Aberystwyth University Method for derivation of the exact π value

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Abstract - Polygon's value 3.14159265358… of Exhaustion method has been in vogue as π of the circle for the last 2000 years. An attempt is made in this paper to replace polygon's approximate value with the exact π value of circle with the help of Prof. C.R. Fletcher's geometrical construction.

Keywords: Circle, diagonal, diameter, Fletcher, π, polygon, radius, side, square

I. INTRODUCTION

The official π value is 3.14159265358… It is considered as approximate value at its last decimal place, always. It implies that there is an exact value to be found in its place. a^2, 4a, ½ab etc are the formulas of square and triangle which are derived based on their respective line-segments. Similarly, radius is a line-segment and a need is there to have a formula with radius alone and without π. The following formulas are discovered (March, 1998) from Gayatri method and Siva method.

1. Area of Circle = $r\left(\frac{7r}{2}-\frac{\sqrt{2}r}{4}\right)=\pi r^2$ and
2. Circumference of Circle = $6r+\frac{2r-\sqrt{2}r}{2}=2\pi r$; where r = radius

$$d-\left[d-\left(d-\frac{\sqrt{2}d}{2}\right)\frac{1}{2}\right]\frac{1}{4}=\frac{\pi d}{4}$$

where d = diameter = side of the superscribed square

In the Fletcher's geometrical construction there are two line-segments. They are radius and corner length. To find out the area of the shaded region in which corner length is present Professor has given $1-\frac{1}{4}\pi$.

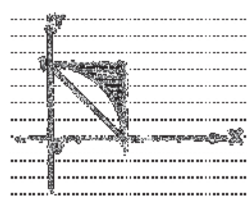

Fig-1: Professor's Diagram (by courtesy)

II. CONSTRUCTION PROCEDURE OF SIVA METHOD

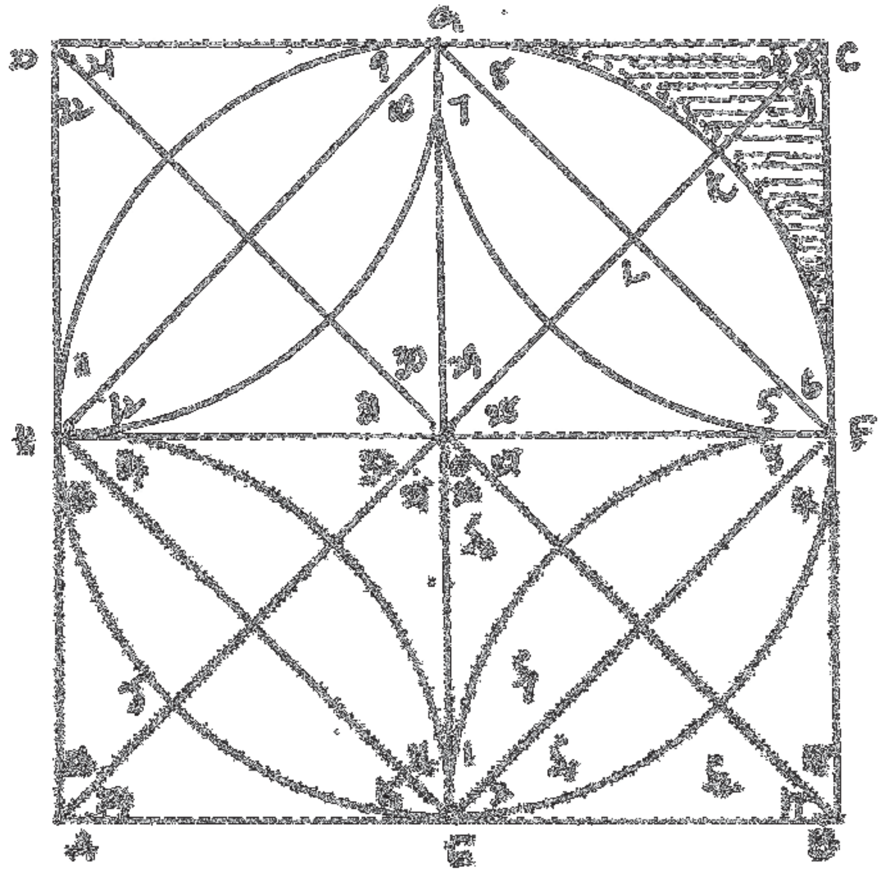

Fig-2: Siva Method

Draw a square ABCD. Draw two diagonals. ‘O’ is the centre. Inscribe a circle with centre ‘O’ and radius ½. Side of the square is 1. E, F, G and H are the midpoints of four sides. Join EG, FH, EF, FG, GH and HE. Draw four arcs with centers A, B, C and D and with radius ½. Now the circle-square composite system is divided into 32 segments of two different dimensions, called S_1 segments and S_2 segments. Number them from 1 to 32. There are 16 S_1 and 16 S_2 segments in the square and $16S_1$ and $8S_2$ segments in the circle.

Square: ABCD, AB = Side = 1, AC = Diagonal = $\sqrt{2}$; Circle : EFGH,

JK = Diameter = 1 = Side; Corner length = $\dfrac{\text{Diagonal} - \text{diameter}}{2} = \dfrac{AC - JK}{2} = \dfrac{\sqrt{2}-1}{2}$; OL = $\dfrac{\sqrt{2}}{4}$;

OK= radius = $\dfrac{1}{2}$; LK = OK – OL = $\dfrac{1}{2} - \dfrac{\sqrt{2}}{4} = \dfrac{2-\sqrt{2}}{4}$ =0.14644660942…

From the diagram of Fletcher the area of the shaded segment cannot be calculated arithmetically. The diagram of the Siva method helps in calculating the area of the shaded segment. How ?

Shaded area of Fletcher is equal to two S_2 segments 19 and 20 of Siva method.

This author, in his present study, has utilized radius/ diameter as usual, and a corner length, in addition, of the construction to find out the arithmetical value to the shaded area. A different approach is adopted here. What is that ? As a first step the shaded area is calculated using four factors. They are of Fig-2.

AC and BD, 2 diagonals $\left(2\sqrt{2}\right)$

KC corner length = $\dfrac{\text{diagonal} - \text{diameter}}{2} = \dfrac{\text{AC} - \text{JK}}{2} = \left(\dfrac{\sqrt{2}-1}{2}\right)$

Area of the square (a^2 = 1 x 1 = 1) and

32 constituent segments of the square.

Their relation are represented here in a formula and is equated to

Professor's formula $$\frac{\frac{a^2}{32}}{2\sqrt{2}\left(\frac{\sqrt{2}-1}{2}\right)} = 1 - \frac{1}{4}\pi \text{ (of Fig.1, where radius = 1)}$$

where, $1-\frac{1}{4}\pi$ has been derived with radius equal to 1, and naturally, the diameter = side of the square = 2. With this, the above formula becomes

$$\frac{\frac{4}{32}}{2\sqrt{2}\left(\frac{\sqrt{2}-1}{2}\right)} = 1 - \frac{1}{4}\pi \qquad \therefore \pi = \frac{14-\sqrt{2}}{4}$$

The accepted value for π is 3.14159265358… With this π, the area of shaded region is equal to $1-\frac{1}{4}\times 3.14159265358 = 0.21460183661...$

And, with the new π value derived above, the area of the shaded region is equal to

$$1-\frac{1}{4}\left(\frac{14-\sqrt{2}}{4}\right) = \frac{2+\sqrt{2}}{16} = 0.21338834764...$$

So, this method creates a dispute now. Which π value is right i.e. is 3.14159265358… or $\frac{14-\sqrt{2}}{4} = 3.14644660942...$?

The study of this method is extended further to decide which π value is the real π value ?

To decide which π is real, a simple verification test is followed here. What is that ? We have a line segment LK = $\frac{2-\sqrt{2}}{4} = 0.14644660942...$

LK is part of the diagonal along with the corner length KC.

So, in the Second step, an attempt is made to obtain the LK length, from the area of the shaded region. How ? Let us take the reciprocal of the area of the shaded region;

$$\frac{1}{\text{Area of the shaded region}} = \frac{1}{0.21460183661 \text{ of official } \pi} = 4.65979236616$$

and with new π

$$\frac{1}{\text{Area of the shaded region}} = \frac{1}{\frac{2+\sqrt{2}}{16}} = \frac{16}{2+\sqrt{2}} = 4.68629150101...$$

Then, this value when divided by 32, we surprisingly get KL length. It may be questioned 'why' one should divided that value. The answer is not simple. Certain aspects have to be believed, without raising questions like what, why and how at times.

Official $\pi = \frac{4.65979236616}{32} = 0.14561851144...$

New $\pi = \frac{4.68629150101}{32} = 0.1464466094...$

0.14644660942… of new π value is in total agreement with LK of Fig.2.

i.e. $\frac{2-\sqrt{2}}{4}$ =0.14644660942… and differs however with 0.14561851144… of official π from 3rd decimal onwards. If this argument is accepted, the present π value 3.14159265358… is not approximate value from its last decimal place, but it is an approximate value from the 3rd decimal.

IV. CONCLUSION

From the beginning to the end of this method, various line-segments are involved. Professor Fletcher's construction is analyzed arithmetically with the line-segments of the Siva method. This arithmetical interpretation has resulted in the derivation of a new π value, equal to $\frac{14-\sqrt{2}}{4}$. The new value is exact, algebraic number.

REFERENCES

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