An Iterative Semi-blind Channel Estimation for MIMO Wireless Communication System

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Abstract- This paper presents a novel iterative approach to channel estimation for multiple input multiple output (MIMO) wireless communication system. The initial channel estimate is obtained using whitening rotation based orthogonal pilot maximum likelihood (OPML) semi-blind channel estimation (SBCE) scheme. The estimation is further enhanced using detected symbols iteratively. The performance of this scheme is compared with whitening rotation based OPML SBCE. The simulation study shows that this approach clearly outperforms the OPML SBCE method to the extent of achieving near optimal performance.

Keywords – Least Square, Multiple Input Multiple output, Singular Value Decomposition, Semi-blind Channel Estimation, Space Time Block Code, Training Based Channel Estimation.

I. Introduction

The demand for high data rate multimedia systems in wireless scenario is a major driving force for research in the area of MIMO wireless communication system. It was shown that, the use of multiple antennas at transmitter and or receiver can improve the capacity [1] - [3]. The simple transmit diversity scheme suggested by Alamouti [4] and the space time coding suggested by V. Tarokh et al.[5] triggered research in this area. Several transmission schemes have been proposed that utilize the MIMO channel in different ways. These schemes can be categorized as spatial multiplexing, spatial diversity (space-time coding) and smart antennas & beamforming techniques [6].

The system's ability to achieve MIMO capacity depends on channel state information. Accurately estimating MIMO channel is much more challenging than SISO channel [7]. There are number of channel estimation schemes suggested in literature. These schemes can be categorized as Training based (TBCE), Blind (BCE) and Semi-Blind (SBCE). Training based schemes are capable of accurately estimating a MIMO channel, provided a large training overhead is made available. Hence there is considerable reduction in system throughput. The least-square (LS) and minimum-mean-square-error (MMSE) techniques are widely used for channel estimation when training symbols are available. The LS method is simpler than MMSE but performance of MMSE scheme is better. MMSE method however, requires knowledge of channel correlation. Blind methods do not require the training overhead. However these methods not only impose high complexity and slow convergence, but also suffer from unavoidable estimation and decision ambiguities. Semi-blind methods offer attractive practical means of implementing MIMO systems. Semi-blind channel estimation schemes, use a few training symbols to provide the initial MIMO channel estimation and make use of blind information to further improve the estimation. Some SBCE schemes also exchange the information between the channel estimator and the data detector iteratively.

Several SBCE solutions have been proposed to minimize the computational cost, and hence the energy spent in channel estimation of MIMO systems. The SBCE schemes suggested in [8], [9] use few training symbols to provide initial estimate and then the data detector and estimator exchange the information iteratively. In [10], [11] and [12] the MIMO channel matrix is decomposed into whitening and rotation matrix. The whitening matrix is estimated using blind symbols and the rotation matrix is estimated using few orthogonal pilot symbols. This Orthogonal Pilot Maximum Likelihood (OPML) estimator shows a 1dB improvement of bit error rate (BER) compared to the conventional least squares (LS) training scheme if the same length of training sequence is used. [12]. A signal perturbation free whitening rotation based semi blind channel estimation is discussed in [14]. In [15] TBCE and SBCE, considering perfect, LS, LMMSE, ML, and MAP estimators are studied in terms of BER and complexity. Subspace based semi-blind channel estimation is discussed in [18], [17]. A linear prediction based semi-blind estimation for FIR MIMO channel is proposed in [18]. Number of semi-blind channel estimation schemes are reported for orthogonal frequency division multiplexing (OFDM) and MIMO-OFDM systems as well [19]-[26].

This paper proposes a new approach to semi-blind channel estimation in which the whitening rotation based schemes of [10], [11] and [12] can be used to obtain initial estimate using few training symbols. The channel estimate can be further enhanced by using the detected symbols iteratively. The detected symbols are used as training symbols in the SBCE. The performance of new scheme is compared with the whitening rotation based SBCE of [10], [11] and [12]. The performance is measured in terms of BER. The new scheme is found to outperform the OPML SBCE scheme and performance equivalent to perfect channel estimate can be achieved with few iterations. This of course comes at the cost of increased time complexity.

The rest of this paper is organized as follows. Section 2 describes the system model. The design of the proposed estimator is given in section 3. Section 4 has the simulation results and discussion on the results. Finally section 5 has the conclusion.

Throughout our discussions we adopt the following notational conventions. Boldface capitals and lower-case letters stand for matrices and vectors, respectively. $\boldsymbol{I}$ denote the identity matrix. $(.)^H$ and $(.)^\dagger$ are conjugate transpose and Moore-Penrose pseudo inverse operators respectively, while $\|.\|_F^2$ and $|.|$ denote the Frobenius norm and magnitude operators respectively. Finally E[.] is the expectation operator.

II. SYSTEM MODEL

Consider a MIMO system with M transmit and N receive antennas. Let us assume the channel to be flat fading with channel matrix $\boldsymbol{H} \in \mathbb{C}^{N \times M}$.Each element h_{ij} in the matrix, represents the flat-fading channel coefficient between the ith receive and jth transmit antenna. Denoting the complex received data by $\boldsymbol{Y} \in \mathbb{C}^{N \times 1}$the equivalent base-band system can be modeled as [10],

$$\boldsymbol{Y}(k) = \boldsymbol{H}\boldsymbol{X}(k) + \boldsymbol{n}(k) \tag{1}$$

k is a time instant of transmission of symbol vector $\boldsymbol{X} \in \mathbb{C}^{M \times 1}$ and $\boldsymbol{n}$ is the additive white Gaussian noise with zero mean and noise power σ_n^2. Also, the sources are assumed to be spatially and temporally independent with identical source power σ_s^2.

Assume that the channel remains constant for K symbol periods. In these K symbols, the first L symbols are used for training. Let these training symbol vectors be $\boldsymbol{X}_p = [\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3 \dots, \boldsymbol{x}_L]$ where $\boldsymbol{X}_p \in \mathbb{C}^{M \times L}$ and its corresponding output $\boldsymbol{Y}_p \in \mathbb{C}^{N \times L}$ is the received training symbol output vectors. The remaining $K-L$ symbol vectors are blind data symbols $\boldsymbol{X}_b$ and their corresponding output $\boldsymbol{Y}_b$ where $\boldsymbol{X}_b \in \mathbb{C}^{M \times K-L}$and $\boldsymbol{Y}_b \in \mathbb{C}^{N \times K-L}$.

III. ESTIMATOR DESIGN

The channel matrix $\boldsymbol{H}$ can be estimated using only training symbols. The LS estimation of $\boldsymbol{H}$ is given by (2),

$$\hat{\boldsymbol{H}}_{ls} = \boldsymbol{Y}_p \boldsymbol{X}_p^\dagger \tag{2}$$

Where $\boldsymbol{X}_p^\dagger$ denotes Moore-Penrose pseudo-inverse of $\boldsymbol{X}_p$.

Training based channel estimation techniques are easy to implement, but because of the redundant training information, spectral efficiency suffers. Efficient usage of bandwidth can be achieved through blind methods but they have poor convergence rate and require long data records to be processed. Hence techniques which use a small amount of training symbols with blind data can provide attractive solution for channel estimation. One such technique called Orthogonal Pilot Maximum Likelihood Semi-blind Channel Estimation (OPML SBCE) is discussed in [10], [11] and [12].

Consider a MIMO channel with number of transmit antenna (M) is less than or equal to number of receive antenna (N) i.e. $N \geq M$. Then MIMO channel matrix can be decomposed as,

$$\boldsymbol{H} = \boldsymbol{W}\boldsymbol{Q}^H \tag{3}$$

Where the whitening matrix, $\boldsymbol{W} \in \mathbb{C}^{N \times M}$and rotation matrix, $\boldsymbol{Q} \in \mathbb{C}^{M \times M}$. If we take SVD of channel matrix then, $\boldsymbol{H}$ can be written as,

$$\boldsymbol{H} = \boldsymbol{P}\boldsymbol{\Sigma}\boldsymbol{V}^H \tag{4}$$

Then, $\boldsymbol{W} = \boldsymbol{P}\boldsymbol{\Sigma}$ and $\boldsymbol{Q} = \boldsymbol{V}^H$. Hence $\boldsymbol{H}\boldsymbol{H}^H = \boldsymbol{W}\boldsymbol{W}^H$. The whitening matrix, $\boldsymbol{W}$ can be estimated using blind data and $\boldsymbol{Q}$ can be estimated using training symbols. The output correlation matrix $\boldsymbol{R}_{yy}$is given as,

$$R_{yy} = E[Y_b Y_b^H] \approx (HX_b)(HX_b)^H + \sigma_n^2 I \quad (5)$$

This can be further simplified as,

$$HH^H = WW^H = \frac{1}{\sigma_s^2}(R_{yy} - \sigma_n^2 I) \quad (6)$$

Assuming σ_n^2 and σ_s^2 is known [10], then from Singular Value Decomposition (SVD) of $\frac{1}{\sigma_s^2}(R_{yy} - \sigma_n^2 I)$ we can write,

$$\frac{1}{\sigma_s^2}(R_{yy} - \sigma_n^2 I) = U\Sigma^2 U^H. \quad (7)$$

Hence from (4), (6) and (7) we can write,

$$W = U\Sigma \quad (8)$$

Thus W is estimated blindly. The unitary matrix Q is estimated using orthogonal pilots. Constrained ML estimator of Q is obtained by minimizing the likelihood function,

$$\left\| Y_p - WQ^H X_p \right\|_F^2 \text{ such that } QQ^H = I \quad (9)$$

It is shown in [11] that estimate of Q is obtained as,

$$\hat{Q} = V_m U_m^H \text{ where } U_m \Sigma_m V_m^H = SVD(W^H Y_p X_p^H) \quad (10)$$

Using the estimate obtained in (3), The blind data is detected. This detected data can be used again in (10) as training symbols X_p to obtain new $\hat{Q}$. This procedure is performed iteratively as depicted in algorithm 1.

Algorithm 1: Iterative Semi-blind Channel Estimation
ITRTV_SBCE ($X_p, Y_p, Y_b, \sigma_n^2, \sigma_s^2$)

1. Compute $R_{yy} = E[Y_b Y_b^H]$
2. Compute $SVD\left(\frac{1}{\sigma_s^2}(R_{yy} - \sigma_n^2 I)\right)$ to obtain U and Σ.
3. Compute $W = U\Sigma$
4. Compute $SVD(W^H Y_p X_p^H)$ to obtain U_m and V_m.
5. Compute $\hat{Q} = V_m U_m^H$.
6. Compute Estimate $\hat{H} = W\hat{Q}^H$
7. Detect X_b using Y_b and $\hat{H}$
8. Make $X_p = X_b$ and $Y_p = Y_b$
9. Repeat steps 4 to 8 for specified number of iterations.

In this new approach, very few training symbols are used to obtain initial estimate of the channel matrix using steps 1 to 6 in the algorithm. Using this channel estimate the blind symbols X_b are detected from Y_b. Using these detected symbols as pilots, U_m and V_m are obtained (step 4 & 5) to get a better estimate of $\hat{Q}$ (step 6) which in turn is used to enhance the estimate $\hat{H}$. This channel estimate can give more correct detection of blind symbols which can act as more reliable pilots. This procedure can be repeated for a fixed number of times or till we get desired accuracy of the channel estimation. Note that the iterations involve computation of SVD of a small size matrix *(MXM)* as against Moore-Penrose pseudo-inverse required in TBCE.

IV. Results and Discussions

Extensive simulations are carried out in MATLAB 7.9 to test the performance of the proposed channel estimators and they are compared with the SVD-OPML estimators of [11] and [12]. The simulation is carried out for 2X4 and 4X8 MIMO systems under flat fading channel. MIMO system with Space Time Block Code (STBC) is used for this study. M-ary Quadrature Amplitude Modulation (M-QAM) or M-ary Phase Shift Keying (M-PSK) modulation can be used. Performance of these systems under the assumption of channel knowledge is discussed by the authors of this paper in [27] [28]. The performance is measured in terms of Bit Error Rate (BER). STBC symbols are transmitted on multiple antennas. For 2 transmitting antennas, G2 (Alamouti) code is used while for 4 transmitting antennas, G4 code is used [30]-[32].

A. 2X4 MIMO System

The BER performance of 2X4 MIMO system using Alamouti coding [4] is plotted. In simulation scenarios, 16-QAM data modulation is used with flat fading Rayleigh MIMO channel. Each transmitted frame consists of 4 orthogonal pilots (L) and 100 blind symbols ($K - L$) over which the channel conditions are assumed to remain constant. The BER is calculated by averaging over 100 rounds of simulation with perfect CE, TBCE, OPML SBCE and ITRTV SBCE for different values of Eb/No. Figure 1 shows the performance of ITRTV SBCE with two iterations whereas figure 2 shows the performance of ITRTV SBCE with three iterations.

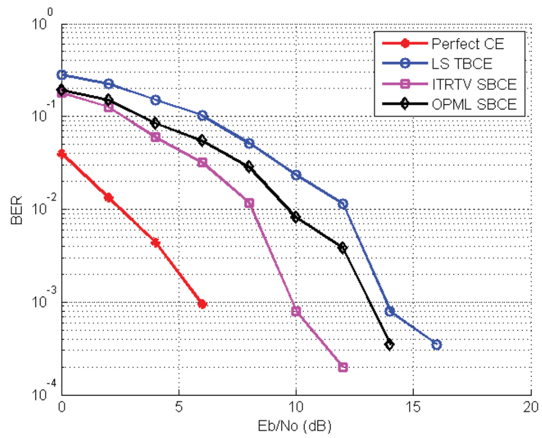


Figure 1. BER Performance of 2X4 MIMO system using Alamouti coding under Perfect CE, LS TBCE and OPML SBCE and ITRTV SBCE. Modulation scheme is 16 QAM and frame length=100. ITRTV scheme uses 2 iterations

The results obtained in plots of figure 1 and 2 indicate that the ITRTV SBCE scheme performs better than TBCE and OPML SBCE schemes. With three iterations, the performance is near optimal i.e. almost comparable with the perfect channel state information. The new scheme has an advantage of 1 to 1.5 dB over OPML SBCE. With two iterations, the advantage is 0.5 dB for low SNR and about 1 dB for high SNR values.

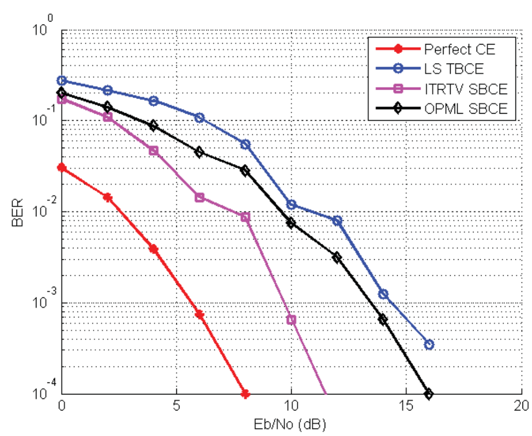
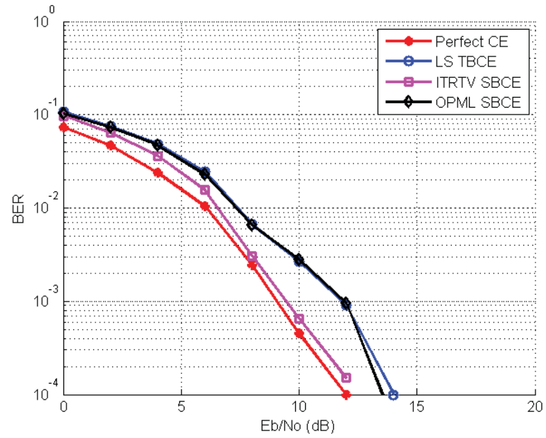


Figure 2. BER Performance of 2X4 MIMO system using Alamouti coding under Perfect CE, LS TBCE and OPML SBCE and ITRTV SBCE. Modulation scheme is 16 QAM and frame length=100. ITRTV scheme uses 3 iterations

The BER performance of these schemes is also studied with a different modulation scheme. Figure 3 and 4 show the performance of the schemes with 16 PSK modulation.

Figure 3. BER Performance of 2X4 MIMO system using Alamouti coding under Perfect CE, LS TBCE and OPML



SBCE and ITRTV SBCE. Modulation scheme is16 PSK and frame length=100. ITRTV scheme uses 2 iterations

The results obtained in plots of figure 3 and 4 indicate that the ITRTV SBCE scheme performs better than TBCE and OPML SBCE schemes for 16 PSK modulation as well. It is also seen that the plots of TBCE and OPML SBCE are almost overlapping i.e. OPML SBCE scheme under MPSK modulation does not offer any performance advantage. But the iterative SBCE scheme can enhance the channel estimate which results into better performance. With three iterations, the performance is near optimal i.e. almost comparable with the perfect channel state information. The new scheme has an advantage of 1 to 2 dB over OPML SBCE. With two iterations, the advantage is less than 0.5 dB for low SNR and about 1 to 2 dB for high SNR values.

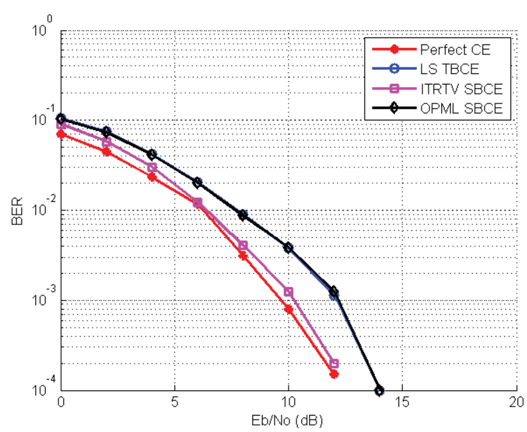


Figure 4. BER Performance of 2X4 MIMO system using Alamouti coding under Perfect CE, LS TBCE and OPML SBCE and ITRTV SBCE. Modulation scheme is 16 PSK and frame length=100. ITRTV scheme uses 3 iterations

B. 4X8 MIMO System

The BER performance of 4X8 MIMO system using G4 STBC coding is also studied. In simulation scenarios, 64-QAM data modulation is used with flat fading Rayleigh MIMO channel. Each transmitted frame consists of 8 orthogonal pilots (L) and 100 blind symbols ($K - L$) over which the channel conditions are assumed to remain constant. The BER is calculated by averaging over 100 rounds of simulation with Perfect CE, LS TBCE, OPML SBCE and ITRTV SBCE for different values of Eb/No. Figure 5 shows the performance of ITRTV SBCE with two iterations whereas figure 6 shows the performance of ITRTV SBCE with three iterations.

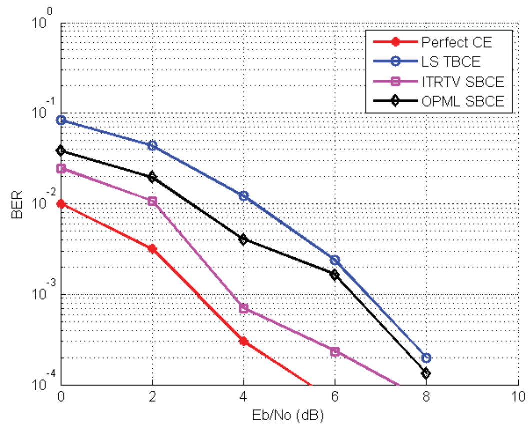


Figure 5. BER Performance of 4X8 MIMO system using G4 STBC coding under Perfect CE, LS TBCE and OPML SBCE and ITRTV SBCE. Modulation scheme is 64 QAM and frame length=100. ITRTV scheme uses 2 iterations

The results obtained in plots of figure 5 and 6 indicate that the ITRTV SBCE scheme performs better than TBCE and OPML SBCE schemes. Since higher modulation scheme is used, more number of iterations will be required to achieve the performance comparable to that of perfect channel

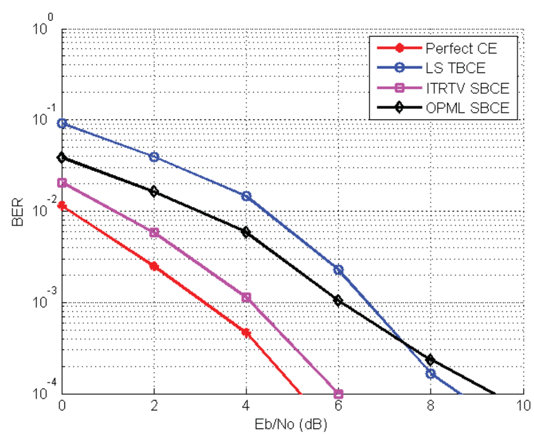


Figure 6. BER Performance of 4X8 MIMO system using G4 STBC coding under Perfect CE, LS TBCE and OPML SBCE and ITRTV SBCE. Modulation scheme is 64 QAM and frame length=100. ITRTV scheme uses 3 iterations

The algorithm was also studied with 64 PSK modulation scheme. Figure 7 and 8 depict the BER performance of the four schemes under consideration for 4X8 MIMO system with 64 PSK modulation. The results obtained in plots of figure 7 and 8 indicate that the ITRTV SBCE scheme performs better than TBCE and OPML SBCE schemes for 16 PSK modulation as well. It can also be seen that the proposed ITRTV SBCE method performs better when MPSK modulation is employed. The method exhibits BER almost equivalent to Perfect CE in second iteration itself. Thus the proposed method converges to the optimum performance faster for MPSK modulation than MQAM modulation.

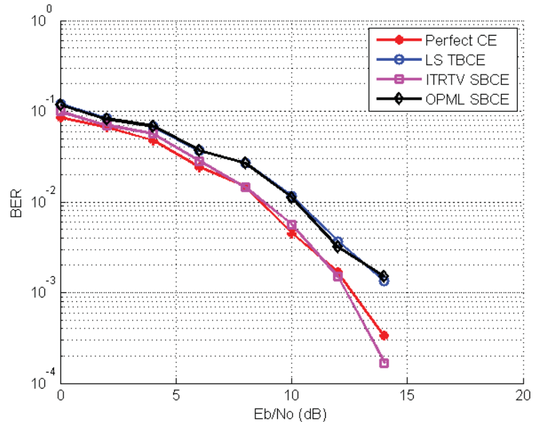


Figure 7. BER Performance of 4X8 MIMO system using G4 STBC coding under Perfect CE, LS TBCE and OPML SBCE and ITRTV SBCE. Modulation scheme is 64 PSK and frame length=100. ITRTV scheme uses 2 iterations

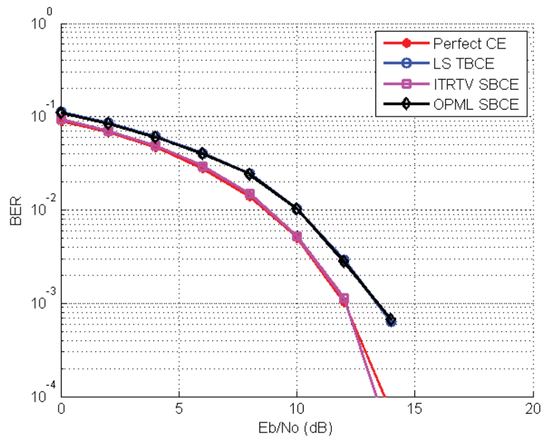


Figure 8. BER Performance of 4X8 MIMO system using G4 STBC coding under Perfect CE, LS TBCE and OPML SBCE and ITRTV SBCE. Modulation scheme is 64 PSK and frame length=100. ITRTV scheme uses 3 iterations

V. Conclusions

ITRTV SBCE scheme for channel estimation in MIMO wireless communication system is considered and studied under different modulation schemes. The proposed scheme makes use of symbols detected with the OPML SBCE to enhance the estimate of the channel further. This scheme performs better than the OPML SBCE. The proposed scheme is found to converge to optimum performance faster for MPSK modulation compared to MQAM modulation. The accuracy of the estimate improves with more number of iterations. This advantage comes at the cost of more computational burden. Future work in this area can be done on reducing this computational cost and these results can be extended to MIMO OFDM systems.

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