate the desired control action.

The main control job is to automatically regulate the position of the ball on the beam by changing the tilt angle of the beam. This is difficult control task because the ball does not stay in one place on the beam but moves along the beam with an acceleration that is proportional to the tilt angle of the beam.

There are many research work has been done to control the ball in desired position on the beam. Some conventional techniques have been developed for BBS, such as Proportional plus Derivative (PD) controller [2]. There are number of advanced control techniques have been designed , such as state observer with state feedback [3], linear quadratic regulator [4], robust stabilization using time scaling and Lyapunov redesign [5], sliding mode controller [6], fuzzy controller [7], variable universe fuzzy controller [8], single input fuzzy logic controller[9], single input interval type-2 fuzzy logic controller [10].

Full state feedback (FSF), or pole placement, is a method employed in feedback control system theory to place the closed-loop poles of a plant in pre-determined locations in the left half of s-plane. Placing poles is desirable because the location of the poles corresponds directly to the eigen values of the system, which control the characteristics of the response of the system. Generally the LQR parameters are chosen manually to find the feedback gain vectors but it cannot give the optimal performance of the system. In this paper Genetic Algorithm is used to find the optimum parameters of LQR with respect to the Integral Time and Absolute Error (ITAE), Integral Square Error (ISE) and Integral Absolute Error (IAE) fitness functions. This optimized full state feedback controller will give the optimum performance of the ball and beam system.

II. MATHEMATICAL MODEL OF BBS

A. Physical structure of BBS

Physical structure of BBS [1] is shown in Fig.1. The beam is supported by a support block at one end and by a lever arm at the other end. The lever arm is attached with a servo gear. This servo gear can make positive and negative angle by rotation in both direction. The tilt angle of the beam is controlled by the gear. Depending on the tilt angle and gravity, ball can freely roll along the beam. The actual position of the ball is measured by a linear potentiometer sensor which is attached with the beam.

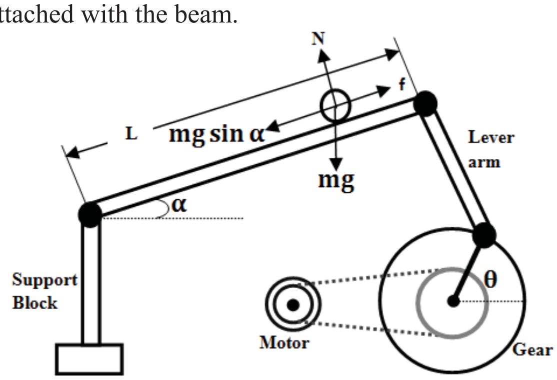

Fig. 1 Physical structure of BBS

B. Mathematical model of BBS

Let the angle between the lines that connects the joint of the lever arm with the center of the gear and the horizontal line be **θ**; the distance between the center of the gear and joint be **d** and the length of the beam be **L**. Then the beam angle **α** can be expressed in terms of the rotation angle of the gear **θ** according to the following equation (1)

$$\alpha = \frac{d}{L}\theta \tag{1}$$

The angle **θ** is connected with the rotational angle of motor shaft through the reduction gear ratio n=4.28. The ball on the beam is subjected to the gravity, inertial, centrifugal and frictional forces. The dynamic equation of the ball on the beam can be described by using Lagrange method:

$$\left(\frac{J}{R^2} + m\right)\ddot{r} + mg\sin\alpha - mr(\dot{\alpha})^2 - \mu mg\cos\alpha = 0 \tag{2}$$

$$\ddot{r} = \frac{-1}{\left(\frac{J}{R^2} + m\right)}\left(mg\sin\alpha - mr(\dot{\alpha})^2 - \mu mg\cos\alpha\right) \tag{3}$$

Where

g is the gravitational acceleration (m/s^2); m is the mass of the ball (Kg); $J = \frac{2}{5}mR^2$ is the ball moment of inertia ($Kg.m^2$); r is the position of the ball on the beam (m); R is the radius of the ball (m); and μ is friction coefficient between ball and beam.

This model can be described by equation (2) and (3). In this dissertation we have assumed that the ball rolls without slipping and friction between the beam and ball is negligible. Our main interest is to keep the angle α close to zero. We can linearize the dynamic equation (3) with respect to α in the neighborhood of zero. Then we get the linear approximation of the system.

$$\ddot{r} = \frac{-mg\sin\alpha}{(\frac{J}{R^2}+m)} \tag{4}$$

As $\alpha \approx 0, \sin\alpha \approx \alpha$; then

$$\ddot{r} = \frac{-mgd}{L(\frac{J}{R^2}+m)}\theta \tag{5}$$

The state space model of ball and beam system can be obtained from (5) and can be expressed by (6) and (7)

$$\dot{X} = AX + BU \tag{6}$$

$$Y = CX + DU \tag{7}$$

Here $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ \dfrac{-mgd}{L(\frac{J}{R^2}+m)} \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$ and D=[0]

The values of the ball and beam parameters are given in Table I.

TABLE I.
VALUE OF THE BBS PARAMETERS

Name of Parameters	Value	Name of Parameters	Value
m (kg)	0.028	d (m)	0.04
R (m)	0.01	$J(kg.m^2)$	$1.12*10^{-6}$
$g (m/s^2)$	9.8	μ	$5*10^{-4}$
L (m)	0.4		

III. DESIGN OF FULL STATE FEEDBACK CONTROLLER

Full state feedback (FSF), or pole placement, is a method employed in feedback control system theory to place the closed-loop poles of a plant in pre-determined locations in the left half of S-plane../../../../DISSERTATION/Ball and Beam/PAPERS FOR WRITING/LQR/Full_state_feedback.htm - cite_note-Sontag1998-0. Placing poles is desirable because the location of the poles corresponds directly to the eigen values of the system, which control the characteristics of the response of the system. The system must be controllable in order to implement this method. The general block diagram for full state feedback controller is given in Fig. 2

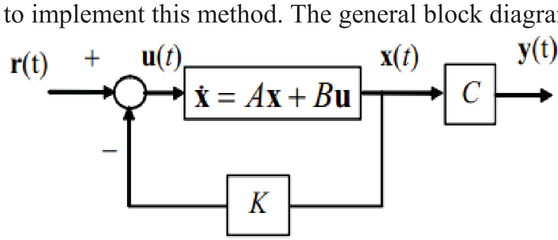

Fig. 2 Full state feedback controller

The dimension of the all variable related to the ball and beam system are given below

X – [2 x 1]; A – [2 x 2]; B – [2 x 1]; U – [1 x 1];

Y – [1 x1]; C – [1 x 2] and D – [2 x 1]

The control u(t) is a function of the all related state vector x(t). Then the state feedback law will be

$$U(t)=f(X(t),t) \tag{8}$$

The linear state feedback law will be

$$U(t)= - K.X(t) \tag{9}$$

Here K=[2 x 1] state gain matrix.
Affine the state feedback law

$$U(t) = -K.X(t) + r(t) \tag{10}$$

Here r(t) is the reference input to the plant. Now the closed loop system can be described as:

$$\dot{X} = AX + B(-KX + r) = (A - BK)X + Br \tag{11}$$

$$\dot{X} = A_{cl}X + Br \tag{12}$$

$$A_{cl} = A - BK \tag{13}$$

$$Y = CX \tag{14}$$

To get the desired performance, we have to place the poles in appropriate place in the left half of s-plane. For this requirement we have to choose the gain matrix K. For optimal performance the feedback gain matrix K must be optimum. For choosing the value of k, Linear Quadratic Regulator is used.

A. Linear Quadratic Regulator

The theory of optimal control is concerned with operating a dynamic system at minimum cost (J) (15). The case where the system dynamics are described by a set of linear differential equations and the cost is described by a quadratic functional is called the LQ problem. One of the main results in the theory is that the solution is provided by the linear-quadratic regulator (LQR) [11].

In effect, the LQR algorithm takes care of the tedious work done by the control systems engineer in optimizing the controller. However, the engineer still needs to specify the weighting factors and compare the results with the specified design goals. Often this means that controller synthesis will still be an iterative process where the engineer judges the produced "optimal" controllers through simulation and then adjusts the weighting factors to get a controller more in line with the specified design goals.

$$J = \int (X^T Q X + U^T R U)dt \tag{15}$$

Where Q and R are positive semi definite and positive definite symmetric constant matrices respectively. The gain vector K is given by (16)

$$K = R^{-1} B^T P \tag{16}$$

Where P is a positive definite symmetric constant matrix obtained from the solution of matrix algebraic Reccatti equation (ARE) (17)

$$A^T P + PA + Q + K^T RK - K^T B^T P - PKB = 0 \tag{17}$$

The main difficulty is to find the right weighting factors, limits the application of the LQR based controller synthesis. In this paper Genetic Algorithm optimization method is used to find the optimal weighting factors.

B. Genetic Algorithm

Genetic Algorithm (GA) [12] is one of the optimization algorithms, which is invented to mimic some of the processes observed in natural evolution. The Genetic Algorithm is stochastic search techniques based on the mechanism of natural selection and natural genetics. That is a general one, capable of being applied to an extremely wide range of problems. The GA, differing from conventional search techniques, start with an initial set of random solutions called population. Each individual in the population is called a chromosome, representing a solution to problem at hand. The chromosomes evolve through successive iterations, called generations. During each generation, the chromosomes are evaluated, using some measures of fitness. To create the next generation, new chromosomes, called offspring, are form by either merging two chromosomes form current generation using a crossover operator or modifying a chromosome using a mutation operator. A new generation is form by selecting, according to the fitness values, some of the parents and offspring; and rejecting others so as to keep the population size constant. Fitter chromosomes have higher probabilities of being selected. After several generations, the algorithms converge to the best chromosome, which hopefully represents the optimum solution to the problem. The working principles of genetic operators [13] are given below.

Reproduction: Reproduction is usually the first operator applied on population. From the population, the chromosomes are selected to be parents to crossover and produce offspring. According to Darwin's evolution theory "survival of the fittest" – the best ones should survive and create new offspring.

Elite count: The number of individuals with the best fitness values in the current generation that are guaranteed to survive to the next generation. These individuals are called *elite children.* The default value of Elite count is 2.

Crossover: Crossover is a genetic operator that combines (mates) two chromosomes (parents) to produce a new chromosome (offspring). The idea behind crossover is that the new chromosome may be better than both of the parents if it takes the best characteristics from each of the parents. Crossover occurs during evolution according to a user-definable crossover probability. Crossover selects genes from parent chromosomes and creates a new offspring.

Mutation: After a crossover mutation takes place. Mutation is a genetic operator used to maintain genetic diversity from one generation of a population of chromosomes to the next. Mutation occurs during evolution according to a user-definable mutation probability, usually set to fairly low value, say 0.01 a good first choice. Mutation alters one or more gene values in a chromosome from its initial state. Mutation is an important part of the genetic search, helps to prevent the population from stagnating at any local optima. Mutation is intended to prevent the search falling into a local optimum of the state space.

IV. Design And Implementation of Full State Feedback Controller For BBS

We have to choose the value of matrix Q and R. Depending the value of A,B,C,D,Q and R we can calculate the value of feedback gain vector K by using equation (16) and (17). Here Q is standard Hermitian matrix and can be defined as:

$Q = \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix}$ and R can be defined as $R = r_1$

For optimum performance of the Ball and Beam system, we have to choose the optimum value of q_1, q_2 and r_1. Genetic Algorithm is used to find the optimal value of q_1, q_2 and r_1. The block diagram for optimized full state feedback control of Ball and Beam system is given in Fig. 3

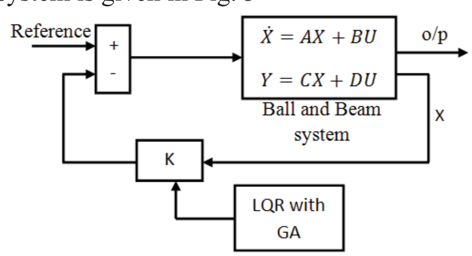

Fig. 3 optimized full state feedback control of Ball and Beam system

To apply Genetic Algorithm, we have to decide the chromosome length and chromosome element. In this case the chromosome length is 3. The elements are q_1, q_2 and r_1. We have chosen three types of fitness function ITAE, ISE and IAE.

$$ITAE = \int_0^\infty t|e(t)|dt \ , \quad ISE = \int_0^\infty [e(t)]^2 dt \ \text{ and } \ IAE = \int_0^\infty |e(t)|dt$$

The chromosome will be like

q_1	q_2	r_1

The convergence curve and ball position response for the fitness function (ITAE) with generation no.=25 and population no.=80 are given in Fig. 4 and Fig. 5 respectively.

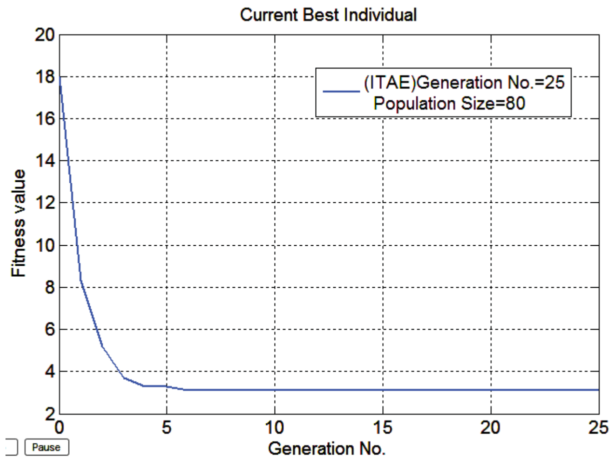

Fig. 4 fitness value (ITAE) for generation no.=25 and population no.=80

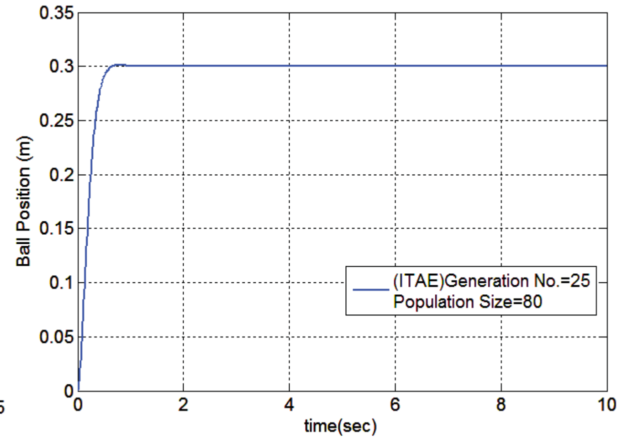

Fig. 5 Position response for ITAE with generation no.=25 and Population no.=80

The convergence curve and ball position response for the fitness function (ISE) with generation no.=25 and population no.=80 are given in Fig. 6 and Fig. 7 respectively.

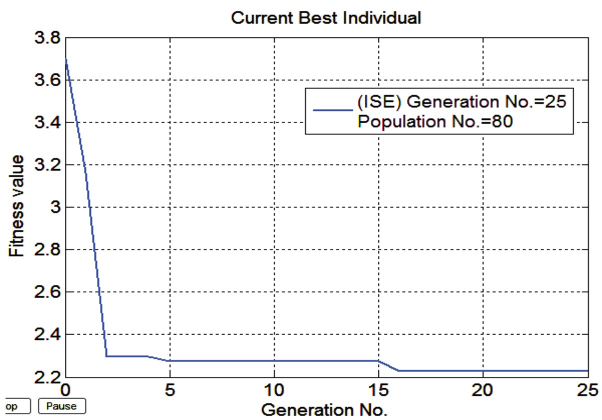

Fig. 6 fitness value (ISE) for generation no.=25 and population no.=80

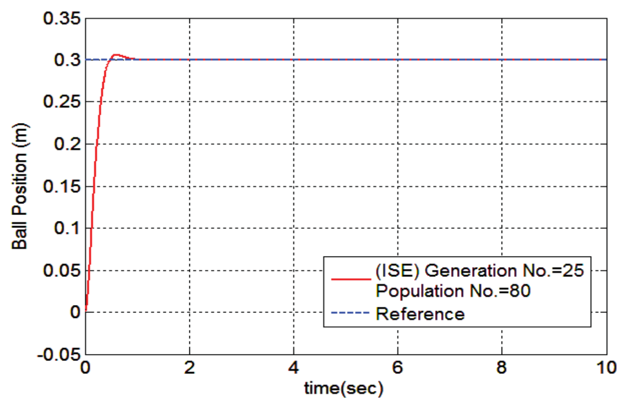

Fig. 7 Position response for ISE with generation no.=25 and Population no.=80

The convergence curve and ball position response for the fitness function (IAE) with generation no.=25 and population no.=80 are given in Fig. 8 and Fig. 9 respectively.

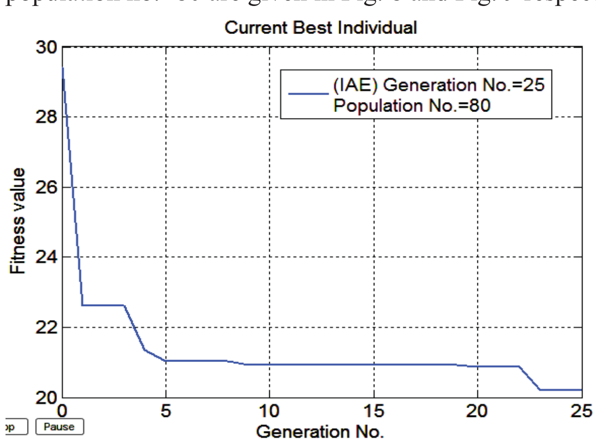

Fig. 8 fitness value (IAE) for generation no.=25 and population no.=80

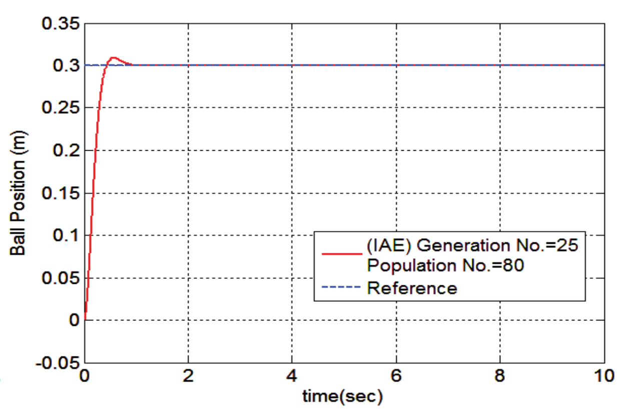

Fig. 9 Position response for IAE with generation no.=25 and Population no.=80

TABLE II. VALUES OF THE BEST CHROMOSOME AND GAIN VECTOR

Fitness Function	q_1	q_2	r_1	K_1	K_2
ITAE (Generation no.=25, Population no.=80)	98.621	1.513	0.134	99.308	20.859
ISE (Generation no.=25, Population no.=80)	97.682	0.629	0.125	98.834	18.581
IAE (Generation no.=25, Population no.=80)	98.525	0.307	0.682	99.260	17.731

TABLE III. PERFORMANCE COMPARISON BASED ON DIFFERENT FITNESS FUNCTION

Fitness Function	Rise time (sec)	Settling time (sec)	Overshoot (%)	Steady state error
ITAE	0.6	0.7	0	0
ISE	0.6	0.8	15	0
IAE	0.55	0.8	20	0

The values of the best chromosome, obtained from different fitness function and the performance comparison for three type of fitness function is given in above Table II. and Table III. From the Table III, we can say that the ball position response for fitness function ITAE is better than the responses for other fitness function. The tracking performance for the ITAE fitness function is given in Fig. 10

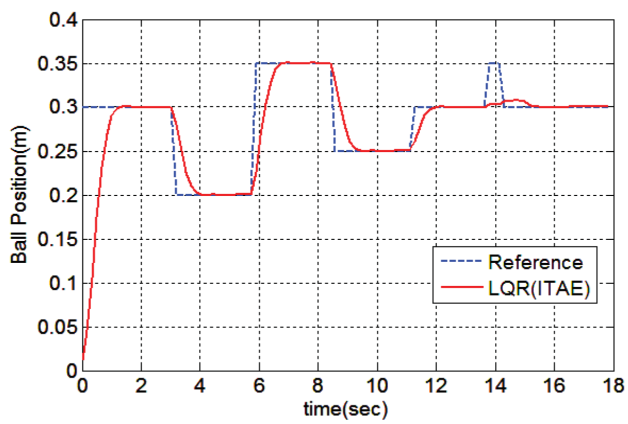

Fig. 10 Tracking response for full state feedback controller with ITAE

The obtained values are q_1=98.6210, q_2=1.5138 and r_1=0.1344So Q=diag(98.6210;1.5138) and R=0.1344. The value of the feedback gain matrix K= [99.3081 20.8595]. After using the full state feedback the poles will move to the left half of S-plane, shown in Fig. 11. Now the close loop transfer function (CLTF) will be

$$CLTF = \frac{69.93}{S^2 + 15.52S + 69.93}$$

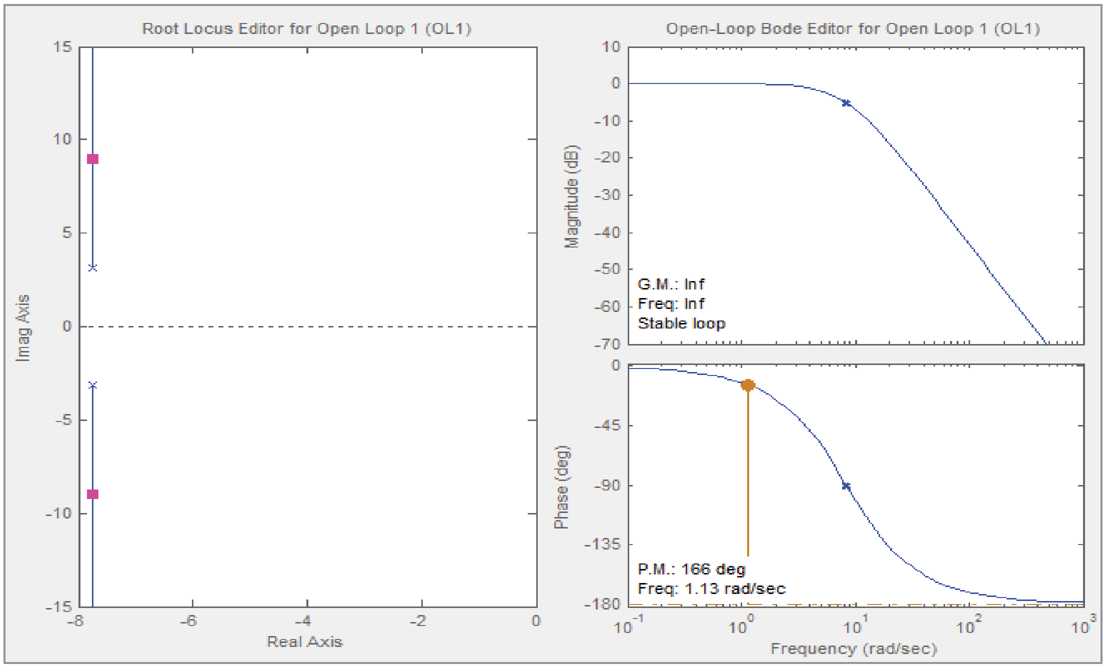

Fig. 11 The close loop pole location

The Fig. 11 shows that, for any gain value, the close loop poles will lie on the left half of the S-plane that confirms the stability of the system and Bode plot (right side of Fig. 11) will also support this. The close loop poles are located at (-7.7581±3.1205i), damping ratio=0.93 and natural frequency=8.36 rad/sec.

V. CONCLUSION

This paper has described an optimized full state feedback controller. Linear Quadratic Regulator has been used to find the feedback gain vector. For finding the optimum gain vector, Genetic Algorithm has been used to find the optimum weighting factors of the linear quadratic regulator. The performance for this optimized full state feedback controller has been analyzed for different fitness functions. Integral time and absolute error (ITAE) fitness function gives the optimal performance of the ball and beam system. The tracking performance is satisfactory in terms of transient and steady state response. The experimental results depict the optimality of the proposed controller.

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