Performance analysis of BPSK system with ZF & MMSE equalization

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Abstract- In mobile communications systems, data transmission at high bit rates is essential for many services such as video, high quality audio and mobile integrated service digital network. When the data is transmitted at high bit rates, over mobile radio channels, the channel impulse response can extend over many symbol periods, which lead to Inter-symbol interference (ISI). Inter-symbol Interference always causes an issue for signal recovery in wireless communication. This can be combated with application of an equalizer. Equalization compensates for Inter-symbol Interference (ISI) created by multipath signal prorogation within time dispersive channels. In this paper we investigate the bit error rate performance characteristics of two types of equalizers namely ZF, MMSE for BPSK system. Multi-tap ISI channel is considered. For the simulation the Matlab/Simulink is chosen as investigation tool. Simulation results show that as the tap length increases in ZF the BER decreases. It's also inferred that the MMSE equalizer out performs ZF equalizer in terms of BER performance characteristics.

Keywords – Binary phase shift keying (BPSK), Inter-symbol interference (ISI), Minimum mean square error (MMSE), Bit error rate (BER), Zero Forcing (ZF), Absolute white Gaussian noise channel (AWGN)

I. INTRODUCTION

A general problem found in high speed communication is inter-symbol interference. ISI occurs when a transmission interferes with itself and the receiver cannot decode the transmission correctly [1]. The all-pass assumption made in the AWGN (or non-dispersive) channel model is rarely practical. Due to the scarcity of the frequency spectrum, we usually filter the transmitted signal to limit its bandwidth so that efficient sharing of the frequency resource can be achieved. Moreover, many practical channels are band-pass and, in fact, they often respond differently to inputs with different frequency components, i.e., they are dispersive. We have to refine the simple AWGN (or non-dispersive) model to accurately represent this type of practical channels. One such commonly employed refinement is the dispersive channel mode,

$$r(t) = u(t) \otimes h_c(t) + n(t);$$

Where u(t) is the transmitted signal, hc(t) is the impulse response of the channel, and n(t) is AWGN with power spectral density N0/2. In essence, we model the dispersive characteristic of the channel by the linear filter hc(t). The simplest dispersive channel is the band limited channel for which the channel impulse response hc(t) is that of an ideal low pass filter. This low-pass filtering smears the transmitted signal in time causing the effect of a symbol to spread to adjacent symbols when a sequence of symbols is transmitted. The resulting interference, inter-symbol interference (ISI), degrades the error performance of the communication system. There are two major ways to mitigate the detrimental effect of ISI. The first method is to design band-limited transmission pulses which minimize the effect of ISI. We will describe such a design for the simple case of band limited channels. The ISI free pulses obtained are called the nyquist pulses. The second method is to filter the received signal to cancel the ISI introduced by the channel impulse response. This approach is generally known as equalization. [2]

II. CHANNEL

Additive white Gaussian Noise (AWGN) is a channel model in which the only impairment to communication is a linear addition of wideband or white noise with a constant spectral density (expressed as watts par hertz of bandwidth) and a Gaussian distribution of amplitude. The model does not account for fading, frequency, selectivity, interference, nonlinearity or dispersion. However, it produces simple and tractable mathematical models which are useful for gaining insight into the Wideband Gaussian noise comes from many natural sources, such as the thermal vibrations of atoms in conductors (referred to as thermal noise or Johnson- Nyquist noise), shot noise, black body radiation from the earth and other warm objects and from celestial sources such as the sun [6]. AWGN does not work will thus the more specified model are used. Fading is deviation of the attenuation that a carried modulated telecommunication signal experiences over certain propagation media. A fading channel is communication Rayleigh fading is caused by multipath reception really fading is statistical model for the effect of propagation environment on a radio signal such as is used by wireless devices.

2.1 Transmission Symbol

Let the transmit symbol be modeled as

x(t)= $\sum_{-\infty}^{\infty} a_n g(t-r)$

Where

T is the symbol Period,

an is the symbol to transmit,

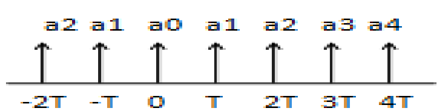
g(t) is the the transmit filter,

n is the symbol index ,

x(t) is the output waveform.

For simplicity assume that transmit pulse shaping filter is not present i.e g (t)=Δ(t).So the transmission symbol can be modeled as discrete time equivalent

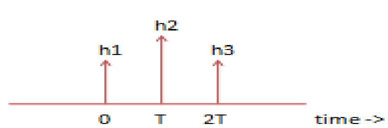
$$x[k]=a_n.$$



2.2 Channel Model:

Consider 3-tap multipath channel of spacing T i.e.

$$h\,[k]=[h_1\ h_2\ h_3]$$



In addition to multipath channel, the received signal gets corrupted by noise n, typically referred to as Absolute White Gaussian Noise (AWGN).The values of the noise follows Gaussian Probability function

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

With mean μ=0 and variance. The received signal is

$$y(k)=x(k)\otimes h(k)+n$$

Where⊗ is the convolution operator. [4]

3. EQUALIZER:

Equalizer is a digital filter that provides an approximate inverse of channel frequency response. Equalization is to mitigate the effects of ISI to decrease the probability of error that occurs without suppression of ISI, but this reduction of ISI effects has to be balanced with prevention of noise power enhancement. [3]

3.1 Adaptive equalization:

Adaptive equalizer is an equalizer that automatically adapts to time-varying properties of the communication channel. It is frequently used with coherent modulations such as phase shift keying, mitigating the effects of multipath propagation and Doppler spreading.

3.2 Blind equalization:

Equalizer minimizes the error between actual output and desired output by continuous Blind is a digital signal processing technique in which the transmitted signal is inferred from the received signal. While making use only of the transmitted signal statistics.

3.3 Zero Forcing Equalizer

Zero Forcing Equalizer is a linear equalization algorithm used in communication systems, which inverts the frequency response of the channel. This equalizer was first proposed by Robert Lucky. The Zero-Forcing Equalizer applies the inverse of the channel to the received signal, to restore the signal before the channel. The name Zero Forcing corresponds to bringing down the ISI to zero in a noise free case. This will be useful when ISI is significant compared to noise. For a channel with frequency response F(f) the zero forcing equalizer C(f) is constructed such that C(f) = 1 / F(f). Thus the combination of channel and equalizer gives a flat frequency response and linear phase F(f)C(f) = 1.If the channel response for a particular channel is H(s) then the input signal is multiplied by the reciprocal of this. This is intended to remove the effect of channel from the received signal, in particular the Inter-symbol Interference (ISI). For simplicity let us consider a 2x2 MIMO channel, the channel is modeled as,

The received signal on the first receive antenna is,

$$y_1 = h_{1,1}x_1 + h_{1,2}x_2 + n_1 = \begin{bmatrix} h_{1,1} h_{1,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1$$

The received signal on the Second receive antenna is,

$$y_2 = h_{2,1}x_1 + h_{2,2}x_2 + n_2 = \begin{bmatrix} h_{2,1} h_{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2$$

Where

y1, y2 are the received symbol on the first and second antenna respectively,

h1,1 is the channel from 1st transmit antenna to 1st receive antenna,

h1,2is the channel from 2nd transmit antenna to 1st receive antenna,

h2,1 is the channel from 1st transmit antenna to 2nd receive antenna,

h2,2 is the channel from 2nd transmit antenna to 2nd receive antenna, x1, x2 are the transmitted symbols and

n1, n2 are the noise on 1st and 2nd receive antennas.

The equation can be represented in matrix notation as follows:

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$$

Equivalently,

$$y = H.x + n$$

To solve for x, we need to find a matrix W which satisfies WH = I. The Zero Forcing (ZF) detector for meeting this constraint is given by,

W = (HHH)-1 HH

Where W - Equalization Matrix and

H - Channel Matrix

This matrix is known as the Pseudo inverse for a general m x n matrix where

$$H^H H = \begin{pmatrix} h^*_{1,1} & h^*_{2,1} \\ h^*_{1,2} & h^*_{2,2} \end{pmatrix} \begin{pmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{pmatrix} = \begin{bmatrix} |h_{1,1}|^2 + |h_{2,1}|^2 & h^*_{1,1}h_{1,2} + h^*_{2,1}h_{2,2} \\ h^*_{1,2}h_{1,1} + h^*_{2,2}h_{2,1} & |h_{1,2}|^2 + |h_{2,2}|^2 \end{bmatrix}$$

Note that the off diagonal elements in the matrix HHH are not zero, because the off diagonal elements are non zero in values. Zero forcing equalizer tries to null out the interfering terms when performing the equalization, i.e. when solving for x1 the interference from x2 is tried to be nulled and vice versa. While doing so, there can be an amplification of noise. Hence the Zero forcing equalizer is not the best possible equalizer. However, it is simple and reasonably easy to implement. [07]

For BPSK Modulation in Rayleigh fading channel, the BER is defined as

$$P_b = \frac{1}{2}\left(1 - \sqrt{\frac{(E_b / N_o)}{(E_b / N_o) + 1}}\right)$$

Where

Pb - Bit Error Rate

Eb / No - Signal to noise Ratio

3.4 MMSE Equalization

A minimum mean square error (MMSE) estimator describes the approach which minimizes the mean square error (MSE), which is a common measure of estimator quality. The main feature of MMSE equalizer is that it does not usually eliminate ISI completely but, minimizes the total power of the noise and ISI components in the output. Let x be an unknown random variable, and let y be a known random variable. An estimator x^ (y) is any function of the measurement y, and its mean square error is

given by

$MSE = E$ {(X^ -X2)},

Where the expectation is taken over both x and y.

The MMSE estimator is then defined as the estimator achieving minimal MSE. In many cases, it is not possible to determine a closed form for the MMSE estimator. In these cases, one possibility is to seek the technique minimizing the MSE within a particular class, such as the class of linear estimators. The linear MMSE estimator is the estimator achieving minimum MSE among all estimators of the form AY + b. If the measurement Y is a random vector, A is a matrix and b is a vector.

Let us now try to understand the math for extracting the two symbols which interfered with each other. In the first time slot, the received signal on the first receive antenna is,

$$y_1 = h_{1,1}\, x_1 + h_{1,2} x_2 + n_1 = [h_{1,1}\ h_{1,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1$$

The received signal on the second receive antenna is,

$$y_2 = h_{2,1}\, x_1 + h_{2,2} x_2 + n_2 = [h_{2,1}\ h_{2,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2$$

Where

y1, y2 are the received symbol on the first and second antenna respectively,

y1, y2 are the received symbol on the first and second antenna respectively,

h1,1 is the channel from 1st transmit antenna to 1st receive antenna,

h1,2 is the channel from 2nd transmit antenna to 1st receive antenna,

h2,1 is the channel from 1st transmit antenna to 2nd receive antenna,

h2,2 is the channel from 2nd transmit antenna to 2nd receive antenna,

x1, x2 are the transmitted symbols and

n1, n2 are the noise on 1st and 2nd receive antennas.

The equation can be represented in matrix notation as follows:

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$$

Equivalently,

$$y = H.x + n$$

The Minimum Mean Square Error (MMSE) approach tries to find a coefficient W which minimizes the

$$E\ \{[Wy\text{-}x]\ [Wy\text{-}x]^H\}$$

Criterion,

Where W - Equalization Matrix

H - Channel Matrix and

n - Channel noise

y- Received signal.

To solve for x, we need to find a matrix W which satisfies WH =I. The Minimum Mean Square Error (MMSE) detector for meeting this constraint is given by,

$$W = [H^H H + NoI)^{-1} H^H$$

This matrix is known as the pseudo inverse for a general m x n matrix

Where

$$H^H H = \begin{pmatrix} h^*_{1,1} & h^*_{2,1} \\ h^*_{1,2} & h^*_{2,2} \end{pmatrix} \begin{pmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{pmatrix} = \begin{bmatrix} |h_{1,1}|^2 + |h_{2,1}|^2 & h^*_{1,1}h_{1,2} + h^*_{2,1}h_{2,2} \\ h^*_{1,2}h_{1,1} + h^*_{2,2}h_{2,1} & |h_{1,2}|^2 + |h_{2,2}|^2 \end{bmatrix}$$

When comparing the eq. (3.6) to the eq. (1.5) in Zero Forcing equalizer, apart from NoI the term both the equations are comparable. In fact, when the noise term is zero, the MMSE equalizer reduces to Zero Forcing equalizer. [08]

II. PROPOSED ALGORITHM

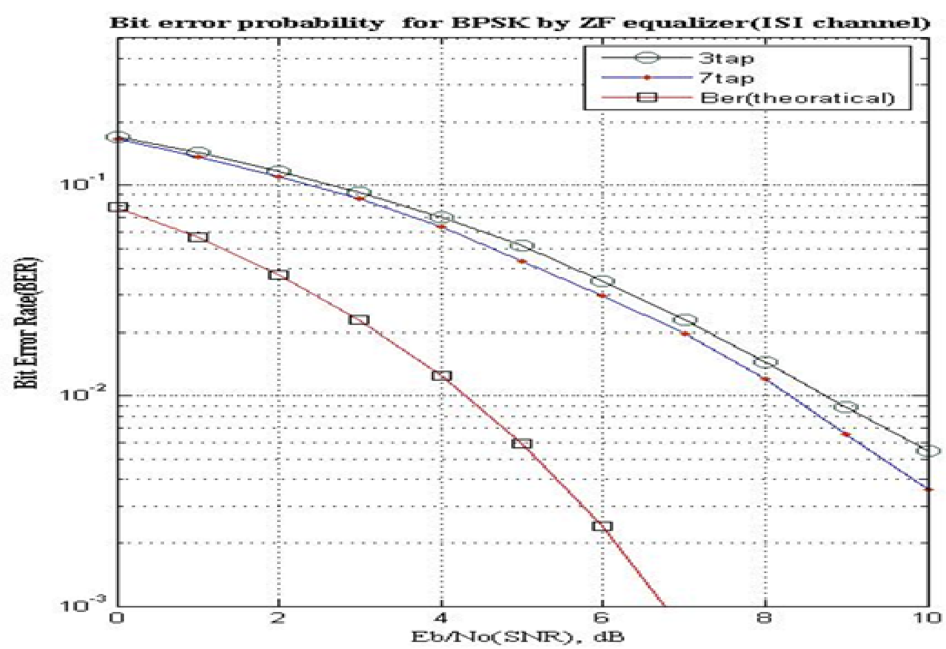
Random digital sequence in binary form of 1's and 0's is generated and BPSK modulation is performed on digital data. Now convolving the symbols with multipath channel. Consider this channel is affected by AWGN noise. Now two different programs are prepared.

- First zero frequency equalization technique is applied for varying tap length and ber is calculated. Then comparison is made with theoretical ber also. Then demodulated the received data and count no. of bit errors for repeating values of Eb/No.
- We performed mmse equalization technique for different tap-lengths and calculated ber .and comparison of ber of mmse, zf & theoretical value is done.

IV. EXPERIMENT AND RESULT

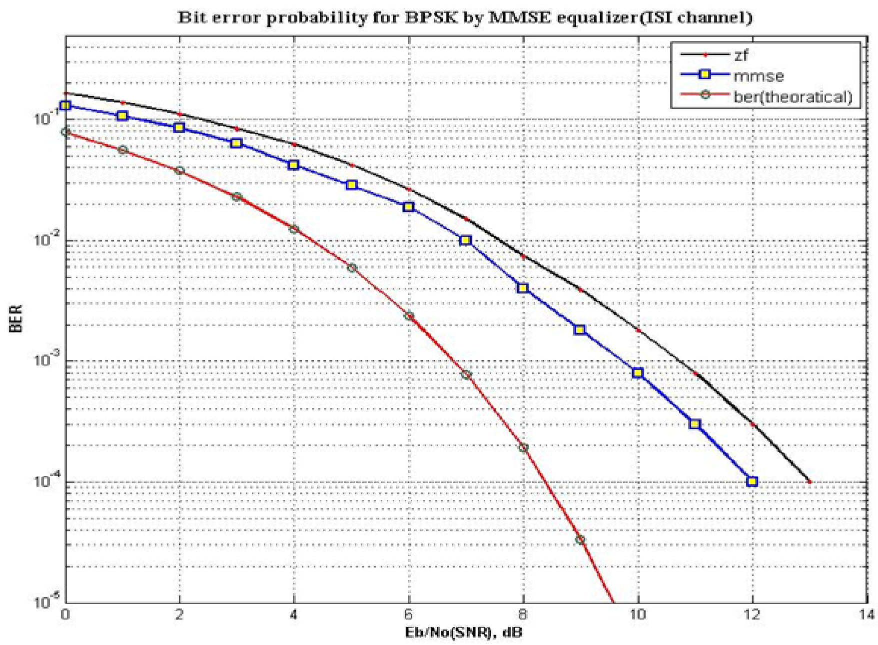
Simulaiton analysis 1 :

This analysis is for Zero frequency equalizer for varying tap length. It's observed from the results that as we increase the tap length of equalizer, the bit error rate decreases. But still there is lot of gap between theoretical and practical ber.



Simulaiton analysis 2 :

This is comparison between zero frequency equalizer and mmse equalizer .From the rsults its observed that with mmse technique bit error rate has decreased by certain amount as that with zf technique for fixed tap length.



V.CONCLUSION

With the aid of Matlab Programming, We came to know that as the tap length increases, the BER decreases with zero forcing equalization for noise free channel. Then further with the help of MMSE equalization technique BER of marginal value (db) is decreased for AWGN channel. Hence we succeeded in improving BER

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